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Collusion via Information Sharing and Optimal Auctions

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Abstract

This paper studies collusion via information sharing in the context of auctions. The model of collusion via information sharing builds on Aumann's (1976) description of knowledge. Robustness of auction mechanisms to collusion via information sharing is defined as the impossibility of an agreement to collude. A cartel can agree to collude on a contract if it is common knowledge within that cartel that the contract is incentive compatible and individually rational. Robust mechanisms are characterized in a number of settings where some, all, or no bidders are bound by limited liability. Finally, the characterization is used in a simple IPV setting to design a mechanism that is both optimal and robust to collusion.

Keywords: Bidder collusion, mechanism design, communication design, no-trade theorem.

JEL codes: D82, D44, C72.

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1 Introduction

Auction bidders can extract rents from knowing how much allocations are worth to them. Given the rent from information, they have an incentive to protect its privacy from the auctioneer. In contrast, with regard to sharing information with their peers, the bidders' incentives are precisely reversed. When bidders' information is pooled, rather than dispersed, they can achieve even higher rents through coordinated action.

This paper focuses on collusion via information sharing among bidders. Information sharing is different from, and stronger than, cheap-talk communication; it implies a prior commitment to disclosure, which is equivalent to granting access to future data. In auctions, information sharing allows bidder cartels to overcome an important obstacle to successful collusion, the asymmetry of information. Hence any sharing technology that can serve as a commitment device will be used by cartels of auction bidders. This paper is concerned with the auctioneer's response to the threat of collusion via information sharing.

The focus on collusion via information sharing is novel to the literature, which has emphasized the cartels' role in the enforcement of actions rather than in the enforcement of information exchange (e.g., Laffont and Martimort, 1997). In contrast to the extant literature, I assume that the members of a cartel can disobey its bidding recommendation but can not withhold the information they had previously committed to share. Consequently, the role of monetary side payments is different. In the extant literature, action-enforcing cartels use the payments to counteract the asymmetries of information and induce truth-telling. In contrast, in this paper cartels employ the payments to induce obedience to the cartel's plan of action.

The objective is three-fold: (1) set up the model of collusion via information sharing, (2) characterize auction mechanisms robust to such collusion, and (3) use the characterization to design a robust mechanism.

The model of collusion via information sharing builds on Aumann's (1976) representation of knowledge as a partition of the type (valuation) space. Aumann's knowledge structures emerge initially when the bidders learn their own values and are subsequently refined as result of information exchange between the bidders. The model offers a notion of *agreeing to collude* and its counterpart, the robustness of an auction mechanism to collusion. For a variety of information sharing technologies as well as transfer schemes, I study whether and how bidder collusion via information sharing can be prevented.

The difficulty in studying this type of collusion is due to the possibility of information sharing and the failure of Revelation Principle it implies.[1] Consequently, the analysis can not benefit from the reduction to a class of direct mechanisms. Moreover, I show that robustness to collusion via information sharing can only be achieved by introducing more complex games of communication between the bidders and the auctioneer.

In the first set of results, I find that depending on the bidders' ability to commit to transfers, mechanisms' robustness to collusion can be immediately achieved, or, in contrast, impossible. In particular, when bidders cannot commit to side-transfers the auctioneer can prevent collusion at virtually no cost by extending the set of messages. For example, in a special case where bidders can obtain hard evidence of collusion, collusion can be easily precluded by rewarding whistle-blowers. In contrast to that result, even moderate information sharing upsets any efficient auction in the case where all bidders can commit to pay side transfers. A direct implication of this negative result is that if both information sharing and enforcement of actions are available to the cartel, there is no benefit of running an auction whatsoever.

Another set of results relates to the intermediate case where the cartel members who are designated to win are the ones who can commit to side payments, as means of rewarding cooperation. The non-designated members from the cartel's plan may be deprived of the reward if they deviate, but no money can be extracted from them. In this setup I derive sufficient and necessary conditions for mechanisms' robustness to collusion via information sharing. The sufficient condition for robustness bears similarity to zero- (fixed-) sum games, as the cartel plays against the set of possible defectors when it shares the spoils from collusion. The cartel chooses the bidding manipulation so as to minimize the sum of gains from possible deviations while maximizing the surplus from collusion. The test is whether the cartel can always generate

[1] Green and Laffont (1986) discuss the necessary and sufficient conditions underlying the Revelation Principle (see e.g., Myerson, 1979). The possibility of certifiable messages, as in collusion via information sharing, violates the necessary condition.

enough surplus and share it with the potential defectors to induce them to comply. If the answer is negative, the mechanism is robust. The sufficient condition is also necessary in open IPV auctions when information sharing is complete.

The characterization offers an insight into designing robust auction mechanisms: one seeks to induce a conflict of interest within a prospective cartel to prevent it from rationally forming in the first place. Specifically, deviations from any collusive plot must yield greater payoffs than the surplus from collusion itself. An example of auction mechanism achieving this is described in Section 5. The auction is simultaneously designed to yield the optimal revenue in a symmetric IPV setting (that is, under the conditions of Revenue equivalence theorem). This is in contrast to Che and Kim (2009), Pavlov (2008), Gruyer (2009), and Feige et al. (2013) who show that robustness to collusion with enforcing cartels may lead to a loss of revenue. In that sense, collusion via information sharing may be seen as less problematic for the auctioneer than collusion via controlled bidding.

With respect to communication surrounding a bidding process and its effect on robustness, our analysis generates several qualitative predictions. First, it is not true in general that the more information is shared within the cartel, the more the cartel can achieve. Indeed on the one hand, better information allows the cartel fine-tune the bidding manipulation to its precise data; but on the other, better information equally expands the possibilities for individual deviations from the cartel's plot. Thus cooperation may break down as bidders become more informed. Second, including extra communication between the bidders and the auctioneer has an ambiguous effect on the robustness, as well. While it may give the necessary leeway for individual cartel members to defect, it also creates a larger menu of possible manipulations for the cartel to choose from. (One of the examples uses an integer game *à-la* Maskin (1999) to rule out whistle-blowing as part of the cartel's manipulation.) The auction designer has to navigate carefully between the competing objectives. Third, and final effect is unambiguous. Eliciting less information about the auction's course and outcome is always to the auctioneer's advantage. The reason is that the less cartels can observe and punish deviations, the easier it is to deviate, the better for robustness.

Related Literature

It has been long understood that standard auction procedures are prone to collusion. However, few alternative auction designs have been proposed. A clever framework for designing collusion-robust auctions was presented by Laffont and Martimort (1997) and applied to auctions by Che and Kim (2009); Pavlov (2008) and others. The approach is to model collusion organized by a side-contractor, as in Mailath and Zemsky (1991); McAfee and McMillan (1992), and subsequently to employ the Revelation principle (e.g., Myerson, 1979). The side contractor is benevolent to the cartel but faces information asymmetry, like the auctioneer. By the Revelation principle, if a mechanism is robust to collusion sustained by a side-contract that satisfies truth-telling and participation constraints, then it is robust to any side contract that is achieved via an arbitrary negotiation process. It is noted that without truth-telling incentive constraints (specifically, under the assumption of complete information within the cartel) the auctioneer cannot do better than charge monopoly prices (see, e.g., Dequiedt, 2007). This negative result holds under the assumption that the cartel is able to enforce actions during the auction, or equivalently, to observe deviations and 'shoot the traitor'. It may not hold if bidders are autonomous and not all deviations are observable by design. This paper considers precisely such possibilities and requires that compliance within cartels be a rational response to the incentives. Formally, I replace truth-telling by obedience incentive constraints and show that the auctioneer may be able to do better much than reducing the auction to monopoly pricing.

Empirically the type of collusion where cartels can not enforce the members' actions was studied in McMillan (1991); Porter and Zona (1993); Hendricks and Porter (1989); Pesendorfer (2000); Levenstein and Suslow (2006), among others. Non-enforcing bidder cartels have also been the subject of multiple theoretical studies. For a review of this literature, see Rachmilevitch (2013) as well as the earlier studies including Graham and Marshall (1987); Robinson (1985); von Ungern-Sternberg (1988). Schummer (2000) studies manipulations by two bidders where one of them rewards the other for misrepresenting his type, which is similar to the intermediate case covered in the present paper. Enforcing and non-enforcing cartels are compared in Marshall and Marx (2007) in the context of first- and second-price auctions.

In another related paper, Marshall and Marx (2009) study the effect of the amount of information revealed by the seller on the viability of collusion. They find that cartels where only the winner pays can collude efficiently in the second-price auction, unless the winner's true identity remains unknown. The present paper complements their analysis by looking at how the auctioneer's strategy in terms of outcome disclosure affects the robustness of mechanisms.

Computer science has tackled several aspects of bidder collusion. Chen and Micali (2012); Deckelbaum and Micali (2016) design indirect auction mechanisms, a feature that relates their work to mine. Indirect mechanisms allow for richer communication than direct mechanisms where the bidders only submit their bids to the auctioneer. The above papers are concerned with the effect of collusion on allocative efficiency, as opposed to seller's revenue (this paper). The mechanisms guarantee that the object is allocated to the highest valuation bidder, despite that a significant part of the seller's revenue may be lost. Goldberg and Hartline (2005) use a collusion model with bribes and describe auctions that are approximately efficient and revenue maximizing in a setup where items are in excess supply.

The literature has also studied other types of collusion, for example, collusion via resale (Garratt et al., 2009) and collusion via repeated interactions (Aoyagi (2003, 2007); Skrzypacz and Hopenhayn (2004); Vergote (2011); Abdulkadiroglu and Chung (2003)). Along with the models of collusion by action enforcing cartels and the present collusion via information sharing, these models ought not to be seen as competing, but complementary tools in the auctioneer's toolbox. The economic environment in which the auction takes place, the nature of object at sale, and of the bidders (people, automata, or corporations, e.g.) are the factors that should all be taken into account when choosing are the relevant collusion model to inform auction design in practice.

