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Abstract

This paper examines the evolutionary stability of behaviour in contests where players' participation can be stochastic. We find, for exogenously given participation probabilities, players exert more effort under the concept of a finite-population evolutionarily stable strategy (FPESS) than under Nash equilibrium (NE). We show that there is ex-ante overdissipation under FPESS for sufficiently large participation probabilities, if, and only if, the impact function is convex. With costly endogenous entry, players enter the contest with a higher probability and exert more effort under FPESS than under NE. Importantly, under endogenous entry, overdissipation can occur for all (Tullock) contest success functions, in particular those with concave impact functions.

JEL Classification: C73; D72.

Keywords: Contests with Stochastic Participation; Overdissipation; Evolutionarily Stable Strategy; Finite Population; Endogenous Entry

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1 Introduction

The issue of dissipation in contests was first raised in the context of political rent-seeking, where lobbying efforts are considered as an unproductive use of resources to contest something valuable (Tullock, 1967; Krueger, 1974). In line with this literature, Posner (1975) posed his famous full dissipation hypothesis according to which competition for a monopoly creates an additional social cost that eats up the entire monopoly rent at stake.

Perhaps more surprisingly, *over*dissipation is not uncommon. For example, overdissipation has featured prominently in political sciences and in experimental economics. Deacon and Rode (2015) discuss the resource curse and point out that "political theories of the resource curse consistently predict over-dissipation [...]." Morgan et al. (2012) review earlier lab experiment evidence of overdissipation (e.g., Millner and Pratt, 1989; Anderson and Stafford, 2003; Abbink et al., 2010, among many others) and also find frequent occurrences of overdissipation in their own experiment, particularly in their small prize treatment.[1] Sheremeta (2013) and Dechenaux et al. (2015) systematically survey the experimental literature on contests. They address the issue of overbidding (individual effort being above the Nash equilibrium prediction) and discuss a number of behavioural explanations.[2]

Theoretically, ex-post overdissipation in Tullock contests has been explained as the incidental outcome of mixed strategy Nash equilibria (Baye et al., 1999), which occur when the impact function is sufficiently convex.[3] In contrast, *ex-ante* dissipation can never be more than full in Nash equilibrium. Under the finite-population evolutionary approach, matters are different. Here, relative performance determines survival. By adopting the indirect

[1]Overdissipation is also well documented in experimental studies of closely related all-pay auctions. See, e.g, Davis and Reilly (1998), Gneezy and Smorodinsky (2006), and Lugovskyy et al. (2010).

[2]Sheremeta (2016) designs an eight-part experiment to test various theoretical explanations of overbidding in rent-seeking contests. He finds significant support for the existing theories, while at the same time suggests impulsivity being the most important factor explaining overbidding in his study. Mago et al. (2016) present a behavioural model that incorporates a nonmonetary utility of winning and relative payoff maximisation. It can explain significant overspending of effort in their controlled laboratory experiment. On the other hand, Chowdhury et al. (2014) find that a deterministic proportional sharing rule results in average effort closer to the Nash prediction than the random lottery rule.

[3]Following Wärneryd (2001), we call the numerator of the contest success function the *impact function*.

evolutionary approach to deterministic Tullock contests, Leininger (2009) provides an endogenous explanation of spiteful preferences, viz. a rationale for relative payoff concerns.

In many real world contests, participants do not know how many other competitors they are facing in the contest. For example, in a job interview, an applicant may not know the number of short-listed interviewees. A lobbyist may not know against how many others he or she is lobbying. Animals competing for food or mating opportunities may not be able to perceive the actual number of competitors or to tune their effort level accordingly. Moreover, entry into such a contest can be endogenous. Thus, not knowing how many other active competitors there are can be an equilibrium feature of a contest.

Adopting an evolutionary approach to stochastic contests is of principal interest. First, the evolutionary approach represents a way to examine whether weaker assumptions on the rationality of players still allow Nash equilibrium to occur.[4] Evolutionary equilibrium, as deployed in this paper, rests on the concept of an evolutionarily stable strategy, ESS. In its original version, such as introduced by Smith and Price (1973), evolutionary equilibrium underpins Nash equilibrium in that any ESS represents a (symmetric) Nash equilibrium. Implicitly, however, Maynard Smith and Price's concept of an ESS assumes an infinite population of players. In finite populations, a staple in the rent-seeking literature, the refinement property does not hold generally.[5] It certainly does not hold in deterministic rent-seeking contests (Hehenkamp et al., 2004; Leininger, 2003).

Second, the evolutionary approach incorporates a competitive element on the selection of behaviour that is present in many real world contests, but absent in Nash equilibrium generally. This selection of behaviour can operate directly at the level of actions or indirectly through the selection of contestants that employ certain actions.

Third, the evolutionary approach in finite populations turns out to be equiv-

[4]The idea of applying evolutionary concepts to economic theory goes at least back to Alchian (1950), who suggested an economic evolutionary approach. This approach interprets "the economic system as an adoptive mechanism," and determines actions and behaviour based on their relative success or profit.

[5]See Ania (2008) and Hehenkamp et al. (2010) who characterise the classes of games where Nash and evolutionary equilibrium coincide when populations are finite.

alent to the rationalistic approach under relative payoff maximisation (Schaffer, 1988, 1989). Hence, our analysis also sheds light on the important case of relative payoff concerns, which both have empirical relevance and find theoretical support through the indirect evolutionary approach.[6]

Last but not least, with regard to relevant applications such as political rent-seeking, Leininger (2003) demonstrates that, in finite populations, ESS behaviour in a contest is identical to *rational* behaviour in a symmetric Nash equilibrium of a corresponding transfer contest where losers have to provide the source of the gain for the winner. Many political rent-seeking contests are of such a nature. A prominent example is welfare policies where the contested prize is financed by "taxing" the losers. Due to this intuitive interpretation of evolutionarily stable behaviour in contests, the present paper sheds light on the issue of overdissipation in rent-seeking.

1.1 An overview of the main results

We consider two scenarios. First, we study stochastic contests with exogenous participation, where players participate in a Tullock contest with an exogenously given probability. Subsequently, we endogenise participation by incorporating the probability of participation in the evolutionary approach.

As to the case of exogenous participation, we first characterise the finite-population evolutionarily stable strategy (FPESS) and then show that individual effort in the FPESS is exactly $n/n-1$ times its Nash equilibrium (NE) counterpart in a contest with n potential contestants.[7] Intuitively, the ex-ante total expected effort increases in the number of players, and in the participation probability. While Lim and Matros (2009) demonstrate that, under NE, overdissipation is possible ex-post, we show that under the economic

[6]Seminal contributions on relative payoff concerns are e.g., Messick and Thorngate (1967), Bolton and Ockenfels (2000), and Fehr and Schmidt (1999) among many others. Herrmann and Orzen (2008) report evidence that can attribute subjects' investment decisions to spiteful preferences rather than fairness or reciprocity. The indirect evolutionary approach (Güth and Yaari, 1992) has been adopted to rent-seeking contests by Leininger (2009). Relatedly, Konrad and Morath (2016) employ the evolutionary approach in finite populations to study determinants of war.

[7]For the Nash equilibrium outcome, we draw on results by Lim and Matros (2009), who characterise and extensively study the Nash equilibrium of stochastic contests with exogenous participation.

evolutionary approach, the FPESS entails ex-ante as well as ex-post overdissipation when the probability of participation is sufficiently large and the impact function is convex. Thus, the economic evolutionary approach can explain ex-ante overdissipation.

When participation is endogenous, a player must incur a fixed cost in order to participate in the contest. We find that the FPESS participation probability is at least as high as the NE participation probability. In particular, for a given entry cost, the FPESS participation probability strictly exceeds the NE participation probability whenever the endogenous entry is truly stochastic in NE. This is in accordance with Morgan et al. (2012) who find statistically significant excessive entry compared to the Nash prediction in their experimental study. Otherwise, endogenous entry is deterministic under both concepts. Moreover, ex-ante both total effort and total entry cost are higher under FPESS than under NE. In this sense, players behave more aggressively under FPESS than under NE along both dimensions, entry and effort.

With endogenous participation, dissipation is measured by the *total expenditure*, which includes effort cost and entry cost of all players. As demonstrated by Fu et al. (2015), under NE, players are indifferent between entering the contest or not, when entry is truly stochastic. Therefore, the NE ex-ante total expenditure is at most the size of the prize. Under FPESS, however, there can be ex-ante overdissipation for all (Tullock) contest success functions due to the increased levels of entry and effort. This represents a striking result as to the best of our knowledge ex-ante overdissipation has only been shown for convex impact functions (see, e.g., Hehenkamp et al., 2004). In the present paper, ex-ante overdissipation can occur for convex as well as concave impact functions. Thus, overdissipation could be the norm rather than an anomaly once we take entry cost into account. This has important policy impactions for contests in R&D, job applications, etc.

1.2 Related literature

Our paper is most closely related to Lim and Matros (2009), Fu et al. (2015), and Hehenkamp et al. (2004).[8] Lim and Matros (2009) (henceforth LM)

[8] It is beyond the scope of the current paper to review the voluminous contest literature. For an excellent in-depth treatment see, e.g., Konrad (2009).

characterise the Nash equilibrium of stochastic contests with exogenous participation probabilities. They show, among other results, that individual effort is single-peaked in the participation probability while total effort is monotonically increasing in the participation probability and the number of players. Our contribution in the exogenous entry part is to establish a precise relationship between the NE effort level and the corresponding FPESS level in stochastic contests. Moreover, we show that overdissipation is possible ex-ante, and identify the range of parameters supporting overdissipation.

