Working Paper in Economics

201904

August 2019

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July 19, 2019

Abstract

This paper shows the recent success of valuation risk (time-preference shocks in Epstein-Zin utility) in resolving asset pricing puzzles rests sensitively on an undesirable asymptote that occurs because the preference specification fails to satisfy a key restriction on the weights in the Epstein-Zin time-aggregator. When we revise the preferences to satisfy the restriction in a simple asset pricing model, the puzzles resurface. However, when estimating a sequence of Bansal-Yaron long-run risk models, we find valuation risk under the revised specification consistently improves the ability of the models to match asset price and cash-flow dynamics.

Keywords: Epstein-Zin Utility; Asset Pricing; Equity Premium Puzzle; Risk-Free Rate Puzzle
JEL Classifications: D81; G12

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1 INTRODUCTION

In standard asset pricing models, uncertainty enters through the supply side of the economy, either through endowment shocks in a Lucas (1978) tree model or productivity shocks in a production economy model. Recently, several papers introduced demand side uncertainty or "valuation risk" as a potential explanation of key asset pricing puzzles (Albuquerque et al., 2016, 2015; Creal and Wu, 2017; Maurer, 2012; Nakata and Tanaka, 2016; Schorfheide et al., 2018). In macroeconomic parlance, valuation risk is typically referred to as a discount factor or time preference shock.[1]

The literature contends valuation risk is an important determinant of key asset pricing moments when it is embedded in Epstein and Zin (1991) recursive preferences. We show the success of valuation risk rests on an undesirable asymptote that permeates the determination of asset prices. The influence of the asymptote is easily identified in a stylized model. In that model, an intertemporal elasticity of substitution (IES) marginally above one predicts an arbitrarily large equity premium and an arbitrarily low risk-free rate, while an IES slightly below one predicts the opposite results. The asymptote significantly affects equilibrium outcomes even when the IES is well above unity.

In a business cycle model, de Groot et al. (2018) show that with Epstein-Zin preferences, time-varying weights in a CES time-aggregator must sum to 1 to prevent an undesirable asymptote from determining equilibrium outcomes. The current specification in the literature fails to impose this restriction. de Groot et al. (2018) propose an alternative (henceforth, the "revised specification") that eliminates the asymptote and ensures that preferences are well-defined when the IES is one.

This paper makes two key contributions to the literature. First, it analytically shows the change to the preference specification profoundly alters the equilibrium determination of asset prices. For example, the same IES and risk aversion (RA) parameters can lead to very different values for the equity premium and risk-free rate and comparative statics, such as the response of the equity premium to the IES, switch sign. Second, it empirically re-evaluates of the role of valuation risk in explaining asset pricing and cash-flow moments. We find after estimating a sequence of models under the revised specification, the role and contribution of valuation risk change dramatically.

For intuition, consider the log-stochastic discount factor (SDF) under Epstein-Zin preferences

$$\hat{m}_{t+1} = \theta \log \beta + \theta(\hat{a}_t - \omega \hat{a}_{t+1}) - (\theta/\psi)\Delta \hat{c}_{t+1} + (\theta - 1)\hat{r}_{y,t+1}, \quad (1)$$

where the first, third, and fourth terms—the subjective discount factor (β), log-consumption growth ($\Delta \hat{c}_{t+1}$), and the log-return on the endowment ($\hat{r}_{y,t+1}$)—are all standard in this class of asset pricing models. The second term captures valuation risk, where $\hat{a}_t$ is a time preference shock. In the current literature, $\omega = 0$. Once we revise the preferences and re-derive the log-SDF, we find $\omega = \beta$.

[1]Time preference shocks have been widely used in the macro literature (e.g., Christiano et al. (2011); Eggertsson and Woodford (2003); Justiniano and Primiceri (2008); Rotemberg and Woodford (1997); Smets and Wouters (2003)).

When we apply this single alteration to the model, the asset pricing predictions are starkly different.