2 The Setup

There is a set $N = \{1, 2, ..n\}$ of bidders, the number of bidders is greater than two. The seller is referred to as player 0, and the extended set of players is $N_0 = N \cup \{0\}$.[2] The auction results in allocation $(x_0, x_1, ...x_n) \in \mathbb{X}$, where x_i is the set of goods assigned to player i. $x_i = \emptyset$, for some $i \in N$, implies that bidder i does not win anything in the auction. Bidder i' valuation v_i maps the allocation into a positive real number,

$$v_i : \mathbb{X} \to \mathbb{R}_+. \tag{1}$$

The image $v_i(\mathbb{X})$ is compact in $\mathbb{R}_+$. The bidders' valuation functions are drawn randomly from the set $\mathcal{V}$, assumed discrete.[3] This setup allows for correlation in value functions, as well as allocation externalities. In the special case of independent private values, the functions v_i, $i \in N$, are drawn independently and each v_i only depends on x_i.

The state space $\Omega = \mathcal{V}^n \times \mathbb{A}_0$ is the product of the space of buyers' valuations and the seller's signals, whose role is discussed below. A typical element of Ω is denoted ω,

$$\omega = (v_1, ..v_n, a_0). \tag{2}$$

We will consider a normal-form representation of the auction game. An *auction mechanism*,

$$\mathcal{M} = (\mathbb{A}, \mathcal{O}, \mathcal{P}), \tag{3}$$

is defined by three elements: the set of actions $\mathbb{A}$, the outcome function $\mathcal{O}$, and the public disclosure operator $\mathcal{P}$.

$\mathbb{A} \equiv \times_{i \in N_0} \mathbb{A}_i$, where $\mathbb{A}_i$ is the set of player i's actions, assumed discrete. In the sim-

[2] The setup can be re-framed to describe procurements, or reverse auctions: the *lowest* bidder wins the procurement contract; the analysis is completely symmetric.

[3] The assumption that $\mathcal{V}$ is discrete implies that valuations are known with a finite degree of precision, e.g., up to one cent of a dollar. Assuming that the space of actions (bids) is also discrete allows us to disregard infinitesimal deviations, and to assure that all infima and suprema in strategies are attained.

plest case of a direct mechanism, $\mathbb{A}_i = \mathcal{V}$, for all $i \in N$.[4] $\mathbb{A}_0$ corresponds to the seller's randomization device.[5]

$\mathcal{O} \equiv (\mathcal{O}_{\mathbb{X}}, \mathcal{O}_1, ..\mathcal{O}_n) : \mathbb{A} \to \mathbb{X} \times \mathbb{R}^n$ is the (pure) outcome function that maps the action set $\mathbb{A}$ into the product of the set of allocations $\mathbb{X}$ and payments that buyers make to the seller $\mathbb{R}^n$.[6] I focus on winner-pay auctions, such that $\mathcal{O}_{\mathbb{X}i}(a) = \emptyset$ implies $\mathcal{O}_i(a) = 0$.[7] This excludes entry fees and all pay auctions, in particular.

$\mathcal{P}(a, \mathcal{O}(a))$, also denoted $(\mathcal{P} \circ \mathcal{O})(a)$, is the public disclosure operator, a projection of actions and outcomes onto a reduced space of information, a projection of $\mathbb{A} \times \mathbb{X} \times \mathbb{R}^n$. $\mathcal{P}$ is what the auctioneer discloses publicly during and after the auction. For instance, $\mathcal{P} \circ \mathcal{O} = \mathcal{O}_{\mathbb{X}}$ implies that the auction is seal-ed bid and only the final allocation is public.

All three elements $\mathbb{A}, \mathcal{O}$, and $\mathcal{P}$ are set by the auction designer, e.g.,. in a way to preclude collusion. Note that public disclosure may be subject to regulation as well as practical limitations. (For instance, the identity on the winner of a takeover bid can not typically be concealed.) To fix ideas, I assume that $\mathcal{P}$ is such that at least the final allocation becomes public.[8]

For a given mechanism $\mathcal{M} = (\mathbb{A}, \mathcal{O}, \mathcal{P})$, bidder i's utility is given by

$$U_i(a) = (v_i \circ \mathcal{O}_{\mathbb{X}} - \mathcal{O}_i)(a), \tag{4}$$

where $\mathcal{O}_{\mathbb{X}}(a) \in \mathbb{X}$ denotes the allocation and $\mathcal{O}_i(a) \in \mathbb{R}_+$ bidder i's payment to the seller. The seller's revenue at a equals $\sum_{i \in N} \mathcal{O}_i(a)$.

[4]Footnote 1 on page 4 discusses how the Revelation Principle relates to the current setup.

[5]For example, if $\mathbb{A}_0$ is the set of all possible orderings of N then the random draw a_0 breaks ties in a single-unit auction.

[6]The outcome is 'purified', or made deterministic, by the appropriately choosing $\mathbb{A}_0$.

[7]$\mathcal{O}_{\mathbb{X}i}(a) = \emptyset$ implies that no items are allocated to bidder i.

[8]Formally, if $(\mathcal{P} \circ \mathcal{O})(a) = (\mathcal{P} \circ \mathcal{O})(a')$ for some $a, a' \in \mathbb{A}$ then $\mathcal{O}_{\mathbb{X}}(a) = \mathcal{O}_{\mathbb{X}}(a')$. $\mathcal{O}_{\mathbb{X}}(a)$ denotes the allocation resulting from a.

3 The Model of Collusion

The strength of cartels' commitment to information sharing, actions and transfers is exogenous. For a given cartel $C \subseteq N$, it is characterized by a *collusion environment*

$$\mathcal{K} = (\mathscr{C}_C, \mathcal{S}_C, T_C)_{C \subseteq N} \tag{5}$$

where $\mathscr{C}_C$ is an information sharing technology, $\mathcal{S}_C$ is a set of feasible bidding manipulations and T_C is a set of feasible transfers. $\mathscr{C}_C$ is discussed in Subsection 3.1, while $\mathcal{S}_C$ and T_C, composing the space of side-contracts, are discussed in Subsection 3.2. The game evolves according to the following time-line.[9]

1 *(ex ante)*. The auction mechanism $\mathcal{M}$ and the collusion environment $\mathcal{K}$ are fixed.

2 (ex ante). Cartel $C \subseteq N$, $|C| \geq 2$, forms exogenously.[10]

3 *(interim-(3))*. Nature draws a random state ω. Players learn their private signals.

4 *(interim-(4))*. Information is shared according to $\mathscr{C}_C$ and a side-contract

$(s_C, t_C) \in \mathcal{S}_C \times T_C$ is signed.

5 (ex post). The auction is run.

6 *(ex post)*. Side transfers t_C are exchanged within the cartel.

With regard to information, I distinguish four stages: *ex ante, interim-(3)*, *interim-(4)* and *ex post*. At the *ex ante* stage, the players have no payoff-relevant information. At the *interim* stage 3 the bidders learn their valuations. At the *interim* stage 4, the bidders receive further information. Finally, *ex post,* bidders observe whatever is

[9]The timing is similar to Laffont and Martimort (1997): first, a prior framework agreement, then a specific contract, and finally the execution of contractual payments. The stage where the grand contract is accepted is omitted, since the bidder can guarantee zero payoff in the auction and hence acceptance is trivial.

[10]Exogenous coalition formation is a common assumption in the auction literature. A notable exception is Biran and Forges (2011), who first study interim coalition formation.

disclosed publicly about the actions and outcomes of the auction (as well as their own actions). I assume that the bidders start with the same prior belief and update their beliefs according to the Bayes rule. Information sharing that occurs at the *interim* stages is discussed in more detail below.

3.1 $\mathscr{C}_C$: Information structure and information sharing

The knowledge of bidder i at stage t of the game is represented by a partition $\mathcal{I}_i^{(t)}$ of the state space Ω (Aumann, 1976). The partition is made up of sets of states that are indistinguishable to the bidder at t. Specifically, $\mathcal{I}_i^{(t)}(\omega)$ is the set of all states that bidder i considers possible at state ω, whereas any subset of $\Omega/\mathcal{I}_i^{(t)}(\omega)$ is assigned zero probability at t. We say that a bidder *knows* an event $\mathcal{E}$ at ω if $\mathcal{I}_i^{(t)}(\omega) \subseteq \mathcal{E}$.

Partitions $\mathcal{I}_i^{(3)}$, $i \in N$, at stage 3 emerge from the bidders' observation of their own values.[11] Partitions $\mathcal{I}_i^{(4)}$, $i \in N$, are the refinements of $\mathcal{I}_i^{(3)}$, $i \in N$, that result from information sharing.

The model of information sharing within cartel $C \subseteq N$ includes a fictitious omniscient mediator. The mediator observes all of the cartel's information such that his knowledge is given by the join of the cartel members' partitions $\vee_{i\in C}\mathcal{I}_i^{(3)}$. The mediator imparts arbitrary bits of his knowledge to the cartel members. The resulting stage-4 partitions are as follows:

$$\mathcal{I}_i^{(4)} = \mathcal{I}_i^{(3)} \vee \mathscr{C}_i\left(\vee_{j\in C}\mathcal{I}_j^{(3)}\right), \tag{6}$$

where $\mathscr{C}_i(\cdot)$ is some (weak) coarsening of the partition.[12,13] The collection of coarsening operators for cartel C is denoted $\mathscr{C}_C = (\mathscr{C}_i)_{i\in C}$.

The coarsening operators $\mathscr{C}_C$ determine the degree of information sharing, from complete information sharing (no coarsening) to null information sharing (maximal coarsening). Formally, information sharing $\check{\mathscr{C}}$ is *complete* in C if $\check{\mathscr{C}}_i(A) \equiv A$, or equivalently, $\mathcal{I}_i^{(4)} = \vee_{j\in C}\mathcal{I}_i^{(3)}$, for all $i \in N$. Information sharing $\hat{\mathscr{C}}$ is *null* in C if $\hat{\mathscr{C}}_i(A) \equiv \{\Omega\}$,

[11] For all $\omega = (a_0, v_1, ..v_n)$ and $\omega = (a_0', v_1', ..v_n')$, $\mathcal{I}_i^{(3)}(\omega) = \mathcal{I}_i^{(3)}(\omega')$ if and only if $v_i = v_i'$.

[12] Partition A of Ω is coarser than partition B of Ω if for any $\omega \in \Omega$, $A(\omega) \supseteq B(\omega)$.

[13] For any bidder i who does not collude, or equivalently, is the sole member of his cartel, $\mathcal{I}_i^{(4)} = \mathcal{I}_i^{(3)}$.

or equivalently, $\mathcal{I}_i^{(4)} = \mathcal{I}_i^{(3)}$. Information sharing $\mathscr{C}^e$ is *efficient* if the bidders learn what the efficient allocation among them is. Table 1 illustrates the various degrees of information sharing.