Fu et al. (2015) (henceforth FJL) study contests with endogenous entry under the Nash equilibrium solution concept. The authors establish the existence of a symmetric Bayesian Nash equilibrium and show that a Tullock contest can be optimal for a contest designer. They further identify the conditions under which the optimum can be achieved by solely setting the right discriminatory power of the contest success function.[9] In contrast, we establish the existence of an evolutionarily stable pair of entry probability and effort level. Moreover, we compare and contrast the FPESS outcome with the NE outcome, and demonstrate a general overdissipation result.

The ESS concept in a finite population was applied to deterministic Tullock contests by Hehenkamp et al. (2004) (henceforth HLP). They show overdissipation can be expected under FPESS in deterministic contests with convex impact functions. One of our contributions is to generalise their overdissipation result to stochastic contests. More importantly, we consider endogenous entry and show that overdissipation can also hold for concave impact functions.

Stochastic participation in contests has also been featured in earlier literature such as Myerson and Wärneryd (2006), Münster (2006), and Fu et al. (2011). Myerson and Wärneryd (2006) study contests in which a player is uncertain about the actual size of the contest. They do not assume a particular distribution of the contest size and it can potentially be infinitely large. In Münster (2006), the size of a contest follows a binomial distribution similar to the current paper. However, Münster (2006) compares risk-neutral players with CARA players under Nash equilibrium. Fu et al. (2011) study the optimal disclosure policy of an effort-maximizing contest organizer in a contest with

[9] In a related paper with endogenous entry, Fu and Lu (2010) study the optimal choice of prize size and entry fee/subsidy for an effort-maximizing contest designer.

stochastic participation. The present paper departs from the above literature by exploring a different equilibrium concept.

In the rest of the paper, we proceed as follows. Section 2 introduces the model and presents the analysis for the case of exogenous entry probabilities. Section 3 is devoted to the case of endogenous entry. Finally, Section 4 concludes. Involved proofs are relegated to the Appendix.

2 Stochastic contests with exogenous entry

We consider a stochastic contest of $n \geq 2$ potential players as in LM. Each potential player is drawn to play, i.e., becomes active, with an independent probability $p \in (0, 1]$. All active players compete for a single prize of value $V > 0$ by selecting an effort level $X_i \in [0, +\infty)$.

Conditional on being active, player i's probability of winning the prize is given by

$$P_i(X_i; \boldsymbol{M}) = \begin{cases} \frac{1}{|\boldsymbol{M}|+1} & \text{if } X_i = X_j = 0, \forall j \in \boldsymbol{M}, \\ \frac{X_i^r}{X_i^r + \sum_{j \in \boldsymbol{M}} X_j^r} & \text{otherwise,} \end{cases}$$

where $0 < r \leq {}^{n}/_{n-1}$ measures the convexity of the impact function and is also known as the discriminatory power of the contest success function, $\boldsymbol{M}$ is the set of active players *except* player i, and $|\boldsymbol{M}|$ denotes the cardinality of $\boldsymbol{M}$. The material payoff of an inactive player is 0. As it is known in the literature, this range of r ensures that an equilibrium exists in pure strategies.

Before studying the FPESS outcome, we reproduce the result below, which is due to Lim and Matros (2009).[10]

Theorem 1 (Lim and Matros, 2009)**.** There exists a unique symmetric pure-strategy Nash equilibrium (NE) where each active player's equilibrium ex-

[10] Although Lim and Matros (2009) assume $0 < r \leq 1$, the theorem indeed holds for $0 < r \leq {}^{n}/_{n-1}$.

penditure is

$$X^{\text{Nash}}(r,V,n,p) = rV\left[\sum_{i=1}^{n-1} C_i^{n-1} p^i (1-p)^{n-1-i} \frac{i}{(i+1)^2}\right] \tag{1}$$

with $C_i^{n-1} = \frac{(n-1)!}{i!(n-i-1)!}$ denoting the binomial coefficient.

2.1 Evolutionarily stable strategy with stochastic participation

We now proceed to characterise the finite-population evolutionarily stable strategy, FPESS. For this purpose, we first adapt Schaffer's (1988) evolutionarily stable equilibrium condition to games with stochastic participation. We *note* that, for notational ease, in what follows we often use the simpler abbreviation ESS. It should be understood that the solution concept we use in this paper is finite-population evolutionarily stable strategy, FPESS.

Let us consider a finite population of $n \geq 2$ players each being drawn to play with an independent probability $p \in (0, 1]$. Instead of playing a fixed size contest, an active player in our context may face $k = 0, \ldots, n-1$ active opponents. Thus the expected payoff of an *active mutant* playing $\bar{s} \in S$ when the rest of the population playing s^{ESS}, denoted by $\bar{\pi}$, is the sum of the payoffs when $\bar{s}$ plays against $k = 0, \ldots, n-1$ ESS strategists weighted by the probability of each case. On the other hand, for an *active* ESS strategist, the mutant will be present with probability p and with probability $(1-p)$ this is not the case. In the former case, the ESS strategist faces the mutant and possibly also $k = 0, \ldots, n-2$ other ESS strategists, while in the latter case the ESS strategist encounters only $k = 0, \ldots, n-2$ other ESS strategists. The expected payoff of an ESS strategist, denoted by π^{ESS}, is thus the weighted average of the payoffs in those individual cases. For s^{ESS} to be evolutionarily stable, following the same idea in Smith and Price (1973) and Schaffer (1988), $p \cdot \pi^{ESS} \geq p \cdot \bar{\pi}$ needs to hold for any $\bar{s} \in S$.[11]

We now apply this adapted definition to Tullock contests with stochastic participation. Let X denote the candidate ESS strategy and $\bar{X}$ the mutant

[11]First, we note that this definition differs from Schaffer's (1988) in that the size of contest is not only variable but also stochastic. Second, in this evolutionary framework the players are merely representatives of behaviours. Their knowledge is inconsequential.

strategy. We consider invasions by a single mutant, say w. l. o. g. player 1. When *active*, the expected payoff of player 1 is

$$\Pi_1(\bar{X}, X, \ldots, X) = V\left[\sum_{\boldsymbol{M}\in\mathcal{P}^{\boldsymbol{N}_1}} p^{|\boldsymbol{M}|}(1-p)^{|\boldsymbol{N}_1\setminus\boldsymbol{M}|}\cdot\frac{\bar{X}^r}{\bar{X}^r+|\boldsymbol{M}|X^r}\right]-\bar{X}, \quad (2)$$

where $\boldsymbol{N}_1$ is the set of player 1's possible opponents and $\mathcal{P}^{\boldsymbol{N}_1}$ is the set of all subsets of $\boldsymbol{N}_1$.

On the other hand, the expected payoff of an *active* ESS strategist, say player 2, is

$$\begin{aligned}\Pi_2(\bar{X}, X, \ldots, X) = {} & \underbrace{p\,V\left[\sum_{\boldsymbol{M}\in\mathcal{P}^{\boldsymbol{N}_2}} p^{|\boldsymbol{M}|}(1-p)^{|\boldsymbol{N}_2\setminus\boldsymbol{M}|}\cdot\frac{X^r}{\bar{X}^r+X^r+|\boldsymbol{M}|X^r}\right]}_{\text{The mutant being active}} \\ & +\underbrace{(1-p)\,V\left[\sum_{\boldsymbol{M}\in\mathcal{P}^{\boldsymbol{N}_2}} p^{|\boldsymbol{M}|}(1-p)^{|\boldsymbol{N}_2\setminus\boldsymbol{M}|}\cdot\frac{X^r}{X^r+|\boldsymbol{M}|X^r}\right]}_{\text{The mutant being inactive}}-X, \quad (3)\end{aligned}$$

where $\boldsymbol{N}_2$ is the set of player 2's possible opponents *except* the mutant player 1, and $\mathcal{P}^{\boldsymbol{N}_2}$ is the set of all subsets of $\boldsymbol{N}_2$.