The asymptote in the current valuation risk specification is related to the preference parameter $\theta \equiv (1-\gamma)/(1-1/\psi)$ that enters the log-SDF, where γ is RA and ψ is the IES. Under constant relative risk aversion (CRRA) preferences, $\gamma = 1/\psi$. In this case, $\theta = 1$ and the log-SDF becomes

$$\hat{m}_{t+1} = \log\beta + (\hat{a}_t - \omega\hat{a}_{t+1}) - \Delta\hat{c}_{t+1}/\psi. \tag{2}$$

The return on the endowment drops out of (1), so the log-SDF is simply composed of the subjective discount factor and consumption growth terms. The advantage of Epstein-Zin preferences is that they decouple γ and ψ, so it is possible to simultaneously have high RA and a high IES. However, there is a nonlinear relationship between θ and ψ, as shown in figure 1. A vertical asymptote occurs at $\psi = 1$: θ tends to infinity as ψ approaches 1 from below while the opposite occurs as ψ approaches 1 from above. When the IES equals 1, θ is undefined. In addition to the vertical asymptote in θ, there is also a horizontal asymptote at $1-\gamma$ as the IES becomes perfectly elastic.

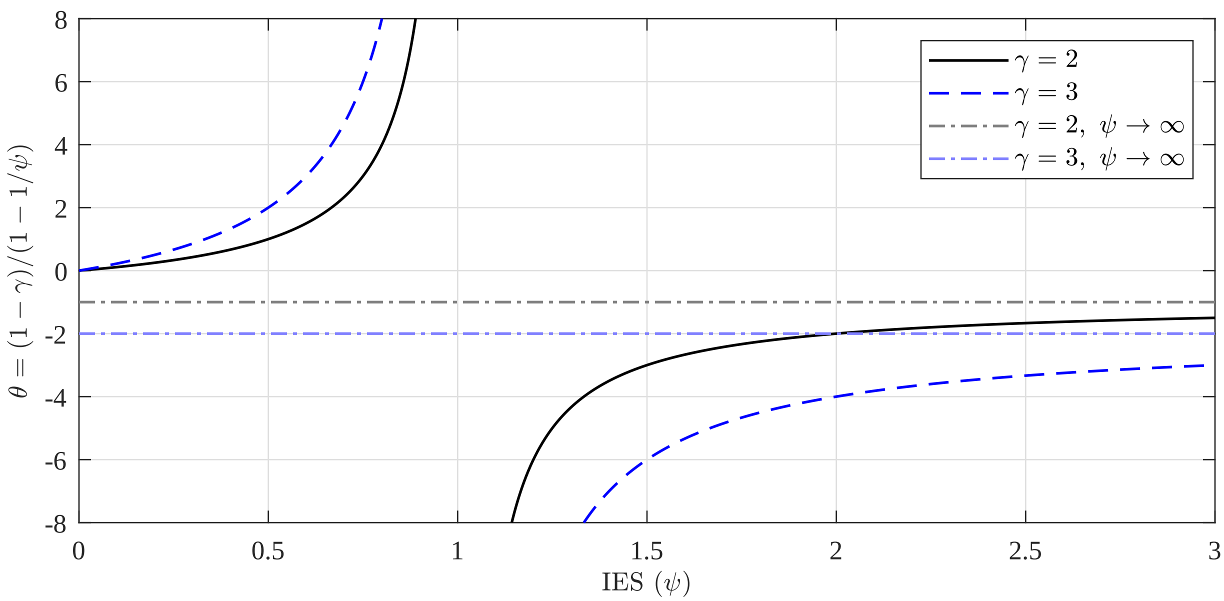

Figure 1: Preference parameter θ in the stochastic discount factor from a model with Epstein-Zin preferences.

Under Epstein and Zin (1989) preferences and the generalization in de Groot et al. (2018) to include valuation risk, the asymptote in figure 1 does not affect asset prices. There is a well-defined equilibrium when the IES equals 1 and asset prices are robust to small variations in the IES. Continuity is preserved because the weights in the time-aggregator always sum to unity. An alternative interpretation is that the time-aggregator maintains the well-known property that a CES aggregator tends to a Cobb-Douglas aggregator as the elasticity approaches 1. The current specification violates the restriction on the weights so the limiting properties of the CES aggregator break down. As a result, the asymptote permeates key asset pricing moments, even when the IES is well above 1.