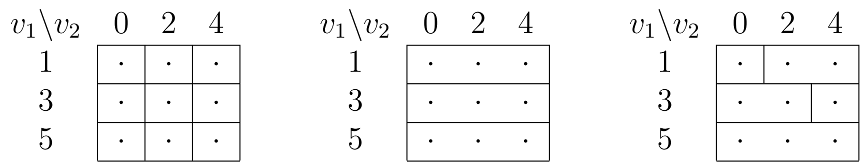

Table 1: *Examples of information sharing between two bidders in a single-unit auction. Bidder 1's* interim-(4) *knowledge partitions when his type* v_1 *takes values 1, 3, or 5 and Bidder 2's type* v_2 *takes values 0, 2, or 4. Left: Complete; Center: Null; Right: A case of efficient information sharing (knowing which bidder has the highest valuation).*

The following definition of common knowledge, as well as the rest of the analysis, focus on stage-4 partitions only. Hence the superscript *(4)* will be omitted: $\mathcal{I}_i^{(4)} \equiv \mathcal{I}_i$, $i \in N$, from here on.

Definition 2 (Aumann, 1976). An event $\mathcal{E} \subseteq \Omega$ is common knowledge in $C \subseteq N$ at $\omega \in \mathcal{E}$ if $\mathcal{I}_C(\omega) \subseteq \mathcal{E}$, where $\mathcal{I}_C \equiv \wedge_{i \in C} \mathcal{I}_i$ is the meet of partitions $\mathcal{I}_i$.

Table 2 provides an illustration to cartel's common knowledge.

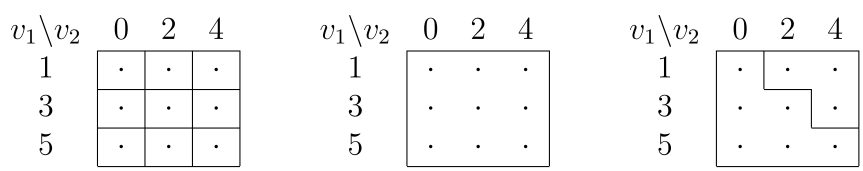

Table 2: *Examples of information sharing between two bidders in a single-unit auction. Common knowledge partitions in the example of Table 1, if Bidder 2's knowledge partitions are symmetric to those of the Bidder 1. Left: Complete; Center: Null; Right: Efficient information sharing.*

A strategy $s_i \in S_i$, $i \in N$, is a $\mathcal{I}_i$-measurable function that maps the state space Ω into the action set $\mathbb{A}_i$. We focus on pure strategies; for a fixed strategy profile

$s_N \equiv (s_i)_{i \in N}$, the state of the world ω uniquely determines the outcome of the auction $(\mathcal{O} \circ s_N)(\omega) \in \mathbb{X} \times \mathbb{R}^n_+$, and the bidder's auction payoffs $U_i(s_N(\omega); \omega) \equiv (U_i \circ s_N)(\omega)$.[14]

The strategy profile that serves as benchmark for comparison with the cartel's manipulations is denoted $s^*_N \equiv (s^*_i)_{i \in N}$. The appropriate benchmark may depend on the application, but typically it is a non-cooperative equilibrium of the mechanism.

The analysis of cartel behavior focuses on the game reduced to the cartel $C \subseteq N$. Payoffs U^*_i of the reduced game are derived from payoffs U_i of the original game where the residual set of bidders N/C play $s^*_{N/C} = (s^*_i)_{i \in N/C}$.[15] Formally,

$$U^*_i \circ s_C \equiv U_i \circ \left(s_C, s^*_{N/C}\right) \tag{7}$$

for all $i \in C$, if $C \neq N$. Otherwise if $C = N$, $(U^*_i \circ s_C)(\omega) = (U_i \circ s_C)(\omega)$. The cartel's collusion surplus ΔW^*_C from playing $s_C = (s_i)_{i \in C}$ as opposed to $s^*_C = (s^*_i)_{i \in C}$ is defined as the change in the cartel's welfare,

$$\Delta W^*_C \circ s_C \equiv \sum_{i \in C} U^*_i \circ s_C - \sum_{i \in C} U^*_i \circ s^*_C. \tag{8}$$

3.2 $(\mathcal{S}_C, T_C)$: Side Contracts

A side contract is a bidding manipulation and a system of ex-post transfers that support it. The contract is offered to the cartel at stage 4 after the information is exchanged. The total payoff to cartel member $i \in C$ is the sum of his auction payoff and the (possibly negative) side transfers:

$$U_i(a; \omega) + t_i(a). \tag{9}$$

[14]Note that we omit a_0 as we write $(\mathcal{O} \circ s_N)(\omega) \equiv \mathcal{O}(s_N(\omega))$ as opposed to $\mathcal{O}(s_N(\omega), a_0)$. Recall that $\omega = (v_1, ..v_n, a_0)$, hence no information is lost from the omission.

[15]In Bernheim et al. (1987), e.g., the residual player set N/C plays Nash equilibrium strategies. See Bierbrauer and Hellwig (2016) for a discussion.

3.2.1 Bidding manipulation ($\mathcal{S}_C$)

To break the cartel's indifference we restrict attention to profitable joint deviations. That is, $\mathcal{S}_C$ is such that $\mathbb{E}\left(\Delta W_C^* \circ s_C\right)(\tilde{\omega}) \mid \tilde{\omega} \in \mathcal{I}_C(\omega) > 0$ for all $s_C \in \mathcal{S}_C$, $\omega \in \Omega$. Bidder i is a *designated* bidder under s_C if $\mathcal{O}_{\mathbb{X}i} \circ \left(s_C, s_{N/C}^*\right) \neq \emptyset$ with positive probability.[16] Let $\mathcal{S}_C^\alpha$ denote the set of all manipulations s_c such that the set of designated bidders, denoted $C^\alpha(s_C)$, is a non-trivial subset of C.

3.2.2 Transfers (T_C)

The feasible transfer set T_C is a set of transfer profiles $t_C = (t_i)_{i \in C}$, where $t_i : \times_{i \in N \cup \{0\}} \mathbb{A}_i \to \mathbb{R}$, for all $i \in C$, are $(\mathcal{P} \circ \mathcal{O}) \times \mathcal{I}_C-$measurable functions. t_i represents the payment *received* by bidder i (recall (9)); the measurability condition implies that payment is conditional on the auction's public outcome and the cartel's knowledge, and in particular on whether any deviations have been detected.[17] The set of feasible transfers is given by

$$T_C \equiv \left\{ t_i(\cdot) : \ \forall i \in \underline{C}, \forall a \in \times_{i \in N_0} \mathbb{A}_i, \, t_i(a) \geq 0, \sum_{i \in C} t_i(a) \leq 0 \right\}, \qquad (10)$$

where $\underline{C} \subseteq C$ is the set of bidders with limited liability, i.e., bidders unable to commit to pay. The robustness analysis in Section 4 considers three cases where the set of bidders whose liability is limited vary. In particular, we look at $\underline{C} = C$, $\underline{C} = \varnothing$, and $\underline{C} = C/C^\alpha(s_C)$, for all $s_C \in \mathcal{S}_C^\alpha$, and the corresponding sets of side contracts denoted $\left(\mathcal{S}_C, T_C^{LL}\right)$, $\left(\mathcal{S}_C, T_C^{UL}\right)$, $(\mathcal{S}_C^\alpha, T_C^\alpha)$. The first two cases correspond to the weakest and the strongest possible cartel, whereas the latter corresponds to an intermediate case where the designated bidders can commit to share their profits with the non-designated ones.

[16] $\mathcal{O}_{\mathbb{X}i}(a) = \emptyset$ implies that no items are allocated to bidder i.

[17] A deviation from strategy sub-profile $s_C \in \times_{i \in C} S_i$ is *undetected* by $C \subseteq N$ at $\omega \in \Omega$ and $a' \in \times_{i \in N^0} \mathbb{A}_i$ if $(\mathcal{P} \circ \mathcal{O} \circ s)(\tilde{\omega}) = (\mathcal{P} \circ \mathcal{O})(a')$, for some $\tilde{\omega} \in \wedge_{i \in C} \mathcal{I}_i(\omega)$. Otherwise, the deviation is *detected*.

3.2.3 Incentive Compatibility and Individual Rationality

The incentive compatibility and individual rationality of side contracts is evaluated at stage 4, when the cartel selects a contract from the feasible set $(\mathcal{S}_C, T_C)$.

A contract (s_C, t_C) is *obedience incentive compatible* at $\omega \in \Omega$ if no member of C can profitably deviate from the collusive manipulation. Specifically, for all $i \in C$ and $s_i' \in S_i$,

$$\mathbb{E}\left[\left((U_i^* + t_i^*) \circ s_C\right) | \mathcal{I}_i(\omega)\right] \;\; \geq \;\; \mathbb{E}\left[\left((U_i^* + t_i^*) \circ \left(s_i', s_{C/i}\right)\right) | \mathcal{I}_i(\omega)\right], \tag{11}$$

where $\mathbb{E}\left[g \,|\, \mathcal{I}_i(\omega)\right] \equiv \mathbb{E}\left[g(\tilde{\omega}) \,|\, \tilde{\omega} \in \mathcal{I}_i(\omega)\right]$, and $t_i^* \circ s_C \equiv t_i \circ \left(s_C, s_{N/C}^*\right)$ for all $i \in C$. I require that the inequality be strict for all deviations that may lead to a change in outcome compared to s_C; the set of such deviations is denoted $S_i'(s_C)$.[18] The incentive compatibility amounts to obedience of the cartel's manipulation.

A contract (s_C, t_C) is *individually rational* at $\omega \in \Omega$ if each member is at least as well off colluding as he is playing non-cooperatively. I.e., for all $i \in C$,

$$\mathbb{E}\left[\left((U_i^* + t_i^*) \circ s_C\right) | \mathcal{I}_i(\omega)\right] \;\; \geq \;\; \mathbb{E}\left[\left(U_i^* \circ s_C^*\right) | \mathcal{I}_i(\omega)\right]. \tag{12}$$

The constraint of individual rationality implies that the contract (s_C, t_C) involves no regret of information sharing. Specifically, it states that the utility of collusion is greater than the utility in the non-cooperative equilibrium conditional on the information at stage 4 when the contract is signed.