As noted by Schaffer (1988), a strategy X is an ESS if, and only if, the relative payoff between a mutant and an ESS strategist, as a function of $\bar{X}$, reaches its maximum value of zero when $\bar{X} = X$. Let

$$\phi(\bar{X}, X) := \Pi_1(\bar{X}, X, \ldots, X) - \Pi_2(\bar{X}, X, \ldots, X). \quad (4)$$

As $p > 0$, the ESS strategy X should solve

$$\max_{\bar{X}\in[0,+\infty)} \phi(\bar{X}, X). \quad (5)$$

2.2 ESS characterisation

To determine the solution to (5), consider the corresponding first order condition:

$$\frac{\partial \phi}{\partial \bar{X}} = V \left[\sum_{\boldsymbol{M} \in \mathcal{P}^{\boldsymbol{N}_1}} p^{|\boldsymbol{M}|}(1-p)^{|\boldsymbol{N}_1 \setminus \boldsymbol{M}|} \cdot \frac{r\bar{X}^{r-1}|\boldsymbol{M}|X^r}{(\bar{X}^r + |\boldsymbol{M}|X^r)^2} \right] - 1 +$$
$$pV \left[\sum_{\boldsymbol{M} \in \mathcal{P}^{\boldsymbol{N}_2}} p^{|\boldsymbol{M}|}(1-p)^{|\boldsymbol{N}_2 \setminus \boldsymbol{M}|} \cdot \frac{r\bar{X}^{r-1}X^r}{(\bar{X}^r + X^r + |\boldsymbol{M}|X^r)^2} \right] = 0. \quad (6)$$

By symmetry, in equilibrium it should hold that

$$X = rV \left[\sum_{\boldsymbol{M} \in \mathcal{P}^{\boldsymbol{N}_1}} p^{|\boldsymbol{M}|}(1-p)^{|\boldsymbol{N}_1 \setminus \boldsymbol{M}|} \cdot \frac{|\boldsymbol{M}|}{(1 + |\boldsymbol{M}|)^2} \right]$$
$$+ prV \left[\sum_{\boldsymbol{M} \in \mathcal{P}^{\boldsymbol{N}_2}} p^{|\boldsymbol{M}|}(1-p)^{|\boldsymbol{N}_2 \setminus \boldsymbol{M}|} \cdot \frac{1}{(2 + |\boldsymbol{M}|)^2} \right]. \quad (7)$$

After simplifying (7), we obtain the following theorem which characterises and establishes the existence of an ESS strategy. In the proof, we demonstrate the simplification and show that the first order condition indeed solves the maximization problem (5).

Theorem 2. There exists a unique ESS in a Tullock contest with $r \leq {}^{n}/{}_{n-1}$ where each potential player becomes active with probability $p \in (0, 1]$. It is given by

$$X^{\text{ESS}}(r, V, n, p) = \frac{n \cdot rV}{n-1} \left[\sum_{i=1}^{n-1} C_i^{n-1} p^i (1-p)^{n-1-i} \frac{i}{(i+1)^2} \right]. \quad (8)$$

Moreover,

$$X^{\text{ESS}}(r, V, n, p) = \frac{n}{n-1} \cdot X^{\text{Nash}}(r, V, n, p). \quad (9)$$

Proof: See Appendix A.1.

A first noteworthy observation is that Theorem 2 contains the existence result

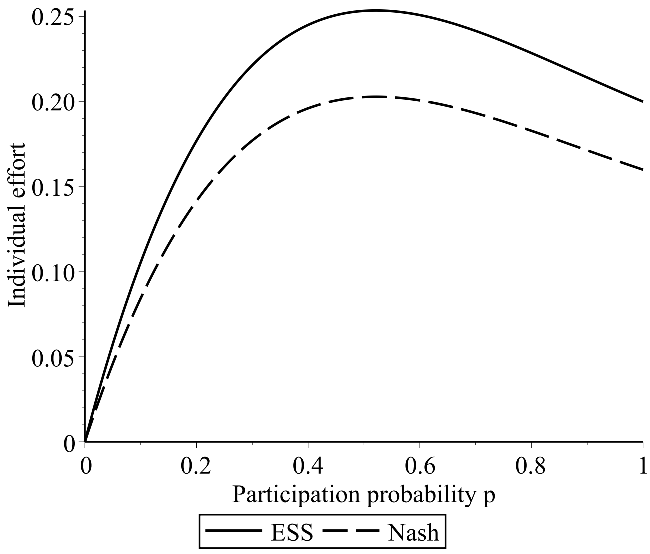


Figure 1: ESS and NE individual effort when $V = r = 1$ and $n = 5$.

of HLP for $p = 1$: the right-hand side of (8) collapses into rV/n, that is, the deterministic case studied by HLP turns to represent the continuous limit of the present model for $p \to 1$.

Furthermore, Theorem 2 establishes a precise relationship between the ESS effort level and the Nash equilibrium (NE) outcome. Namely, the ESS effort level is $n/n-1$ times of the NE effort level. This generalises the main result in HLP who demonstrated $X^{\text{ESS}}(r, V, n, p) = n/n-1 \cdot X^{\text{Nash}}(r, V, n, p)$ for the case of $p = 1$. We show that this precise relationship holds more generally, i.e., for all $p \in (0, 1]$. Thus, the relative aggressiveness of ESS behaviour is not affected by participation uncertainty. Note also when the population size grows, the difference between ESS and NE effort shrinks. Indeed, in accordance with Crawford (1990), ESS converges to the NE level as n goes to infinity.

We also note that, as expected, active players exert more effort when the prize size V is larger and when the discriminative power r of the contest technology is higher. Due to the precise relationship (9), it follows from LM that $X^{\text{ESS}}(r, V, n, p)$ is also single-peaked in p. That is, individual effort in general reaches its maximum at an interior participation rate $p \in (0, 1)$.

Figure 1 draws an example of the equilibrium individual effort as a function of the participation probability under ESS and under NE, respectively. Note that both NE and ESS individual effort levels are single peaked and the ESS effort lies everywhere above the NE counterpart.

2.3 Total effort

The equilibrium level of total effort is a variable of significant interest. In the rent dissipation debate, it is important to weigh equilibrium total effort against the prize value. If the former exceeds the latter, there is overdissipation of economic rent. Therefore, it is instructive to study the ex-ante expected total effort in ESS, denoted by T^{ESS}.

As the number of players follows the binomial distribution $B(n, p)$, the expected value is np. Consequently, we obtain

$$\begin{aligned} T^{\text{ESS}}(r, V, n, p) &:= np \cdot X^{\text{ESS}}(r, V, n, p) \\ =& rV \frac{n}{n-1} \left[\sum_{i=1}^{n} C_i^n p^i (1-p)^{n-i} \left(1 - \frac{1}{i}\right) \right]. \end{aligned}$$

It follows from Theorem 2 that the total ESS effort also corresponds to $n/n-1$ of its NE counterpart denoted by $T^{\text{Nash}}(r, V, n, p)$.

Although the relationship between individual effort and the participation probability p is non-monotonic (see Figure 1), the ex-ante expected *total* effort increases in both n and p.

Theorem 3. Let $0 < r \leq n/n-1$ and $V > 0$ be given. Then,

i) for any $n \geq 2$, the expected total effort increases in p;

ii) for any $p \in (0, 1)$, the expected total effort increases in n.

Proof: See Appendix A.2.

Part i) of Theorem 3 shows that total expected effort increases in players' participation probability. This observation simply follows the precise relationship between ESS and NE outcomes. Intuitively, the positive effect of p

on the expected number of players, np, dominates its potentially negative effect on individual effort.

That total effort increases in n is less obvious. While LM show that T^{Nash} increases in n, the term $n/n-1$ decreases. Nevertheless, as part ii) of Theorem 3 states, ex-ante total effort in ESS indeed increases in the number of potential contestants. In other words, the increase in NE total effort dominates the decrease in players' aggressiveness, which is measured by $n/n-1$.

Interestingly, while in HLP total effort is independent of the number of contestants, n, it is not so in truly stochastic contests, i.e. if $p < 1$. To understand this observation, note that the number of contestants and the participation probability both positively affect the competitiveness of a contest. As shown in HLP, total effort under ESS in deterministic contests is rV for all $n \geq 2$. That is, for $p = 1$, the number of contestants does not strictly increase total effort.[12] In stochastic contests ($p < 1$), however, the competitiveness of a contest does strictly increase with n, and consequently total effort also increases in n.

2.4 Overdissipation

From the players' perspective, an important question is whether they gain ex ante from playing the contest, that is, whether the expected revenue from winning the prize is more or less than the expected cost of effort. In a Nash equilibrium, ex ante no player will incur effort costs that exceed the expected revenue from winning the contest. The reason is that by exerting zero effort the player can always break even. This is however not true when ESS is considered. As shown for deterministic contests by HLP, ex-ante overdissipation can be an equilibrium outcome under ESS. The next result generalises their result to stochastic contests.

Theorem 4. $T^{\text{ESS}} > V$ for p sufficiently large, if, and only if, $r \in (1, n/n-1)$.

Proof: See Appendix A.3.

[12] This doses not apply to total NE effort in deterministic contests, which is $rV(n-1)/n$ and therefore, increases in n.

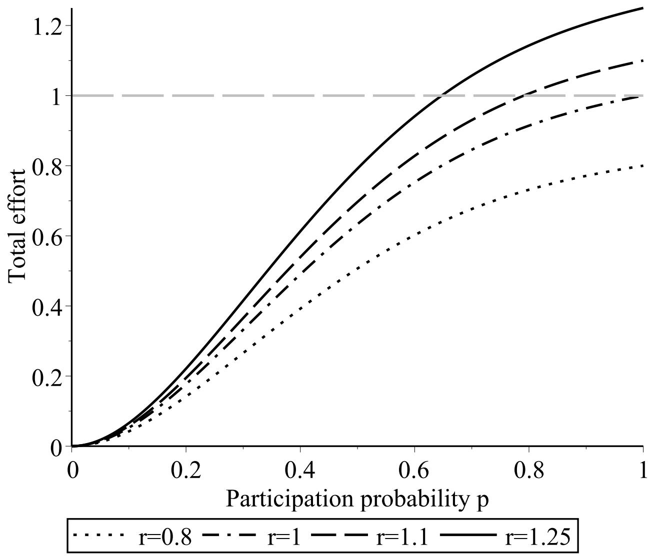


Figure 2: ESS total effort under discriminatory power when $V = 1$ and $n = 5$.

Theorem 4 shows that ex-ante overdissipation is present if, and only if, the impact function is convex *and* the probability of participation, p, is sufficiently high. Like in the deterministic case studied by HLP, overdissipation can be explained by spiteful behaviour in the presence of increasing returns to expenditures ($r > 1$). However, because total effort drops when the probability of participation decreases (Theorem 3), participation has to be sufficiently likely to entail overdissipation. Moreover, due to the monotonicity of total effort in both participation probability (p) and the discriminatory power of the contest technology (r), there is a substitution relationship between p and r such that the higher $r \in (1, n/n-1]$, the larger the overdissipation interval of participation probabilities.

Consider an example where $V = 1$ and $n = 5$. Figure 2 displays the ESS total effort for different values of the contest success function's discriminatory power, namely $r = 0.8$, $r = 1$, $r = 1.1$, and $r = 1.25$. To facilitate comparison, it also shows the full dissipation line. As can be seen in the figure, ex-ante overdissipation occurs when the impact function is convex, i.e., $r > 1$, *and* when the participation probability is sufficiently large. For $r \leq 1$, ex-ante total effort never exceeds the prize value for all p. For $r > 1$, the overdissipation interval increases with r.

3 Stochastic contests with endogenous entry

In this section we endogenise the probability of participation. Let there be $n \geq 2$ potential contestants. Following FJL, a strategy of player i is an ordered pair (p_i, X_i), where $p_i \in [0,1]$ denotes player i's entry probability and $X_i \in [0, +\infty)$ the effort level.[13] The material payoffs are the same as before except that upon entering the contest a player must pay a fixed cost of entry $0 < c < V$.

3.1 (Bayesian) Nash equilibrium

For convenience, we begin with restating results on the symmetric Nash equilibrium outcome which is due to FJL.

Theorem 5 (Fu, Jiao, and Lu, 2015)**.** There exists a unique symmetric equilibrium with pure-strategy bidding of the entry-bidding game.

i) If $c \leq \frac{n-(n-1)r}{n^2} V$ then the entry probability is $p^{\text{Nash}} = 1$.

ii) Otherwise, it is implicitly determined by

$$\frac{1 - \left(1 - p^{\text{Nash}}\right)^n}{np^{\text{Nash}}} V - X^{\text{Nash}} \left(r, V, n, p^{\text{Nash}}\right) = c, \qquad (10)$$

where individual effort level $X^{\text{Nash}}(r, V, n, p)$ is given by (1).

Proof: See Appendix A.4.

Intuitively, when the entry cost is below the expected payoff from entering the contest, it pays to enter, and hence the entry probability is one. However, when the entry cost becomes high enough such that all players entering cannot be sustained, equilibrium competitiveness of the contest has to decrease. This is achieved by a reduced entry probability to keep it worthwhile for players to play the contest. Moreover, the equilibrium entry probability

[13] For brevity, we do not consider mixed effort levels. According to FJL, the existence of pure strategy Nash equilibrium effort is ensured given the parameter values in the present paper.

cannot be too low as this will lead to expected payoff from entering being higher than the entry cost. Thus, in Nash equilibrium with truly stochastic entry all players should be indifferent between entering and abstaining, and the required symmetric entry probability is implicitly determined by the indifference condition (10). This intuition similarly applies to the ESS case, except that players would be concerned of relative payoff rather than absolute payoff.

3.2 ESS outcome

To characterise the ESS outcome, consider now a mutant, say player 1, which enters with probability q and spends $\bar{X}$. Conditional on entering, the mutant's expected payoff from the contest is given by (2) as before. The expected payoff of an ESS strategist conditional on entering, say player 2, now depends on the mutant's entry probability q:

$$\begin{aligned}
&\Pi_2(\bar{X}, X, \dots, X; q) \\
=& \underbrace{q\, V \left[\sum_{\boldsymbol{M} \in \mathcal{P}^{\boldsymbol{N}_2}} p^{|\boldsymbol{M}|}(1-p)^{|\boldsymbol{N}_2 \setminus \boldsymbol{M}|} \cdot \frac{X^r}{\bar{X}^r + X^r + |\boldsymbol{M}| X^r} \right]}_{\text{The mutant being active}} \\
&+ \underbrace{(1-q)\, V \left[\sum_{\boldsymbol{M} \in \mathcal{P}^{\boldsymbol{N}_2}} p^{|\boldsymbol{M}|}(1-p)^{|\boldsymbol{N}_2 \setminus \boldsymbol{M}|} \cdot \frac{X^r}{X^r + |\boldsymbol{M}| X^r} \right]}_{\text{The mutant being inactive}} - X.
\end{aligned}$$

Let the ex-ante relative payoff of the mutant be denoted by Φ. After taking into account entry probabilities and entry cost, we have

$$\Phi(q, \bar{X}; p, X; c) = q \left[\Pi_1(\bar{X}, X, \dots, X) - c \right] - p \left[\Pi_2(\bar{X}, X, \dots, X; q) - c \right]. \tag{11}$$

For a pair (p, X) to constitute an evolutionarily stable strategy, no mutant can invade, i.e., obtain a higher material payoff than ESS strategists. Thus, ESS requires $\Phi(q, \bar{X}; p, X; c) \leq 0$ for all $(p, X) \in [0, 1] \times [0, +\infty)$. We hence consider the maximisation problem,

$$\max_{(p,X) \in [0,1] \times [0,+\infty)} \Phi(q, \bar{X}; p, X; c). \tag{12}$$

After applying symmetry, the first order condition with respect to $\bar{X}$ reduces to (7), provided that $p > 0$. On the other hand, the first order derivative with regard to the participation probability q is

$$\begin{aligned}\frac{\partial \Phi}{\partial q} =& V\left[\sum_{\boldsymbol{M}\in\mathcal{P}^{\boldsymbol{N}_1}} p^{|\boldsymbol{M}|}(1-p)^{|\boldsymbol{N}_1\setminus\boldsymbol{M}|}\cdot\frac{\bar{X}^r}{\bar{X}^r+|\boldsymbol{M}|X^r}\right]-\bar{X}-c\\ &-pV\left[\sum_{\boldsymbol{M}\in\mathcal{P}^{\boldsymbol{N}_2}} p^{|\boldsymbol{M}|}(1-p)^{|\boldsymbol{N}_2\setminus\boldsymbol{M}|}\cdot\frac{X^r}{\bar{X}^r+X^r+|\boldsymbol{M}|X^r}\right]\\ &+pV\left[\sum_{\boldsymbol{M}\in\mathcal{P}^{\boldsymbol{N}_2}} p^{|\boldsymbol{M}|}(1-p)^{|\boldsymbol{N}_2\setminus\boldsymbol{M}|}\cdot\frac{X^r}{X^r+|\boldsymbol{M}|X^r}\right].\end{aligned} \tag{13}$$

Similarly, symmetry implies

$$\frac{\partial \Phi}{\partial q} = \frac{V}{n-1}\frac{1-(1-p)^{n-1}}{p} - X - c = 0. \tag{14}$$

The two necessary first order conditions (7) and (14) identify the candidate ESS equilibrium with endogenous entry. The next theorem shows they are also sufficient.

Theorem 6. There exists an evolutionarily stable strategy ($p^{\text{ESS}}, X^{\text{ESS}}$) such that the entry probability p^{ESS}

i) is 1 if $c \leq \frac{n-(n-1)r}{n(n-1)}V$,

ii) otherwise, it is implicitly determined by

$$\frac{1-(1-p^{\text{ESS}})^{n-1}}{(n-1)p^{\text{ESS}}}V - X^{\text{ESS}}(r,V,n,p^{\text{ESS}}) = c, \tag{15}$$

where ESS individual effort $X^{\text{ESS}}(r,V,n,p^{\text{ESS}})$ is given by (8).

Proof: See Appendix A.5.

Similar to part i) of Theorem 5, players enter with probability 1 in the ESS when the entry cost is sufficiently small. In this degenerated case, individual effort $X^{\text{ESS}}(r,V,n,p^{\text{ESS}}=1)$ takes the value of rV/n. When the entry cost becomes larger than the critical value identified in the theorem, the participation probability p drops below 1. Although individual effort is already higher in ESS than in NE, what matters in ESS is relative fitness and hence the marginal advantage of entering can be higher. Note that under ESS the critical value of the entry cost is higher than under NE. In other words, for a given entry cost, full participation is more likely under ESS than under NE.