4 Robustness to Collusion

Our analysis of the robustness of mechanisms to bidder collusion relies on the following definitions.

[18]The requirement of strict incentive compatibility in such cases can be interpreted as a trembling-hand refinement; the cartel should not accept a contract where the outcome is upset if one of the member's breaks the indifference unfavorably. If we did not impose this refinement then the results of Theorems 1 and 2 would be unchanged, while the inequality in Theorems 3 and 4 would become strict.

Definition 3. A cartel $C \subseteq N$ *can agree to collude* in $(\mathcal{M}, \mathcal{K}_C)$, if there exists a side contract, and a state $\omega \in \Omega$ such that it is common knowledge among the cartel that the side contract is incentive compatible and individually rational at ω.

Definition 4. An auction $\mathcal{M}$ is *robust to collusion via information sharing* $\mathcal{K} \equiv (\mathcal{K}_C)_{C \subseteq N}$ if no cartel $C \subseteq N$ can agree to collude.

By definition, the robustness to collusion via information sharing depends not only on how much information is shared ($\mathscr{C}$) in $\mathcal{K}$, but also on the bidders' power to commit to exchanging ex post transfers (T) that help sustain the manipulation during the auction.[19] This section shows that depending on the components $\mathscr{C}$ and T of the collusive environment $\mathcal{K}$ robustness can be virtually free to achieve or impossible.

We will first consider the case where the bidders cannot commit to ex-post transfers at all. Put differently, the set of feasible contracts is $\left(\mathcal{S}_C, T_C^{LL}\right)$. The following theorem states that any mechanism can be made robust to collusion via non-null information sharing at an arbitrary small cost.

Theorem 1. *Suppose bidders' types are independent. Consider an arbitrary mechanism $\mathcal{M}$, its equilibrium s^* and collusion environment $\mathcal{K} = \left(\mathscr{C}_C, \mathcal{S}_C, T_C^{LL}\right)_{C \subseteq N}$ where $\mathscr{C}_C$, for all $C \subseteq N$ is strictly non-null. There exists a mechanism $\mathcal{M}'$ such that (1) s^* constitutes an equilibrium of $\mathcal{M}'$ (2) $\mathcal{M}'$ is robust to $\mathcal{K}$ and (3) the cost of implementing $\mathcal{M}'$ as opposed to $\mathcal{M}$ is arbitrarily small.*

Proof. See Appendix A.1.

In a special case where bidders can present hard evidence of collusion, such as records of collusive negotiations, the proof can be substantially simplified. We construct such a proof based on the integer game (e.g., Maskin, 1999) below.

Proof in a special case. If $\mathcal{M}$ is (already) robust, then let $\mathcal{M}' = \mathcal{M}$. If $\mathcal{M}$ is not robust, $\mathcal{M}'$ is constructed by expanding the original action set $\mathbb{A}_i$, for all $i \in N$, to allow bidders to present any evidence of collusion. The whistle-blower becomes the dictator: he solely determines the final allocation and receives a small reward of $\epsilon > 0$. If there is more than one whistle-blower, the dictator is chosen according to the highest integer given alongside the report of collusion. Since the whistle-blower's liability

[19] This observation mirrors some of the previous analysis, for example Laffont and Martimort (1997).

is limited, it becomes profitable to report collusion to the seller, even if the bidder is already getting his best value with the manipulation (due to ϵ). The integer game guarantees that whistle blowing cannot be part of an incentive compatible manipulation. ■

Let us now consider the opposite case where all the bidders can commit to exchanging transfers ex post, such that the relevant space of side contracts is $\left(\mathcal{S}_C, T_C^{UL}\right)$. A negative result obtains in this setup. In particular, the following theorem states that no efficient auction, including such mechanisms as the first- and second-price auctions, is robust to collusion, under efficient information sharing.

Theorem 2. *Suppose $\mathcal{M}$ is an efficient auction. $\mathcal{M}$ is not robust to $\mathcal{K} = \left(\mathscr{C}_C^e, \mathcal{S}_C, T_C^{UL}\right)_{C \subseteq N}$.*

Proof. See Appendix A.2.

The proof shows that the grand cartel can enforce any manipulation due to the following argument. Any profitable deviation from the collusive strategy must involve a change in allocation, which is public, or a decrease in auction payment. Since transfers are conditional on the public outcome and are unbounded from below, the cartel can impose fines that are large enough to preclude deviations resulting in allocation changes. Decreasing payment will not be feasible if the cartel initially selects the manipulation that leads to the lowest possible revenue among those that lead to the efficient allocation. A simple implication of Theorem 2's result is that action-enforcing cartels make robustness impossible in this setup.

The quasi-universal robustness stated in Theorem 1 is in sharp contrast with the negative result of Theorem 2. This contrast emphasizes the role of ex post transfers; depending on the possibility to commit to money transfers ex post robustness can be immediate or impossible. In intermediary cases, robustness obtains under certain conditions. Below, we look at designated bidder scenarios spanned by the set of side contracts $(\mathcal{S}_C^\alpha, T_C^\alpha)$ (see (10)). Recall that in this case the bidders designated to win, and only they, can commit to side-payments ex post. The next theorem states that an auction is robust to collusion if no cartel can restrict the non-designated bidders' payoff to be less than the cartel's own surplus under the chosen manipulation.

Theorem 3. *Suppose the net defection value $V_\omega^{\mathcal{M}} \geq 0 \ \forall \omega \in \Omega$. $\mathcal{M}$ is robust to*

$\mathcal{K} = (\mathscr{C}_C, \mathcal{S}^{\alpha}_C, T^{\alpha}_C)_{C\subseteq N}$. *The net defection value* $V^{\mathcal{M}}_{\omega}$ *is defined as follows:*

$$\min_{\substack{C\subseteq N,\\ s_C\in\mathcal{S}_C}} \max_{\substack{s'_{C/C^{\alpha}}\\ \in S'_{C/C^{\alpha}}(s_C)}} \mathbb{E}\left[\left(\sum_{i\in C/C^{\alpha}} \left(U^*_i \circ \left(s'_i, s_{C/i}\right) - U^*_i \circ s^*_C\right) - \Delta W^*_C \circ s_C\right) | \mathcal{I}_C(\omega)\right]. \quad \textbf{(13)}$$

$V^{\mathcal{M}}_{\omega}$ is the lowest net defection payoff the cartels achieve when the defecting bidders try to maximize it. It is similar to a fixed-sum game where the surplus from collusion is to be shared.

Proof. See Appendix A.3.

Let us look at the contra-positive first. Consider the following equivalent statement: If an auction is non-robust then the defection value is negative in at least one state of nature. Non-robustness implies that there exists a cartel that can agree to collude, i.e., at least one cartel can find a feasible side contract to satisfy common knowledge of individual rationality and incentive compatibility at some state. As an illustration, suppose that this is true of a cartel with just two bidders, the designated leader and non-designated follower. To prevent the follower's deviation the contract must be such that the reward for cooperation outweighs the benefit of whatever the follower can achieve by deviating. But for individual rationality to hold on the leader's side, the leader's extra surplus from collusion, less the reward he pays to the follower, must make the leader at least as well off as he was in the non.cooperative equilibrium. Thus, the surplus from collusion must exceed the follower's best payoff from deviation. In sum, this implies that the net defection value is negative at the state where the bidders agreed to collude, of equivalently $\exists\omega\in\Omega$ such that $V^{\mathcal{M}}_{\omega} < 0$.

The intuition behind the theorem relates to the No-trade result of Milgrom and Stokey (1982):[20] Agreement to collude fails whenever the cartel cannot agree to disagree on the price of collusion. Consider a cartel member's cooperation a 'good' that the buyer (designated leader) considers purchasing from the seller (non-designated follower); the value of cooperation is private information of its seller. In a robust mechanism, where there can be no trade, the leader would have to pay more to induce cooperation than what he gains from it. In the other direction, any reward that

[20]The No-trade Theorem that is also derived from Aumann's (1976) model.

leaves the leader with a positive surplus would be insufficient to shut down incentives to deviate from the collusive manipulation of bids.

A simple yet important corollary of Theorem 3 follows.

Corollary. *Suppose mechanism* $\mathcal{M} = (\mathbb{A}, \mathcal{O}, \mathcal{P})$ *satisfies the conditions of Theorem 3 and hence is robust to* $\mathcal{K}$*. If a new mechanism* $\mathcal{M}' = (\mathbb{A}, \mathcal{O}, \mathcal{P}')$ *is obtained by reducing the public disclosure of outcomes, then* $\mathcal{M}'$ *is also robust to* $\mathcal{K}$*.*

Proof. See footnote 24 on page 26.

The corollary suggests that reducing public disclosure can only hinder collusion. For example if an open auction is robust to collusion, then it will stay robust if only the the final allocation is revealed. This is due to the fact that the ex post side transfers that cartels exchange are conditional on coarser information in the latter mechanism, and hence the cartel cannot possibly sustain more manipulations compared to the former mechanism. Indeed, one of the most prominent cases of collusion occurred in the FCC spectrum auction whose design featured extensive public disclosure (see Perry and Reny, 1999). Marshall and Marx (2009) reach a similar conclusion and develop further insights on the effect of outcome disclosure.

The final result shows that the sufficient condition of Theorem 3 is also necessary for the robustness of open IPV auctions when information sharing is complete.

Theorem 4. *Suppose* $\mathcal{M}$ *is open*[21] *and robust to* $\mathcal{K} = \left(\check{\mathscr{C}}_C, \mathcal{S}_C^\alpha, T_C^\alpha\right)_{C \subseteq N}$ *in an independent private valuation setting. Then* $V_\omega^{\mathcal{M}} \geq 0\ \forall \omega \in \Omega$ *(see (13)).*

Proof. See Appendix A.4.

The condition on the net defection value $V_\omega^{\mathcal{M}}$ offers a robustness test for existing auction mechanisms in a natural situation where cooperation can be rewarded in cartels. Importantly, it can also be used to inform robust auction design. The next section constructs a mechanism where the net defection value is always non-negative.