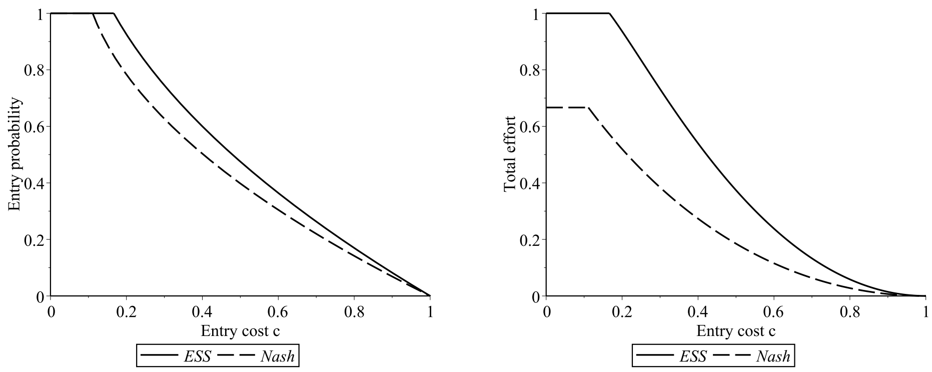
Consider part ii). In the proof, we establish that $\Phi(q, \bar{X}; p^{\text{ESS}}, X^{\text{ESS}}; c) \leq 0$ for all possible pairs of $(q, \bar{X})$. The intuition follows from the exogenous case. Suppose, as a first step, that a mutant's entry probability is exogenously fixed at q. Then no other effort level than $X^{\text{ESS}}(r, V, n, p^{\text{ESS}})$ can give the mutant a higher relative payoff. In other words, for any possible exogenously given mutant entry probability, $X^{\text{ESS}}(r, V, n, p^{\text{ESS}})$ leads to the highest relative payoff. On the other hand, given that all other players and the mutant play $X^{\text{ESS}}(r, V, n, p^{\text{ESS}})$, the mutant's relative payoff stays at zero for all entry probabilities. We then formally show that this means the mutant cannot obtain a higher relative payoff than ESS strategists with any combination of $q \in [0, 1]$ and $\bar{X} \geq 0$.

By comparing part i) in Theorem 5 and 6, we note that the set of entry costs that will result in full participation ($p = 1$) under NE is a proper subset of such entry costs under ESS. In this sense, deterministic contests are more likely to occur under ESS than under NE. In addition, the next result shows that truly stochastic entry has a strictly higher probability under ESS than under NE.

Theorem 7. For a given $0 < c < V$, $p^{\text{ESS}} \geq p^{\text{Nash}}$. This inequality holds strictly when $\frac{n-(n-1)r}{n^2}V < c < V$, i.e., when $p^{\text{Nash}} < 1$.

Proof: See Appendix A.6.

The higher entry probability can also be explained by NE behaviour with regard to relative payoff maximisation, which would for example result from spiteful preferences. Although, at the NE probability level, entering results in a net loss in absolute payoff, it reduces opponents' payoff even more. Therefore, equilibrium entry is more aggressive under ESS. Furthermore, the total effort expenditure is higher in ESS than in NE, when entry is



(a) Equilibrium entry probability

(b) Total effort

Figure 3: A comparison of ESS and NE outcomes with $n = 3$, $r = 1$ and $V = 1$

endogenous.

Theorem 8. For a given $0 < c < V$, $T^{\text{ESS}}(r, V, n, p^{\text{ESS}}) > T^{\text{Nash}}(r, V, n, p^{\text{Nash}})$.

Proof: Theorem 3 and $p^{\text{ESS}} \geq p^{\text{Nash}}$ imply that

$$T^{\text{ESS}}(r, V, n, p^{\text{ESS}}) \geq T^{\text{ESS}}(r, V, n, p^{\text{Nash}}) = \frac{n}{n-1} T^{\text{Nash}}(r, V, n, p^{\text{Nash}}).$$

Q.E.D.

The intuition behind Theorem 8 follows from two observations. First, players enter more often under ESS than under NE. Second, for a given entry probability, players exert higher total effort in an ESS than in a NE. Thus, although *a priori* it is not clear whether endogenous entry leads to higher or lower total expected effort in an ESS than in NE, Theorem 7 and the monotonicity of ESS total effort in the probability of participation imply that total effort is higher in an ESS than in NE.

To illustrate the difference between ESS and NE outcomes, Figure 3 presents the case with three players. We set $r = 1$, $V = 1$, and vary the entry cost $c \in [0, 1]$. For $c \in [0, \frac{n-(n-1)r}{n^2} V] = [0, 1/9]$, the entry probability is 1 under both NE and ESS. For $c \in (\frac{n-(n-1)r}{n^2} V, \frac{n-(n-1)r}{n(n-1)} V] = (1/9, 1/6]$ the ESS entry probability remains at 1 while the NE entry probability falls below 1. As the entry cost keeps increasing, both entry probabilities decrease. As can be seen in the left panel of Figure 3, the ESS entry probability lies (at least weakly) above the NE entry probability for all levels of the entry cost. The right panel of Figure 3 depicts total effort in ESS and in NE. In accordance with Theorem 8, total effort is always higher in an ESS than in NE.

3.3 Total resource spending

With endogenous entry, it should be acknowledged that the total expenditures incurred by the players consist of total effort cost *and* total entry cost. Theorems 7 and 8 together imply that players unequivocally spend more in ESS than in NE: not only do they exert higher effort upon entering, they incur higher entry cost on average. We also note that each player's absolute payoff in a Nash equilibrium with a non-degenerated entry probability has to be zero because of the indifference condition on the entry decision. As a result, players obtain a negative absolute payoff under ESS when entry is truly stochastic.

This finding has immediate implications for the issue of dissipation. Comparing the *total* resource expenditure with the prize size, it is clear that under Nash equilibrium the total resource expenditure can never exceed the total prize value. With stochastic entry, the Nash equilibrium total resource expenditure has an even lower upper bound. This is because with a probability of $(1-(1-p)^n)$ no player enters and hence none of the players will obtain the prize. As a result, total resource spending in NE can be no higher than $(1-(1-p)^n)V$. Under ESS, however, this is different. In the exogenous entry case, we have established that total effort exceeds the size of the prize if, and only if, the impact function is convex *and* the entry probability is sufficiently close to 1.

Let us now redefine overdissipation being the total resource expenditure exceeding the prize size V. We then characterise the overdissipation result under endogenous entry. Note that the total resource expenditure in a given contest, $R^{\mathrm{ESS}}(r,V,n,c)$, is given by

$$R^{\mathrm{ESS}}(r,V,n,c) = np^{\mathrm{ESS}} \cdot (X^{\mathrm{ESS}}(r,V,n,p^{\mathrm{ESS}}) + c),$$

since on average, np^{ESS} players enter the contest, each of which incur a resource expenditure of $X^{\text{ESS}}(r, V, n, p^{\text{ESS}}) + c$.

By Theorem 6, we know that $p^{\text{ESS}} = 1$ when $c \leq \frac{n-(n-1)r}{n(n-1)}V$. Consequently, in this case $R^{\text{ESS}} = n \cdot (rV/n + c) = rV + nc$, which exceeds V if $c > \frac{1-r}{n}V$. Note that for convex impact functions, no additional condition on c is needed since $rV + nc$ cannot be less than V. It follows that overdissipation occurs when $c \in \left(\max\left\{\frac{1-r}{n}V, 0\right\}, \frac{n-(n-1)r}{n(n-1)}V\right)$, a non-empty interval for all $r \in (0, n/n-1)$. We have thus demonstrated that, for arbitrary impact functions of the Tullock type, there exists a non-empty range of entry cost such that overdissipation occurs. In particular, overdissipation can emerge for concave impact functions.

Consider now the case $c > \frac{n-(n-1)r}{n(n-1)}V$. By (15), we have

$$R^{\text{ESS}}(r, V, n, c) = np^{\text{ESS}} \cdot \frac{1 - (1 - p^{\text{ESS}})^{n-1}}{(n-1)p^{\text{ESS}}}V = \frac{n\left(1 - \left(1 - p^{\text{ESS}}\right)^{n-1}\right)}{n-1}V.$$

Hence, overdissipation occurs whenever $p^{\text{ESS}} > 1 - n^{\frac{-1}{n-1}}$. In a game with a given set of parameters, the entry cost needs to be small enough to make entry probability stay above $1 - n^{\frac{-1}{n-1}}$. The exact critical value of this entry cost depends on model parameters. However, as p^{ESS} ranges from 0 to 1, when the entry cost is below a certain value, $p^{\text{ESS}} > 1 - n^{\frac{-1}{n-1}}$ will be met. We note again that overdissipation is present regardless of the discriminatory power of the contest success function. This represents a substantial generalisation of the existing overdissipation result, which relies on the convexity of the impact function. Theorem 9 summarizes our finding.

Theorem 9. Let entry cost be part of the resource expenditure. Under ESS, overdissipation occurs for all (Tullock) contest success functions for a range of entry cost. In particular, overdissipation is present for $c \in \left(\max\left\{\frac{1-r}{n}V, 0\right\}, \hat{c}\right)$ where $\hat{c}$ is implicitly defined by $p^{\text{ESS}}(r, V, n, \hat{c}) = 1 - n^{\frac{-1}{n-1}}$.

Overdissipation under ESS was first shown by HLP for deterministic contests and is extended to stochastic contests with exogenous entry in Theorem 4 of the present paper. However, both results require convex impact functions as a necessary condition. Theorem 9 extends the overdissipation result to all

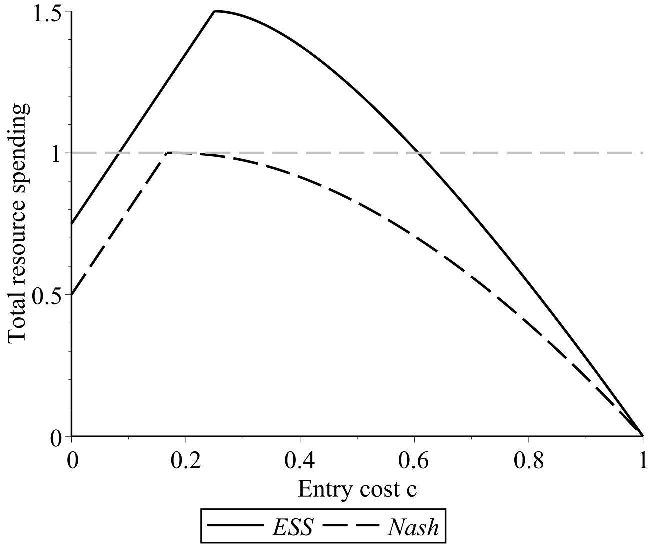


Figure 4: Total resource spending (effort and entry cost) under ESS and under NE with $n = 3$, $V = 1$ and $r = 3/4$.

(Tullock) contest success functions, in particular those with concave impact functions.

To gain the intuition behind this generalisation, consider a concave impact function. By Theorem 5, we see $p^{\text{Nash}} = 1$ when $c \leq \frac{n-(n-1)r}{n^2}V$. In particular, consider an entry cost weakly above $\frac{n-(n-1)r}{n^2}V$. By properties of a mixed strategy Nash equilibrium, players are *indifferent* between entering and not entering, although in equilibrium p^{Nash} tends to be 1. In such a case, the chance of the prize not being won by any player is 0. In other words, for an individual player, the expected benefit of participating in such a contest is exactly V/n. Consequently, the NE individual resource expenditure - effort and entry cost - should also be exactly V/n. This means there is exactly full dissipation under NE. As players act more aggressively under ESS, there is overdissipation, even when the impact function is concave. By continuity, overdissipation is also present when the entry cost is not too far from this special level of $\frac{n-(n-1)r}{n^2}V$.

Figure 4 plots the total resource expenditure under NE and ESS, respectively, for parameter values $n = 3$, $V = 1$, and $r = 3/4$. Notice that, as the entry cost increases, the entry probability decreases and eventually approaches zero. As a result, when the entry cost gets too high, the chance of the prize not being won by any player, $(1-p)^n$, gets very large and the total resources spent to contest for the prize will be quite low. This holds true under both equilibrium concepts. In the limit, as c approaches 1, total resource spending converges to 0.

Observe also that overdissipation does not occur under Nash equilibrium for any $c \in [0,1]$. On the other hand, there is overdissipation under ESS for an intermediate level of entry cost, namely, for $c \in (0.0833, 0.6067)$.[14] In particular, in this example, the impact function is concave as $r = 3/4$.

4 Concluding remarks

In this paper, we have studied evolutionarily stable behaviour in contests where participation can be stochastic. We established a precise relation between FPESS and NE effort under exogenous stochastic entry as well as under endogenous costly entry. We find players exert higher effort and enter more often under FPESS than under NE. In this sense, the evolutionary force of "survival of the fittest" selects more competitive behaviour in situations of conflicts. However, this also results in lower absolute payoff for players under FPESS than under the "rational" NE setting.

Moreover, we have substantially generalised the existing overdissipation result in the contest literature, which assumes exogenous deterministic entry and convexity of the contest impact function. First, we have established that this overdissipation result generalises to the case of exogenous stochastic entry, still assuming convexity of the impact function. Subsequently, we have shown that under endogenous entry and with entry cost being taken into account, the more important aspect of the contest is the entry cost rather than the discriminatory power of the contest success function. Overdissipation can also occur for concave impact functions. An interesting and challenging extension of these findings under endogenous entry would be to consider heterogeneous players with different marginal effort costs. Applications of ESS to finite heterogeneous populations of players have occurred in oligopoly theory; for two different approaches see Tanaka (1999) and Leininger and Moghadam (2018).

[14] This interval can be characterised using Theorem 9.

A Appendix

A.1 Proof of Theorem 2

The plan of the proof is as follows. We first simplify (7) and establish its relationship with respect to (1). Second, we verify that the solution to the first order condition indeed maximises the relative payoff (5).

A.1.1 Simplification

We derive the symmetric effort level implied by the first order condition (7):

$$
\begin{aligned}
X &= rV \left[\sum_{i=1}^{n-1} \frac{C_i^{n-1} p^i (1-p)^{n-1-i}}{(i+1)^2} \cdot i + \sum_{i=0}^{n-2} \frac{C_i^{n-2} p^{i+1} (1-p)^{n-2-i}}{(i+2)^2} \right] \\
&= rV \left[\sum_{i=1}^{n-1} \frac{C_i^{n-1} p^i (1-p)^{n-1-i}}{(i+1)^2} \cdot i + \sum_{i=1}^{n-1} \frac{C_{i-1}^{n-2} p^i (1-p)^{n-1-i}}{(i+1)^2} \right] \\
&= rV \sum_{i=1}^{n-1} p^i (1-p)^{n-1-i} \frac{i \cdot C_i^{n-1} + C_{i-1}^{n-2}}{(i+1)^2} \qquad (16) \\
&= \frac{n}{n-1} rV \sum_{i=1}^{n-1} p^i (1-p)^{n-1-i} C_i^{n-1} \frac{i}{(i+1)^2} \qquad (17) \\
&= \frac{n}{n-1} \cdot X^{\text{Nash}},
\end{aligned}
$$

where, from (16) to (17), we have used the identity $C_{i-1}^{n-2} = {}^{iC_i^{n-1}}/_{n-1}$. The NE individual effort, X^{Nash}, is given by Theorem 1.

A.1.2 Second order derivative of relative payoff w.r.t. effort

Before proceeding to the maximisation problem (5), we first derive the second order derivative in this section and establish Lemma 1 in Section A.1.3.

From the first order derivative (6), we have $\frac{\partial^2 \phi}{\partial \bar{X}^2}(\bar{X}, X) =$

$$
rVX^r \bar{X}^{r-2} \sum_{k=1}^{n-1} C_k^{n-1} p^k (1-p)^{n-1-k} k \frac{(r-1)(\bar{X}^r + kX^r) - 2r\bar{X}^r}{(\bar{X}^r + kX^r)^3}
$$

$$+ prVX^r\bar{X}^{r-2}\sum_{k=0}^{n-2} C_k^{n-2}p^k(1-p)^{n-2-k}\frac{(r-1)(\bar{X}^r+(k+1)X^r)-2r\bar{X}^r}{(\bar{X}^r+(k+1)X^r)^3}$$

$$=rVX^r\bar{X}^{r-2}\sum_{k=0}^{n-2} C_k^{n-2}p^{k+1}(1-p)^{n-2-k}(n-1)\frac{(r-1)(k+1)X^r-(r+1)\bar{X}^r}{(\bar{X}^r+(k+1)X^r)^3}$$

$$+ rVX^r\bar{X}^{r-2}\sum_{k=0}^{n-2} C_k^{n-2}p^{k+1}(1-p)^{n-2-k}\frac{(r-1)(k+1)X^r-(r+1)\bar{X}^r}{(\bar{X}^r+(k+1)X^r)^3}$$

$$=rnVX^r\bar{X}^{r-2}\sum_{k=0}^{n-2} C_k^{n-2}p^{k+1}(1-p)^{n-2-k}\frac{(r-1)(k+1)X^r-(r+1)\bar{X}^r}{(\bar{X}^r+(k+1)X^r)^3}, \quad (18)$$

where in the first step we have used an index transformation and the identity $(k+1)C_{k+1}^{n-1} = (n-1)C_k^{n-2}$, and where the second step follows from taking the sum of the two previous expressions.

A.1.3 Relative payoff of zero-effort mutant

Lemma 1. Let $p \in (0,1]$, $r \leq n/n-1$, and X be given by (8). Then $\phi(0, X) \leq 0$, with strict inequality for $n > 2$ and $p < 1$.

Proof: Let $p \in (0,1]$ and $r \leq n/n-1$ be arbitrarily given. Then $\phi(0, X) \leq 0$ if, and only if, $\Pi_2(0, X, \ldots, X) \geq \Pi_1(0, X, \ldots, X) = V(1-p)^{n-1}$. Using the identity $C_\kappa^\nu/\kappa+1 = C_{\kappa+1}^{\nu+1}/\nu+1$ for $\kappa \leq \nu$, $\Pi_2(0, X, \ldots, X) \geq V(1-p)^{n-1}$ can be rewritten as follows:

$$V\sum_{k=0}^{n-2} C_k^{n-2}p^k(1-p)^{n-k-2}\frac{1}{k+1} \geq X + V(1-p)^{n-1}$$

$$\sum_{k=0}^{n-2} C_{k+1}^{n-1}p^{k+1}(1-p)^{n-k-2} \geq$$

$$r\sum_{k=1}^{n-1} C_{k+1}^{n}p^{k+1}(1-p)^{n-k-1}\frac{k}{k+1} + (n-1)p(1-p)^{n-1}$$

$$\underbrace{\sum_{k=1}^{n-1} C_k^{n-1}p^k(1-p)^{n-k-1}}_{=1-(1-p)^{n-1}} \geq r\sum_{k=2}^{n} C_k^n p^k(1-p)^{n-k}\frac{k-1}{k} + (n-1)p(1-p)^{n-1}.$$

The last inequality holds for all $r \leq {}^{n}/_{n-1}$ if it holds for $r = {}^{n}/_{n-1}$, i.e., if

$$(n-1)(1-(1-p)^{n-1}) + n\sum_{k=2}^{n} C_k^n p^k (1-p)^{n-k}\frac{1}{k}$$
$$\geq n\left[1-(1-p)^n - np(1-p)^{n-1}\right] + (n-1)^2 p(1-p)^{n-1}.$$

This condition reduces to

$$\sum_{k=2}^{n} C_k^n p^k (1-p)^{n-k}\frac{n}{k} + np(1-p)^{n-1} + (1-p)^n \geq 1. \qquad (19)$$

Since ${}^{n}/_{k} \geq 1$ for $k = 2, \ldots, n$, the left hand side of (19) is bounded below by $\sum_{k=0}^{n} C_k^n p^k (1-p)^{n-k} = 1$. Thus, (19) represents a true statement. Moreover, the inequality is strict if $p < 1$ and $n > 2$.