[21] *An auction mechanism* $\mathcal{M}$ *is open if* $\mathcal{P} \circ \mathcal{O} = \mathcal{O}$*.*

5 Auction with Target Bids

In this section, I use the result of Theorem 3 to construct an auction that is robust to collusion via information sharing. We assume that one indivisible unit is on sale and one bidder can be designated. The mechanism, dubbed *Auction with Target Bids* (ATB), is a modification of Myerson (1981) or Riley and Samuelson (1981) optimal auction with an extended message space.

The auction proceeds in two rounds. In the first round, each bidder places a preliminary bid β_i and chooses a target bidder. In the second round, the final bids are calculated and the Vickrey assignment is made.

Round 1. Each bidder i submits a preliminary bid $\beta_i \geq 0$ and a target bidder identity $\tau(i) \in N$ (self-targeting is permitted). The preliminary bid of the seller is $\beta_0 = c^{-1}(v_0)$, where $c(x) \equiv x - \frac{1-F(x)}{f(x)}$ is assumed to be strictly increasing. (We assume here that bids and values are continuous).

Round 2. The final bids b_i of $i \in N$ are determined as follows:

$$
\begin{aligned}
b_0 &= \beta_0, && (14)\\
b_i &= \min_{j \in N^0_{-i} \cup \{\tau(i)\}} \left\{\beta_j : \beta_j \in \left[\min\left\{\beta_i, \beta_{\tau(i)}\right\}; \max\left\{\beta_i, \beta_{\tau(i)}\right\}\right]\right\}, && (15)
\end{aligned}
$$

The object is allocated to the highest bidder who pays the second highest bid. Equal bids are treated as one. In case of a tie, the bidder with the *lower* preliminary bid wins. If there is a tie in preliminary bids, too, then the winner is chosen uniformly at random. First, we observe that:

Lemma. *Truthful bidding is weakly dominant within the class of self-targeting strategies.*

Proof. Follows from Vickrey (1961).

Due to the presumed symmetry of types and strategies, the bidders are indifferent between targets and may therefore choose a target at random. In the absence of information about the target bidder's valuation targeting another bidder is too risky as it often leads to a bid lower than one's true willingness to pay. Any serious bidder is better off playing the 'Vickrey' strategy of the lemma above, i.e., self-targeting and bidding his true value. The next proposition states that this is, in fact, an equilibrium.

Proposition 1. *Let bidders' values be drawn independently according to the probability distribution function $f : \mathcal{V} \to \mathbb{R}_+$. Suppose f is non-increasing, has no atoms and full support on $\mathcal{V}$, and $c(x) \equiv x - \frac{1-F(x)}{f(x)}$ is strictly increasing (Myerson's (1981) regularity condition). The profile of strategies where all bidders self-target and bid their true valuations constitutes a strict Bayes-Nash equilibrium of the Auction with Target Bids.*

Proof. See Appendix A.5.

An immediate corollary of the proposition is the following.

Corollary. *The equilibrium outcome of the Auction with Target Bids coincides with the outcome the Vickrey auction with the optimal reserve price unless a tie in valuations occurs.*

In the sense of this corollary, the Vickrey auction is the direct-revelation counterpart of the auction with target bids. The case of ties in bidders' valuations must be excluded from the equivalence statement because the auctions have different outcomes: the ATB treats equal bids as one, unlike the Vickrey auction. Note that in the continuous case ties occur with zero probability, hence revenue equivalence holds almost surely.

Proposition 2. *The Auction with Target Bids is robust to collusion via information sharing $\mathcal{K} = (\mathscr{C}_C, \mathcal{S}_C^\alpha, T_C^\alpha)_{C \subseteq N}$.*

Proof. See Appendix A.6.

The robustness of the auction with target bids to collusion is due to the way whistle-blowing is incorporated into the auction procedure. By introducing bid targeting the mechanism combines two pieces of information: the identity of a colluding bidder (whistle-blower's information) and his preliminary bid (auctioneer's information) to increase the whistle blower's chances to win the auction. The mechanism thus creates scope for profitable deviations from the cartel's manipulation of bids.

6 Discussion

This paper introduces a model of collusion via information sharing, characterizes auction mechanisms robust to such collusion, and designs a robust mechanism, the Auction with target bids.

Collusion via information sharing is a type of collusion where auction bidders can commit to exchanging information. The power of commitment with respect to both the information and payments determines whether and when collusion can be prevented. Theorems 1 and 2 outline the limits of auction design in face of collusion via information sharing: robustness to collusion can be impossible or quasi costless depending on the cartels' commitment power. In a natural intermediate case a necessary and sufficient condition is provided in Theorems 3 and 4. In general, as soon as the cartels' commitment power is limited, there is scope for collusion-robust auction design – and the design does not reduce to the choice of reserve prices and assignment rules as per Revelation principle. In a departure from direct mechanisms we show that managing communication is instrumental in precluding collusion. For example, *public* elicitation of *all* the information about the auction's course can only reinforce cooperation within bidder cartels. In contrast, communicating *some* of that information through a *private* channel can introduce conflicts of interest within a cartel and lead to collusion failure.

Specifically, as a way of managing communication in the context of bidder collusion via information exchange, the designer may consider introducing seller-to-buyer communication into the auction mechanism. To illustrate, the seller's information is used by a member of a cartel to construct a profitable deviation in the Auction with target bids. In practice, private and partial seller-to-buyer communication has been used in auctions if not to preclude collusion then to serve a related purpose of reinforcing competition. For example, a common practice in notary real estate auction in France is that the seller would contact several high bidders after the first round of offers and communicate the currently winning bid and ask if they wanted to make a better offer. Another instrument whose use is exemplified in the Auction with target bids is extending the space of bidders' actions to include messages other than bids. Thereby, whistle-blower leniency becomes an integral part of auction design. This, too, mirrors the real-life leniency programs and whistle blower rewards used by competition authorities and regulators around the world (e.g., US Security and Exchange Commission, European Commission, Bundeskartellamt). Similar to auction design, their goal is to introduce a conflict of interest between the members of a cartel and thus to preclude cartel formation in the first place.

A Appendix

A.1 Proof of Theorem 1.

Consider an arbitrary collusion environment and an auction mechanism $\mathcal{M}$. If the original mechanism $\mathcal{M}$ is robust then the theorem holds trivially. If $\mathcal{M}$ is not robust, one can expand $\mathcal{M}$ by adding prior stages and extra payments. Let $\Pi \equiv -\max_{i\in\mathbb{X}} \max_{v_i\in\mathcal{V}} \max_{x\in\mathbb{X}} \{v_i(x)\}$

1. Each bidder is presented with a decision problem. The auctioneer draws two random allocations x_1^i, x_2^i from $\mathbb{X}$ and a number m_1^i, m_2^i from the interval $\left[0, \max_{x,\omega,i} u_i(x;\omega)\right]$, uniformly and independently across $i \in N$. In a private communication with bidder i the auctioneer asks if i prefers x_1^i and m_1^i, or x_2^i and m_2^i. Bidder i's decision is denoted $c_i(x_1^i, x_2^i, m_1^i, m_2^i) \in \{(x_1^i, m_1^i), (x_2^i, m_2^i)\}$.

2. Each bidder $i \in N$ sends a private message to the auctioneer. The message is of the following form: "The true state is in $\mathcal{E}_i \subseteq \Omega$, and the integer is k_i". Bidder i's message is denoted $(\mathcal{E}_i, k_i)$.

   - Example: bidder i communicates his information about the other bidders' types by sending message $(\mathcal{E}_i, k_i)$ such that $\mathcal{E}_i = \cup_{v_i\in\mathcal{V}}\mathcal{I}_i(\omega)$. If the bidder knows nothing then $\cup_{v_i\in\mathcal{V}}\mathcal{I}_i(\omega) = \Omega$, and his (void) message is (Ω, k_i).

3. Payments.

   (a) We say that decisions $\left(c_j\left(x_1^j, x_2^j, m_1^j, m_2^j\right)\right)_{j\neq i}$ are consistent with proposition $(\mathcal{E}_i, k_i)$ if and only if $\exists\omega \in \mathcal{E}_i$ such that the decision $c_j\left(x_1^j, x_2^j, m_1^j, m_2^j\right)$ is rational $\forall j \neq i$. If $\left(c_j\left(x_1^j, x_2^j, m_1^j, m_2^j\right)\right)_{j\neq i}$ are consistent with $(\mathcal{E}_i, k_i)$ then bidder i,

$i \in N$, gets the following "betting payoff"

$$R \cdot \Pr{}^{-1} \left[\left(c_j \left(x_1^j, x_2^j, m_1^j, m_2^j \right) \right)_{j \neq i} \text{ are consistent with } (\mathcal{E}_i, k_i) \right] - R \tag{16}$$

where $\Pr[\cdot]$ is the prior probability, and $R > 0$; otherwise, he pays a penalty $\Pi \gg R$.

- NB: Any bidder sending a void message gets a zero payoff, since $\Pr \left[\left(c_j \left(x_1^j, x_2^j, m_1^j, m_2^j \right) \right)_{j \neq i} \text{ are consistent with } (\mathcal{E}_i, k_i) \right] = 1$.

(b) Moreover, if $\mathcal{E}_j$, for some j, is informative about the type of bidder i, then bidder i pays Π.

4. With a small exogenous probability $\pi > 0$ one of the bidders is selected at random and only his choice $c_i (x_1^i, x_2^i, m_1^i, m_2^i)$ is implemented. With the complementary probability $1 - \pi$ mechanism $\mathcal{M}$ is played.

The auctioneer withholds all information, including payments, until the auction $\mathcal{M}$ is run and final allocations are decided.[22]

Given the payoffs at stage 4, it is a dominant strategy of any bidder i to submit his true preference $c_i (x_1^i, x_2^i, m_1^i, m_2^i)$.

For any cartel member with non-trivial information the best reply is to report it. Truthful reporting maximizes the reward at stage 3 and does not affect the cartel member's payoff at any other stage due to the assumption of type independence. This implies that a side contract can only be incentive compatible if all information shared is reported in stage 2.

Since information sharing is non-null and hence at least one cartel member submits a non-trivial report, at least one (other) cartel member is punished. this implies a violation of individual rationality of the contract.

We have thus shown that there can be no incentive compatible and individually rational contract in this augmented auction. Finally, when no information is shared all bidders send the trivial message (Ω, k_i) to avoid punishment (due to the presumed independence of types they cannot win this lottery).