A.1.4 Maximisation problem

We now show that, for $0 < r \leq {}^{n}/_{n-1}$, X^{ESS} as in (17) indeed solves the maximisation problem (5). To this end, consider the second order derivative (18).

Clearly, for $r \leq 1$, (18) is negative and hence (5) is globally concave. Consider the remaining case $1 < r \leq {}^{n}/_{n-1}$. To show that X^{ESS} solves the maximisation problem (5) in this case, we proceed in three steps.

First, note that the second order derivative is positive when $\bar{X}$ is sufficiently small. To see this, suppose $\bar{X} < \left(\frac{(r-1)(n-1)}{r+1}\right)^{\frac{1}{r}} X^{\text{ESS}}$. It follows that $(r-1)(k+1)(X^{\text{ESS}})^r > (r+1)\bar{X}^r$ for all $k = 0, \ldots, n-2$, and hence (18) is positive.

Second, the second order derivative is negative when $\bar{X} > \left(\frac{r-1}{r+1}\right)^{\frac{1}{r}} X^{\text{ESS}}$. This is because, in this case, $(r-1)(k+1)(X^{\text{ESS}})^r < (r+1)\bar{X}^r$ for all $k = 0, \ldots, n-2$.

Third, the second order derivative is negative when evaluated at the candidate ESS strategy and it switches sign only once for $\bar{X} \in (0, \infty)$. To verify this, evaluate (18) when $\bar{X} = X^{\text{ESS}}$. Then (18) simplifies to

$$Vrn\left(X^{\text{ESS}}\right)^{-2}\sum_{i=1}^{n-1}p^i(1-p)^{n-1-i}\frac{C_{i-1}^{n-2}[(r-1)(i+1)-2r]}{(i+1)^3}. \tag{20}$$

Note that $(r-1)(i+1) < 2r$ for $i = 1\dots n-1$ if $r \le {}^{n}/_{n-2}$. The latter holds true because of $r \le {}^{n}/_{n-1}$. Thus, we have established that (20) is negative.

Now observe that because $r \le 2$, (18) clearly decreases monotonically in $\bar{X}$ as long as (18) remains positive. On the other hand, by inspection, we see that once $\bar{X}$ becomes large enough to turn (18) negative, it stays negative for all larger $\bar{X}$. As a result, (5) is first convex and then concave for $\bar{X} \in (0,\infty)$. Hence, the only two candidates for a global maximum of (5) are 0 and X^{ESS}.

Finally, by Lemma 1, a mutant cannot strictly increase its relative fitness by playing 0. This completes the proof. *Q.E.D.*

A.2 Proof of Theorem 3

Since $T^{\text{ESS}}(r,V,n,p) = \frac{n}{n-1}T^{\text{Nash}}(r,V,n,p)$, part i) follows directly from Theorem 6 in LM.

To establish part ii), we note that

$$\begin{aligned}
&\frac{1}{rV}\left[T^{\text{ESS}}(r,V,n+1,p) - T^{\text{ESS}}(r,V,n,p)\right] \\
=&\frac{1}{rV}\left[\frac{n\left[T^{\text{Nash}}(r,V,n+1,p) - T^{\text{Nash}}(r,V,n,p)\right]}{n-1} - \frac{T^{\text{Nash}}(r,V,n+1,p)}{n(n-1)}\right] \\
=&\frac{n\sum_{i=2}^{n+1}C_{i-1}^{n}p^i(1-p)^{n-i+1}\frac{1}{i(i-1)}}{n-1} - \frac{\sum_{i=2}^{n+1}C_{i}^{n+1}p^i(1-p)^{n+1-i}\frac{i-1}{i}}{n(n-1)} \\
=&\frac{1}{n(n-1)}\sum_{i=2}^{n+1}C_i^{n+1}p^i(1-p)^{n-i+1}\left(\frac{1}{i-1}\frac{n^2}{n+1} - \frac{i-1}{i}\right) \\
=&\frac{1}{n(n-1)}\sum_{i=2}^{n+1}C_i^{n+1}p^i(1-p)^{n-i+1}\frac{((n+1)i-1)(n+1-i)}{i(i-1)} > 0,
\end{aligned}$$

where the result for $\left(T^{\text{Nash}}(r,V,n+1,p) - T^{\text{Nash}}(r,V,n,p)\right)$ on page 596 in LM was used. *Q.E.D.*

A.3 Proof of Theorem 4

First, consider the special case $p = 1$. Then, ex-ante total effort becomes $T^{\text{ESS}}(r, V, n, 1) = \frac{nrV}{n-1}\left(1 - \frac{1}{n}\right) = rV$. That is, ex-ante total effort is larger than the prize value V in deterministic contests if, and only if, $r \in (1, {}^{n}\!/_{n-1})$. Finally, by continuity and monotonicity of T^{ESS} in p *and* r, there is overdissipation for p sufficiently close to 1, if, and only if, $r \in (1, {}^{n}\!/_{n-1})$. *Q.E.D.*

A.4 Proof of Theorem 5

First, we note that α, M and Δ in FJL are equal to 1, n and c, respectively. Hence, $\bar{r}$ in FJL becomes ${}^{n-1}\!/_{n-2}$, and their r_0 is larger than ${}^{n}\!/_{n-1}$. It follows then $0 < r \leq {}^{n}\!/_{n-1} < \min\{r_0, \bar{r}\}$. By Theorem 4 in FJL, there exists a unique symmetric equilibrium with pure-strategy bidding characterised by their Lemma 2. Second, it is verified that the break-even condition in their Lemma 2 reduces to (10) in the current paper. *Q.E.D.*

A.5 Proof of Theorem 6

To prepare the proof, Lemma 2 below collects some auxiliary results. Subsequently, we prove the main statement of Theorem 6.

A.5.1 Auxiliary results

Lemma 2. a) Let $\Phi(q, \bar{X}; p, X; c)$ and $\phi(\bar{X}, X)$ be given by (11) and (4), respectively. Further, let $p, q \in (0, 1]$ and $X \geq 0$ be arbitrary. Then maximizing $\Phi(q, \bar{X}; p, X; c)$ w.r.t. $\bar{X}$ is equivalent to maximizing $\phi(\bar{X}, X)$ w.r.t. $\bar{X}$.

b) Fix $r \leq {}^{n}\!/_{n-1}$, and let (p, X) represent a solution to (15) and (8), respectively. Then, we have $\Phi(q, 0; p, X; c) \leq 0$, where the inequality holds strictly if $n > 2$ and $p < 1$.

c) Fix an arbitrary $q \in (0, 1]$. Then we have

$$\Phi(q, \bar{X}; p^{\text{ESS}}, X^{\text{ESS}}; c) \leq \Phi(q, X^{\text{ESS}}; p^{\text{ESS}}, X^{\text{ESS}}; c), \text{ for all } \bar{X} \geq 0.$$

Proof: To show a), consider the corresponding first order derivatives,

$$\frac{\partial \Phi}{\partial \bar{X}} = q\frac{\partial \Pi_1(\bar{X}, X, \ldots, X)}{\partial \bar{X}} - p\frac{\partial \Pi_2(\bar{X}, X, \ldots, X; q)}{\partial \bar{X}}, \text{ where}$$

$$\begin{aligned}
\frac{\partial \Pi_1(\bar{X}, X, \ldots, X)}{\partial \bar{X}} &= V \sum_{M \in \mathcal{P}^{N_1}} p^{|M|} (1-p)^{|N_1 \setminus M|} \frac{|M| \, r\bar{X}^{r-1} X^r}{\left(\bar{X}^r + |M| \, X^r\right)^2} - 1, \\
\frac{\partial \Pi_2(\bar{X}, X, \ldots, X; q)}{\partial \bar{X}} &= qV \sum_{M \in \mathcal{P}^{N_2}} p^{|M|} (1-p)^{|N_2 \setminus M|} \frac{r\bar{X}^{r-1} X^r}{\left(\bar{X}^r + (|M|+1) X^r\right)^2} \\
&= \frac{q}{p} \frac{\partial \Pi_2(\bar{X}, X, \ldots, X)}{\partial \bar{X}}.
\end{aligned}$$

Note that the last equality follows from differentiating (3) with respect to $\bar{X}$, given the entry probability is exogenous at p.