[22]This is to ensure that cartel members cannot retaliate during the auction $\mathcal{M}$.

The cost of making the mechanism collusion proof this way is due to stage-2 choices being implemented with probability π. By decreasing the probability the cost can be set to an arbitrarily low level. ■

A.2 Proof of Theorem 2.

The impossibility stated in Theorem 2 amounts to existence, under the said conditions, of a cartel $C \subseteq N$, side contract $(s_C, t_C) \in \mathcal{S}_C \times T_C$, and state of nature $\omega \in \Omega$ such that it is common knowledge among the cartel that the side contract is incentive compatible and individually rational at ω. Consider the all-inclusive cartel $C = N$ and its side contract (s_C, t_C) defined as follows.

1. The manipulation s_C is such that the efficient allocation is achieved and any action leading to a smaller payment changes the allocation.

2. The system of side transfers t_C is given by:

$$\forall i \in C \qquad t_i^* \left(a_C\right) = -\max_{\substack{v_i \in \mathcal{V} \\ x \in \mathbb{X}}} \left\{v_i \left(x\right)\right\}, \tag{17}$$

   for all a_C that C detects to be a deviation from s_C at ω,[23] and $t_i\left(a_C\right) = 0$ otherwise.

Individual Rationality Compared to the equilibrium profile s_C^*, payments to the auctioneer decrease, while the allocation does not change and when s_C is played. There are no side payments on-path, hence the individual rationality constraints (12) hold.

Obedience To check that the incentive constraints (11) hold we show that there is no profitable deviation form s_C. Consider a deviation leading to change in allocation x. Since allocations are public, any such deviation is detected and punished according to (17). Consider a deviation that does not change the allocation. Such deviation does not change the bidder's auction payoff, and hence is not profitable.

[23]Footnote 17 on page 14 explains when deviations are detected.

It remains to observe that the expected surplus from the manipulation is strictly positive, hence $s_C \in \mathcal{S}_C$, and the system of transfers satisfies budget balance and the measurability condition, hence $t_C \in T_C$. ■

A.3 Proof of Theorem 3.

The proof is by contradiction. Suppose the auction is non-robust to collusion via information sharing. Then there exist $C, (\mathcal{I}_i)_{i\in C}, \omega, (s_C, t_C)$ such that the members of C agree to collude. Hence, it is C's common knowledge at $\omega \in \Omega$ that $(s_C, t_C) \in \mathcal{S}_C^\alpha \times T_C^\alpha$ is an interim incentive compatible (11) and individually rational (12) side contract.[24] Observe that the common knowledge partition $\mathcal{I}_C \equiv \wedge_{i\in C}\mathcal{I}_i$ is weakly coarser than $\mathcal{I}_i$ for any $i \in C$. Hence, the common knowledge of incentive compatibility implies for all $i \in C$ and $s_i' \in S_i'(s_C)$, if the latter are non-empty,[25]

$$\mathbb{E}\left[(U_i^* + t_i^*) \circ \left(s_i', s_{C/i}\right) |\mathcal{I}_C(\omega)\right] \quad < \quad \mathbb{E}\left[(U_i^* + t_i^*) \circ s_C \,|\, \mathcal{I}_C(\omega)\right]. \tag{18}$$

By the assumption of limited liability for $i \in C/C^\alpha$, $t_i(a) \geq 0$, $\forall a \in \mathbb{A}$. Hence (18) can be rewritten, for all $i \in C/C^\alpha$ and $s_i' \in S_i$, as

$$\mathbb{E}\left[U_i^* \circ \left(s_i', s_{C/i}\right) |\mathcal{I}_C(\omega)\right] \quad < \quad \mathbb{E}\left[(U_i^* + t_i^*) \circ s_C \,|\, \mathcal{I}_C(\omega)\right]. \tag{19}$$

Summing the incentive compatibility conditions (19) across $i \in C/C^\alpha$ and rearranging the terms we obtain, for all $i \in C/C^\alpha$ and $s_i' \in S_i$,

[24] Note that with reduced public outcome disclosure in a mechanism $\mathcal{M}'$ the respective set of feasible transfers T_C' is a subset of T_C, since contracts are measurable with respect to public outcomes. Hence, if no agreement is possible in $\mathcal{S}_C \times T_C$ then no agreement is possible in $\mathcal{S}_C \times T_C'$.

[25] If all $C, (\mathcal{I}_i)_{i\in C}, \omega, (s_C, t_C)$ where C agree to collude are such that $S_i'(s_C)$ is empty for at least some $i \in C$ then $\max_{s'_{C/C^\alpha} \in S'_{C/C^\alpha}(s_C)} \mathbb{E}\left[\left(\sum_{i\in C/C^\alpha} U_i^* \circ \left(s_i', s_{C/i}\right) - \Delta W_C^* \circ s_C - \sum_{i\in C/C^\alpha} U_i^* \circ s_C^*\right) |\mathcal{I}_C(\omega)\right] = -\infty$ and thus $V_\omega = -\infty$, a contradiction to $V_\omega \geq 0$ for all ω, obtains immediately.

$$\sum_{i\in C/C^{\alpha}} \mathbb{E}\left[U_i^* \circ \left(s_i', s_{C/i}\right) - U_i^* \circ s_C \,|\, \mathcal{I}_C(\omega)\right] \;<\; \sum_{i\in C/C^{\alpha}} \mathbb{E}\left[t_i^* \circ s_C \,|\, \mathcal{I}_C(\omega)\right]. \tag{20}$$

The individual rationality condition (12) for a designated bidder $i \in C^{\alpha}$ can be rewritten as follows

$$-\mathbb{E}\left[t_i^* \circ s_C \,|\, \mathcal{I}_i(\omega)\right] \;\leq\; \mathbb{E}\left[\left(U_i^* \circ s_C - U_i^* \circ s_C^*\right) \,|\, \mathcal{I}_i(\omega)\right]. \tag{21}$$

Since (12) is common knowledge at ω and $\mathcal{I}_C$ weakly coarser than $\mathcal{I}_i$, (21) implies

$$-\mathbb{E}\left[t_i^* \circ s_C \,|\, \mathcal{I}_C(\omega)\right] \;\leq\; \mathbb{E}\left[\left(U_i^* \circ s_C - U_i^* \circ s_C^*\right) \,|\, \mathcal{I}_C(\omega)\right]. \tag{22}$$

Summing (22) across $i \in C^{\alpha}$ we obtain

$$-\sum_{i\in C^{\alpha}} \mathbb{E}\left[t_i^* \circ s_C \,|\, \mathcal{I}(\omega)\right] \;\leq\; \sum_{i\in C^{\alpha}} \mathbb{E}\left[\left(U_i^* \circ s_C - U_i^* \circ s_C^*\right) \,|\, \mathcal{I}_C(\omega)\right]. \tag{23}$$

Given the budget balance constraint, $\sum_{i\in C/C^{\alpha}} t_i(\cdot) \leq -\sum_{i\in C^{\alpha}} t_l(\cdot)$, equation (23) implies

$$\sum_{i\in C/C^{\alpha}} \mathbb{E}\left[t_i^* \circ s_C \,|\, \mathcal{I}_C(\omega)\right] \;\leq\; \sum_{i\in C^{\alpha}} \mathbb{E}\left[\left(U_l^* \circ s_C - U_l^* \circ s_C^*\right) \,|\, \mathcal{I}_C(\omega)\right]. \tag{24}$$

If follows from (20) and (24) that

$$\max_{s'_{C/C^{\alpha}}} \sum_{i\in C/C^{\alpha}} \mathbb{E}\left[U_i^* \circ \left(s_i', s_{C/i}\right) \,|\, \mathcal{I}_C(\omega)\right] \;<\; \mathbb{E}\left[\sum_{i\in C} U_i^* \circ s_C - \sum_{i\in C^{\alpha}} U_l^* \circ s_C^* \,|\, \mathcal{I}_C(\omega)\right], \tag{25}$$

where the maximization in $s'_{C/C^{\alpha}}$ is over $S'_{C/C^{\alpha}}(s_C) \equiv \times_{i\in C/C^{\alpha}} S_i'(s_C)$. By (8), $\Delta W_C^* \circ s_C \equiv \sum_{i\in C} U_i^* \circ s_C - \sum_{i\in C} U_i^* \circ s_C^*$, hence

$$\max_{s'_{C/C^\alpha}} \sum_{i\in C/C^\alpha} \mathbb{E}\left[U_i^* \circ \left(s'_i, s_{C/i}\right) | \mathcal{I}_C(\omega)\right] \;<\; \mathbb{E}\left[\Delta W_C^* \circ s_C + \sum_{i\in C/C^\alpha} U_i^* \circ s_C^* \,|\, \mathcal{I}_C(\omega)\right], \quad (26)$$

which implies that $V_\omega^{\mathcal{M}} < 0$. We have thus obtained a contradiction to $V_\omega^{\mathcal{M}} \geq 0 \; \forall \omega \in \Omega$.
■

A.4 Proof of Theorem 4.

Suppose that $V_{\omega^0} < 0$ for some $\omega^0 \in \Omega$. Then, for some $C \subseteq N$ and $s_C^0 \in \mathcal{S}_C^\alpha$

$$\max_{s'_{C/C^\alpha}} \sum_{i\in C/C^\alpha} \mathbb{E}\left[U_i^* \circ \left(s'_i, s^0_{C/i}\right) | \check{\mathcal{I}}\left(\omega^0\right)\right] \;<\; \mathbb{E}\left[\Delta W_C^* \circ s^0_C + \sum_{i\in C/C^\alpha} U_i^* \circ s_C^* \,|\, \check{\mathcal{I}}\left(\omega^0\right)\right]. \quad (27)$$

Consider a state of nature $\omega = (a_0, v_1, ..v_n)$ such that, for all non-designated bidders $i \in C/C^\alpha$, $\mathbb{E}\left(U_i \circ s_N^*\right) | \check{\mathcal{I}}(\omega) = 0$. Let $s_i(\omega) = s_i^0(\omega^0)$ for all $i \in C$.[26] That is, we consider state ω where the non-designated bidders' types are such that their expected equilibrium payoffs are zero. Thereby the actions prescribed to the non-designated bidders are the same in ω as in the original state ω^0.

Since values are independent, inequality (27) also holds when ω^0 replaced by ω,

$$\max_{s'_{C/C^\alpha}} \sum_{i\in C/C^\alpha} \mathbb{E}\left[U_i^* \circ \left(s'_i, s_{C/i}\right) | \check{\mathcal{I}}(\omega)\right] \;<\; \mathbb{E}\left[\Delta W_C^* \circ s_C + \sum_{i\in C/C^\alpha} U_i^* \circ s_C^* \,|\, \check{\mathcal{I}}(\omega)\right]. \quad (28)$$

We will show that cartel C can agree to collude in ω (see Definition 3 on p. 16).

Consider the side contract (s_C, t_C), where the bidding manipulation s_C is given above and the transfer function t_C is given by (29) - (33). Specifically, when no deviation from s_C is detected (see footnote 17 on page 14):

[26] In the private value setting, there are no allocation externalities, state ω is the one where types of non-designated bidders are the lowest in the sense that $\mathbb{E}\left(U_i \circ s_N^*\right) | \check{\mathcal{I}}(\omega) = 0$.

$$\forall i \in C/C^{\alpha} \qquad t_i\left(a_N\right) = \delta_i + \varepsilon, \tag{29}$$
$$\forall i \in C^{\alpha} \qquad t_i\left(a_N\right) = -\mathbb{E}\left[U_i^* \circ s_C - U_i^* \circ s_C^* \,|\, \check{\mathcal{I}}\left(\omega\right)\right], \tag{30}$$

where C^{α} denotes the set of designated bidders under s_C, ε is defined below, and δ_i, $i \in C/C^{\alpha}$, is i's maximal auction payoff in deviation from s_i,

$$\delta_i \;=\; \max_{s_i' \in S_i} \mathbb{E}\left[U_i^* \circ \left(s_i', s_{C/i}\right) \,|\, \check{\mathcal{I}}\left(\omega\right)\right]. \tag{31}$$

When deviation from s_C is detected at a_N:

$$\forall i \in C/C^{\alpha} \qquad t_i\left(a_N\right) = 0, \tag{32}$$
$$\forall i \in C^{\alpha} \qquad t_i\left(a_N\right) = -K_i, \tag{33}$$

where $K_i = \max_{s_i'} \mathbb{E}\left[U_i^* \circ \left(s_i', s_{C/i}\right) - U_i^* \circ s_C \,|\, \check{\mathcal{I}}\left(\omega\right)\right] + \epsilon$ for a small positive ϵ.

First, we need to verify that the transfers defined in (29) - (33) are feasible, i.e., that $t_C \in T_C^{\alpha}$. In all cases, the non-designated bidders' transfer is non-negative, thus it is left to verify that the budget is balanced. In (32) - (33), the budget balance holds trivially. In the case of (29) - (30), we have:

$$-\sum_{i \in C^{\alpha}} \mathbb{E}\left[t_i^* \circ s_C \,|\, \check{\mathcal{I}}\left(\omega\right)\right] \;=\; \mathbb{E}\left[\Delta W_C^* \circ s_C + \sum_{i \in C/C^{\alpha}} U_i^* \circ s_C^* \,|\, \check{\mathcal{I}}\left(\omega\right)\right] \tag{34}$$
$$> \sum_{i \in C/C^{\alpha}} \mathbb{E}\left[U_i^* \circ \left(\hat{s}_i, s_{C/i}\right) \,|\, \check{\mathcal{I}}\left(\omega\right)\right] \tag{35}$$
$$= \sum_{i \in C/C^{\alpha}} \delta_i, \tag{36}$$

where (34) follows from (30) and (8), (35) from (27), (36) is by the definition of δ_i (31).

Hence we can find $\varepsilon > 0$ such that

$$-\sum_{i\in C^{\alpha}}\mathbb{E}\left[t_i^*\circ s_C\,|\,\check{\mathcal{I}}(\omega)\right] \;=\; \mathbb{E}\left[\Delta W_C^*\circ s_C+\sum_{i\in C/C^{\alpha}}U_i^*\circ s_C^*\,|\,\check{\mathcal{I}}(\omega)\right] \tag{37}$$

$$\geq \sum_{i\in C/C^{\alpha}}\left(\mathbb{E}\left[U_i^*\circ\left(\hat{s}_i,s_{C/i}\right)\,|\,\check{\mathcal{I}}(\omega)\right]+\varepsilon\right) \tag{38}$$

$$= \sum_{i\in C/C^{\alpha}}\left(\delta_i+\varepsilon\right) \tag{39}$$

$$= \sum_{i\in C/C^{\alpha}}\mathbb{E}\left[t_i^*\circ s_C\,|\,\check{\mathcal{I}}(\omega)\right], \tag{40}$$

implying that the budget balance holds. (Equation (38) is by definition (30)). Since the auction is open and information sharing is complete, deviations are detected. Hence, the incentive compatibility condition (11) for bidders $i\in C^{\alpha}$ holds by the construction of transfer in (33), for all s_i',

$$\mathbb{E}\left[\left(U_i^*+t_i^*\right)\circ\left(s_i',s_{C/i}\right)\,|\,\check{\mathcal{I}}(\omega)\right] \;<\; \mathbb{E}\left[U_i^*\circ s_C\,|\,\check{\mathcal{I}}(\omega)\right]. \tag{41}$$

Similarly, the individual rationality condition (12) for bidders $i\in C^{\alpha}$,

$$\mathbb{E}\left[\left(U_i^*+t_i^*\right)\circ s_C\,|\,\check{\mathcal{I}}(\omega)\right] \;\geq\; \mathbb{E}\left[U_i^*\circ s_C^*\,|\,\check{\mathcal{I}}(\omega)\right], \tag{42}$$

holds due to (30). We can then derive the **incentive compatibility** condition (11) for bidders $i\in C/C^{\alpha}$, as follows

$$\forall s_i'\in S_i \qquad \mathbb{E}\left[\left(U_i^*+t_i^*\right)\circ\left(s_i',s_{C/i}\right)\,|\,\check{\mathcal{I}}(\omega)\right]$$

$$= \mathbb{E}\left[U_i^*\circ\left(s_i',s_{C/i}\right)\,|\,\check{\mathcal{I}}(\omega)\right] \tag{43}$$

$$\leq \delta_i<\delta_i+\varepsilon \tag{44}$$

$$= \mathbb{E}\left[t_i^*\circ s_C\,|\,\check{\mathcal{I}}(\omega)\right] \tag{45}$$

$$= \mathbb{E}\left[\left(U_i^*+t_i^*\right)\circ s_C\,|\,\check{\mathcal{I}}(\omega)\right], \tag{46}$$

where (43) follows from (32), (44) from (31), (45) from (29), and (46) holds since $i \in C/C^{\alpha}$ is non-designated by definition of C^{α}.

The individual rationality condition (12) for bidders $i \in C/C^{\alpha}$,

$$\mathbb{E}\left[(U_i^* + t_i^*) \circ s_C \,|\, \check{\mathcal{I}}(\omega)\right] \;\; \geq \;\; \mathbb{E}\left[U_i^* \circ s_C^* \,|\, \check{\mathcal{I}}(\omega)\right], \tag{47}$$

is satisfied because (i) the utility on the left-hand side is non-negative as i is not designated and only the winners pay, (ii) $t_i \geq 0$ due to limited liability for $i \in C/C^{\alpha}$, and (iii) the right-hand side is 0 by construction of the state of nature ω.

We have shown that if the $V_{\omega^0} \leq 0$ for some $\omega^0 \in \Omega$ then a cartel can agree to collude in ω. Hence $V_\omega \geq 0$ for all $\omega \in \Omega$ is necessary for an auction to be robust to collusion via information sharing. ■

A.5 Proof of Proposition 1.

Let us refer to $s_i^V(\omega) = (i, v_i)$, the strategy of self-targeting and bidding the true valuation, as the Vickrey strategy. Proposition 1 says that the profile of Vickrey strategies constitutes a Bayes-Nash equilibrium:

$$s_i^* = s_i^V, \, \forall i \in N. \tag{48}$$

Proof

First, suppose $v_i < b_0$, the types of bidder i that are below the reserve price. It is easy to see that for these types there is no profitable deviation from the Vickrey strategy. The auction rules do not allow for a price lower than b_0. Therefore any strategy that leads to winning with a positive probability makes the bidder type v_i strictly worse off than s_i^V. Any strategy that never leads to winning is payoff-equivalent to s_i^V. Therefore, there is no profitable deviation from the Vickrey strategy for type $v_i < b_0$. For the rest of the proof consider $v_i \geq b_0$.

The proof proceeds in three steps. Steps 1 and 2 identify best replies within two distinct classes of strategies: self-targeting (step 1) and other-targeting (step 2). In both classes bidding the true valuation in the first round is best reply to the residual profile of Vickrey strategies. Next, I calculate payoffs to both strategies under the equilibrium assumption. Finally, step 3 compares the payoffs and concludes that the optimal self-targeting is best reply to $s^V_{N/i}$. Thus we obtain that the Vickrey profile is a non-cooperative equilibrium.

Step 1. Self-targeting

By Lemma 1, truthful bidding is weakly dominant in the class of strategies where the bidder self-targets. The corresponding payoffs ex post $\left(U_i \circ s^V_N\right)(\omega)$ and in expectation $\mathbb{E}^{\,|\,\mathcal{I}_i(\omega)} U_i\left(s^V, s^V_{N/i}\right)$ are as follows:

$$\left(U_i \circ s^V_N\right)(\omega) = \left[v_i - \max\left\{v_k,\, k \in N^0_{-i}\right\}\right]_+ \tag{49}$$

$$\mathbb{E}^{\,|\,\mathcal{I}_i(\omega)}\left(U_i \circ s^V_N\right)(\omega) = \begin{cases} \int_0^{v_i}\left(v_i - \max\left\{w, b_0\right\}\right) dF^{n-1}(w), & \text{if } v_i \geq b_0 \\ 0, & \text{if } v_i < b_0 \end{cases} \tag{50}$$

$$= (v_i - b_0)\, F^{n-1}(b_0) + \int_{b_0}^{v_i} (v_i - w)\, dF^{n-1}(w) \tag{51}$$

Step 2. Other-targeting

At this step, we consider the strategies where another bidder is targeted. First, observe that in the class of other-targeting strategies, any choice of target, random or deterministic, is optimal due to the ex ante symmetry. Suppose $T(i) = j \neq i$. Conditional on targeting j, $\beta_i(\omega) = v_i$, $\forall \omega \equiv (v, \xi) \in \Omega$ is optimal. To prove the claim, fix the preliminary bid β_i and consider all possible constellations between β_i, the target's (preliminary and final)[27] bid b_j and the highest residual (preliminary and final) bid $\hat{b}$. Then, I find that in those constellations where i wins, the price is less than β_i.