We can hence rewrite

$$\begin{aligned}
\frac{\partial \Phi}{\partial \bar{X}} &= q\frac{\partial \Pi_1(\bar{X}, X, \ldots, X)}{\partial \bar{X}} - q\frac{\partial \Pi_2(\bar{X}, X, \ldots, X)}{\partial \bar{X}} \\
&= q\left[\frac{\partial \Pi_1(\bar{X}, X, \ldots, X)}{\partial \bar{X}} - \frac{\partial \Pi_2(\bar{X}, X, \ldots, X)}{\partial \bar{X}}\right],
\end{aligned}$$

where the term in brackets coincides with the first order derivative of (5). Accordingly, all results of the exogenous entry case that relate to the sign of the first and higher order derivatives of (5) w.r.t. effort level $\bar{X}$ extend to the endogenous entry case, where (12) is to be maximized.

b) Fix $r \leq {}^{n}/_{n-1}$, and let (p, X) be a solution to (15) and (8). We first show $\Pi_1(0, X, \ldots, X) \leq c$ and $\Pi_2(0, X, \ldots, X) = c$. Then, $\Phi(q, 0; p, X; c) \leq 0$ follows directly from these two results.

First note that $\Pi_1(0, X, \ldots, X) \leq c$ is equivalent to $V(1-p)^{n-1} \leq c$. By (15), we need to show

$$V(1-p)^{n-1} \leq \frac{V}{n-1} \frac{1-(1-p)^{n-1}}{p} - X.$$

Multiplying both sides by ${}^{(n-1)p}/_{V}$ and using (8), this condition becomes

$$(n-1)p(1-p)^{n-1} \leq 1-(1-p)^{n-1} - prn\sum_{k=1}^{n-1} C_k^{n-1} p^k (1-p)^{n-1-k} \frac{k}{(k+1)^2}.$$

Since ${}^{nC_k^{n-1}}/_{k+1} = C_{k+1}^n$, the above can be rewritten as

$$\begin{aligned} r\sum_{k=1}^{n-1} C_{k+1}^n p^{k+1} (1-p)^{n-1-k} \frac{k}{k+1} &= r\sum_{k=2}^{n} C_k^n p^k (1-p)^{n-k} \frac{k-1}{k} \\ &\leq 1-(1-p)^n - np(1-p)^{n-1}. \end{aligned}$$

This inequality holds for arbitrary $r \leq {}^{n}/_{n-1}$ if it holds for $r = {}^{n}/_{n-1}$, i.e. if

$$n\sum_{k=2}^{n} C_k^n p^k (1-p)^{n-k} \frac{k-1}{k} \leq (n-1) \underbrace{\left[1-(1-p)^n - np(1-p)^{n-1}\right]}_{\sum_{k=2}^n C_k^n p^k (1-p)^{n-k}}$$

or equivalently if

$$\sum_{k=2}^{n} C_k^n p^k (1-p)^{n-k} \leq \sum_{k=2}^{n} C_k^n p^k (1-p)^{n-k} \frac{n}{k}.$$

Because ${}^{n}/_{k} \geq 1$, for $k = 2, \ldots, n$, the above inequality is satisfied. Moreover, it holds strictly if $n > 2$ and $p < 1$.

Second, to show $\Pi_2(0, X, \ldots, X) = c$, note that (3) implies $\Pi_2(0, X, \ldots, X)$

$$= V\sum_{k=0}^{n-2} C_k^{n-2} p^k (1-p)^{n-k-2} \frac{1}{k+1} - X = \frac{V}{n-1} \frac{1-(1-p)^{n-1}}{p} - X = c,$$

where the last equality holds because of (15).

c) Let $q \in (0, 1]$ be arbitrary. By a), we can write the first order condition with respect to $\bar{X}$ as

$$q\left[\frac{\partial \Pi_1}{\partial \bar{X}}(\bar{X}, X, \ldots, X) - \frac{\partial \Pi_2}{\partial \bar{X}}(\bar{X}, X, \ldots, X)\right] = 0. \tag{21}$$

As $q > 0$, it follows that (21) is equivalent to the first order condition of (5) with the entry probability being exogenously set at p^{ESS}. Since the first and higher order derivatives of (12) w.r.t. $\bar{X}$ with a fixed q represent multiples of the corresponding derivatives of (5), the remainder of the proof can be established along the lines of the proof for Theorem 2. The only difference to be taken into account is when we evaluate the relative fitness of a zero effort mutant. To compare the relative payoff of $\bar{X} = 0$ with that of $\bar{X} = X^{\text{ESS}}$, note that b) implies $\Phi(q, 0; p^{\text{ESS}}, X^{\text{ESS}}; c) \leq 0$. *Q.E.D.*

A.5.2 Proof of the main statement

Part i): Let $c \leq \frac{n-(n-1)r}{n(n-1)} V$ and note that $X^{\text{ESS}}(r, V, n, p) = {}^{rV}/_{n}$, for $p = 1$. We need to show that $(p^{\text{ESS}}, X^{\text{ESS}}) = (1, {}^{rV}/_{n})$ constitutes an ESS.

First, consider a mutant using strategy $(q, \bar{X}) = (0, 0)$. The mutant obtains zero absolute payoff by staying out while ESS strategists - due to the absence of the the mutant - obtain $\frac{V}{n-1} - \frac{rV}{n} \geq c$. Thus, in terms of relative payoff, the mutant cannot strictly improve from the ESS strategy pair, $p = 1, X = {}^{rV}/_{n}$, which yields a zero relative payoff.

Now consider an arbitrary pair $(q, \bar{X})$. We first decompose $\Phi(q, \bar{X}; p, X; c)$ as follows:

$$\Phi(q, \bar{X}; p, X; c) = q\Phi(1, \bar{X}; p, X; c) + (1 - q)\Phi(0, \bar{X}; p, X; c).$$

This can be verified by using (11) and the observation that

$$\Pi_2(\bar{X}, X, \ldots, X; q) = q\Pi_2(\bar{X}, X, \ldots, X; 1) + (1 - q)\Pi_2(\bar{X}, X, \ldots, X; 0).$$

Note also that $\Phi(0, \bar{X}; p, X; c) = \Phi(0, 0; p, X; c)$ as effort exerting is conditional on entry. Then we have

$$\begin{aligned} \Phi(q, \bar{X}; 1, {}^{rV}/_{n}; c) =& q\Phi(1, \bar{X}; 1, {}^{rV}/_{n}; c) + (1 - q)\Phi(0, 0; 1, {}^{rV}/_{n}; c) \\ \leq& q\Phi(1, {}^{rV}/_{n}; 1, {}^{rV}/_{n}; c) + (1 - q)\Phi(0, 0; 1, {}^{rV}/_{n}; c) \\ \leq& q \cdot 0 + (1 - q) \cdot 0 = 0, \end{aligned}$$

where the first inequality follows because ${}^{rV}/_{n}$ is an ESS in the exogenous case and the second inequality follows from the above discussion that a mutant cannot increase relative payoff by playing $(0, 0)$.

Part ii): Let $(q, \bar{X})$ be an arbitrary mutant strategy and let the candidate ESS strategy $(p^{\text{ESS}}, X^{\text{ESS}})$ satisfy the first order conditions (8) and (15). We show that the mutant's relative payoff $\Phi(q, \bar{X}; p^{\text{ESS}}, X^{\text{ESS}}; c)$ assumes its maximum at 0 for $(q, \bar{X}) = (p^{\text{ESS}}, X^{\text{ESS}})$.

First, note by Lemma 2.c), $\Phi(q, \bar{X}; p^{\text{ESS}}, X^{\text{ESS}}; c) \leq \Phi(q, X^{\text{ESS}}; p^{\text{ESS}}, X^{\text{ESS}}; c)$. Further, following equations (13), (14), and (15), $\Phi(q, X^{\text{ESS}}; p^{\text{ESS}}, X^{\text{ESS}}; c)$ is constant in q. Hence, $\Phi(q, \bar{X}; p^{\text{ESS}}, X^{\text{ESS}}; c) \leq \Phi(p^{\text{ESS}}, X^{\text{ESS}}; p^{\text{ESS}}, X^{\text{ESS}}; c)$. *Q.E.D.*

A.6 Proof of Theorem 7

Let $c > \frac{n-(n-1)r}{n^2}V$. From Theorem 5, $p^{\text{Nash}} < 1$ and

$$\frac{1-(1-p^{\text{Nash}})^n}{p^{\text{Nash}}} = n\frac{c + X^{\text{Nash}}(r, V, n, p^{\text{Nash}})}{V}.$$

Evaluating (14) at $p = p^{\text{Nash}}$ and using the above result lead to

$$\begin{aligned}&\frac{V}{n-1}\left[n\frac{c + X^{\text{Nash}}}{V} - (1-p^{\text{Nash}})^{n-1}\right] - \frac{n}{n-1}X^{\text{Nash}} - c\\ =&\frac{c - V(1-p^{\text{Nash}})^{n-1}}{n-1} > 0,\end{aligned}$$

where the inequality holds because c is equal to the expected equilibrium surplus by bidding X^{Nash} and $V(1-p^{\text{Nash}})^{n-1}$ is the payoff of the deviation to bidding 0 in the NE. The inequality indeed holds strictly because of equation (22) on page 594 in LM. It hence follows that $p^{\text{ESS}} > p^{\text{Nash}}$.

If $c \leq \frac{n-(n-1)r}{n^2}V$, by Theorem 5 and 6, $p^{\text{Nash}} = p^{\text{ESS}} = 1$. *Q.E.D.*

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