[27]Under the equilibrium assumption, bidders in N/i self-target, which implies that their preliminary bids become final.

Let $\hat{b} \equiv \left\{b_k,\ k \in N^0_{-\{i,j\}}\right\}$, the highest bid among all players (including the seller), when i and his target j are excluded. (Under the equilibrium assumption all preliminary bids become final). i wins if and only if $\hat{b} < \beta_i \leq \beta_j = v_j$ and his winning payoff is $v_i - \hat{b} > 0$ since the price is $\hat{b}$.

Payoff ex post $\left(U_i \circ \left(\overrightarrow{s_i}, s^V_{N/i}\right)\right)(\omega)$, where $\overrightarrow{s_i}$ denotes the other-targeting strategy, equals

$$\begin{cases} v_i - \hat{b}, & \text{if } \hat{b} < \beta_i < v_j \\ 0, & \text{otherwise} \end{cases} \tag{52}$$

where $T(i) = j$, $\hat{b} = \left\{b_k,\ k \in N^0_{-\{i,j\}}\right\}$. Payoff in expectation:

$$\mathbb{E}\left[U_i \circ \left(\overrightarrow{s_i}, s^V_{N/i}\right) \mid \mathcal{I}_i(\omega)\right] = \int_{\beta_i}^{+\infty} \int_0^{\beta i} (v_i - \max\{w, b_0\})\, dF(v_j)\, dF^{n-2}(w) \tag{53}$$

$$= (1 - F(\beta_i)) \left((v_i - b_0) F^{n-2}(b_0) + \int_{b_0}^{\beta_i} (v_i - w)\, dF^{n-2}(w)\right) \tag{54}$$

Observe that since bidder i targets up and the rest of the bidders self-target, the seller's bid is $b_0 = \beta_0 = c^{-1}(v_0)$, as in Myerson (1981).

Step 3.

To obtain that $\mathbb{E}\left[U_i \circ s^V_N \mid \mathcal{I}_i(\omega)\right] > \mathbb{E}\left[U_i \circ \left(\overrightarrow{s_i}, s^V_{N/i}\right) \mid \mathcal{I}_i(\omega)\right]$ for all i, v_i consider the difference in the expected payoffs between the two strategies for type $v_i \geq b_0$:

$$\mathbb{E}\left[U_i\left(s^V_i(v_i), s^V_{N/i}\right) - U_i\left(\overrightarrow{s_i}(v_i), s^V_{N/i}\right) \mid \mathcal{I}_i(\omega)\right] \tag{55}$$

substituting for (51) and (54):

$$\begin{aligned} = \quad & (v_i - b_0) F^{n-1}(b_0) + \int_{b_0}^{v_i} (v_i - w)\, dF^{n-1}(w) \\ & - (1 - F(\beta_i)) \left((v_i - b_0) F^{n-2}(b_0) + \int_{b_0}^{\beta_i} (v_i - w)\, dF^{n-2}(w)\right) \end{aligned} \tag{56}$$

rearranging terms:

$$
\begin{aligned}
= \quad & \left(v_i - b_0\right) F^{n-2}\left(b_0\right)\left(F\left(b_0\right) - 1 + F\left(\beta_i\right)\right) \\
& + \int_{b_0}^{\beta_i} \left(v_i - w\right)\left(\frac{n-1}{n-2} F\left(w\right) - 1 + F\left(\beta_i\right)\right) dF^{n-2}\left(w\right) \\
& + \int_{\beta_i}^{v_i} \left(v_i - w\right) dF^{n-1}\left(w\right) \qquad (57)
\end{aligned}
$$

since $v_i \geq \beta_i \geq b_0$:[28]

$$
\begin{aligned}
\geq \quad & \left(v_i - b_0\right) F^{n-2}\left(b_0\right)\left(F\left(b_0\right) - 1 + F\left(b_0\right)\right) \\
& + \int_{b_0}^{\beta_i} \left(v_i - w\right)\left(\frac{n-1}{n-2} F\left(w\right) - 1 + F\left(w\right)\right) dF^{n-2}\left(w\right) \\
& + \int_{\beta_i}^{v_i} \left(v_i - w\right) dF^{n-1} \qquad (58)
\end{aligned}
$$

rearranging terms we obtain:

$$
\begin{aligned}
= \quad & \left(v_i - b_0\right) F^{n-2}\left(b_0\right)\left(2F\left(b_0\right) - 1\right) \\
& + \int_{b_0}^{\beta_i} \left(v_i - w\right)\left(\frac{2n-3}{n-2} F\left(w\right) - 1\right) dF^{n-2}\left(w\right) \\
& + \int_{\beta_i}^{v_i} \left(v_i - w\right) dF^{n-1} > 0 \qquad (59)
\end{aligned}
$$

The inequality holds as soon as $F\left(b_0\right) \geq \frac{1}{2}$, which follows from the assumption that f is non-increasing.[29] The inequality in (59) implies that the Vickrey strategy yields a greater expected payoff, and s^V is thus a best reply to $s^V_{N/i}$, and s^V_N is a strict Bayes-Nash equilibrium. ■

[28] $\beta_i > v_i$ is never best reply.

[29] $F\left(b_0\right) \geq \frac{1}{2} \Leftrightarrow b_0 \equiv c^{-1}\left(v_0\right) \geq F^{-1}\left(\frac{1}{2}\right)$ (since c is strictly increasing($\Leftrightarrow c^{-1}\left(v_0\right) \geq c^{-1}\left(0\right) \geq F^{-1}\left(\frac{1}{2}\right) \equiv x_{med} \Leftrightarrow 0 \geq c\left(x_{med}\right) = x_{med} - \frac{1-F(x_{med})}{f(x_{med})} = x_{med} - \frac{1}{2f(x_{med})} \Leftrightarrow x_{med} f(x_{med}) \leq \frac{1}{2} \Leftarrow f$ is non-increasing.

A.6 Proof of Proposition 2.

Proof. To prove that the ATB auction is robust to collusion via information sharing, we show that it satisfies the sufficient condition of Theorem 3. That is, we show that for all states $\omega \in \Omega$ the value below is non-negative:

$$\min_{\substack{C \subseteq N, \\ s_C \in \mathcal{S}_C}} \max_{\substack{s'_{C/C^\alpha} \\ \in S'_{C/C^\alpha}(s_C)}} \mathbb{E}\left[\left(\sum_{i \in C/C^\alpha} U_i^* \circ \left(s'_i, s_{C/i}\right) - \Delta W_C^* \circ s_C - \sum_{i \in C/C^\alpha} U_i^* \circ s_C^*\right) | \mathcal{I}_C(\omega)\right]. \quad (60)$$

In the setting with private values (no externalities), the non-designated bidders' payoff under s_C is zero, $\mathbb{E}\left[\sum_{i \in C/C^\alpha} U_i^* \circ s_C | \mathcal{I}_C(\omega)\right] = 0$. Moreover, since $|C^\alpha| = 1$,(60) is equivalent to

$$\min_{\substack{C \subseteq N, \\ s_C \in \mathcal{S}_C}} \max_{\substack{s'_{C/\{l\}} \\ \in S'_{C/\{l\}}(s_C)}} \mathbb{E}\left[\left(\sum_{i \in C/\{l\}} U_i^* \circ \left(s'_i, s_{C/i}\right) - U_l^* \circ s_C + U_l^* \circ s_C^*\right) | \mathcal{I}_C(\omega)\right], \quad (61)$$

where l is the designated bidder in a cartel C given the manipulation $s_C = (\beta_i, \tau_i)_{i \in C} \in S_C^\alpha$. Consider cartel member $i \neq l$ and deviation $s'_i = (\beta'_i, \tau'_i) = (v_i, l)$ where i bids his true value and targets the designated bidder l. This deviation affects the outcome of the auction if[30] and only if[31] bidder i wins as a result of his deviation; let us denote this event $\mathcal{E}_i$. In $\mathcal{E}_i$, β_l (part of $s_C(\omega)$) must be the highest preliminary bid and i's beta-bid $\beta'_i(v_i) = v_i$ must be the second-highest (otherwise i's final bid could not have been the highest, which is necessary for winning).

For any $\omega \in \mathcal{E}_i$ such that $\left(\mathcal{O}_{\mathbb{X}l} \circ \left(s_C, s_{N/C}^*\right)\right)(\omega) = \emptyset$ (bidder l is not the winner), $\left(U_l^* \circ s_C\right)(\omega) = 0$ and hence

$$U_i^* \circ \left(s'_i, s_{C/i}\right) - U_l^* \circ s_C + U_l^* \circ s_C^* = U_i^* \circ \left(s'_i, s_{C/i}\right) + U_l^* \circ s_C^* \geq 0.$$

For any $\omega \in \mathcal{E}_i$ such that $\left(\mathcal{O}_{\mathbb{X}l} \circ \left(s_C, s_{N/C}^*\right)\right)(\omega) \neq \emptyset$ (bidder l is the winner), $s_C(\omega)$ must

[30] If i never wins by targeting l then s_C does not create a surplus, $s_C \notin \mathcal{S}_C$, and the sufficient condition holds trivially.

[31] One can verify that it is not possible that the deviation leads to l winning with a price different from the price under s_C.

be such that l targets himself and pays the second-highest final bid in $\left(\mathcal{O} \circ \left(s_C, s^*_{N/C}\right)\right)(\omega)$, let us denote it b_2. Then i's contribution to the collusion surplus is $\max\{v_i - b_2, 0\}$, which is what he gets from deviation to s'_i

$$U^*_i \circ \left(s'_i, s_{C/i}\right) - U^*_l \circ s_C + U^*_l \circ s^*_C = \max\{v_i - b_2, 0\} - \max\{v_i - b_2, 0\} = 0.$$

The statement follows immediately. ■

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