Taken at face value, the asymptote that occurs with the current specification can resolve the equity premium (Mehra and Prescott, 1985) and risk-free rate (Weil, 1989) puzzles in our baseline model with *i.i.d.* cash-flow risk. When we revise the preference specification to satisfy the restriction, valuation risk has a smaller role, RA is implausibly high, and the puzzles resurface because there is no longer an asymptote. However, when estimating a sequence of long-run risk models using a simulated method of moments procedure, we find valuation risk under the revised specification consistently improves the ability of the models to match asset price and cash-flow dynamics.

We begin by estimating the Bansal and Yaron (2004) long-run risk model (without time-varying uncertainty) without valuation risk and find it significantly under-predicts the standard deviation of the risk-free rate, even when these moments are targeted. When we introduce valuation risk, it accounts for roughly 40% of the equity premium, but at the expense of over-predicting the standard deviation of the risk-free rate. After targeting the risk-free rate dynamics, valuation risk only accounts for about 5% of the equity premium. Therefore, we find it is crucial to target these dynamics to accurately measure the contribution of valuation risk. While valuation risk (with or without the targeted risk-free rate moments) improves the fit of the long-run risk model, the model still fails a test of over-identifying restrictions. This is because the model fairs poorly in matching the low predictability of consumption growth from the price-dividend ratio, the high standard deviation of dividend growth, and the weak correlation between dividend growth and equity returns in the data.

We consider two extensions that improve the model's fit: (1) an interaction term between valuation and cash-flow risk (a proxy for general equilibrium demand effects) following Albuquerque et al. (2016) (henceforth, "Demand" model) and (2) stochastic volatility on cash-flow risk as in Bansal and Yaron (2004) (henceforth, "SV" model). In a horse race between these extensions, we find the Demand model wins and passes the over-identifying restrictions test at the 5% level. However, the two extensions are complements and the combined model passes the test at the 10% level. This is because the demand extension lowers the correlation between dividend growth and equity returns, while the SV extension offsets the effect of higher valuation risk on risk-free rate dynamics.

It is common in the asset pricing literature to estimate models using a simulated method of moments procedure (eg., Adam et al., 2016; Albuquerque et al., 2016; Andreasen and Jørgensen, 2019). We build on the existing methodology in two ways. One, we run Monte Carlo estimations of the model and calculate standard errors using different sequences of shocks, whereas estimates in the literature are typically based on a particular sequence of shocks. This approach allows us to obtain more precise estimates and account for differences between the asymptotic and sampling distributions of the parameters. Two, we use a rigorous two-step procedure to find the global optimum that uses simulated annealing to obtain candidate draws and then recursively applies a nonlinear solver to each of the candidates. We find that without applying such rigor to this class of models, the algorithm would settle on local optima and potentially lead to incorrect inferences.

Related Literature This paper builds on the growing literature that examines the role of valuation risk in asset pricing models. Maurer (2012) and Albuquerque et al. (2016) were the first. They adopt the current preference specification and find valuation risk accounts for key asset pricing moments, such as the equity premium. Albuquerque et al. (2016) also focus on resolving the correlation puzzle (Campbell and Cochrane, 1999). Schorfheide et al. (2018) use a Bayesian mixed-frequency approach that allows them to target entire time series, rather than specific moments. They focus on one model with three SV processes. We examine in-depth the role of valuation risk by estimating a sequence of increasingly rich models. A notable difference from our results is that Schorfheide et al. (2018) find a limited role for the general equilibrium demand channel. Creal and Wu (2017) focus on bond premia. They also use the current specification, but valuation risk is tied to consumption and inflation and does not have an independent stochastic element. They find the slope of the yield curve is largely explained by valuation risk, given an IES estimate equal to 1.02.

Nakata and Tanaka (2016) and Kliem and Meyer-Gohde (2018) study term premia in a New Keynesian model using the current specification. The former calibrate the IES to 0.11 and generate a negative term premia. The latter estimate the IES with a prior in the $[0, 1]$ range and obtain a value of 0.09. Both findings are a consequence of the asymptote, as we show analytically. In contrast with the literature, Rapach and Tan (2018) and Bianchi et al. (2018) use the revised specification and estimate a real business cycle model. They find valuation risk can still explain a large portion of the term premium because demand shocks interact with the production side of the economy.[2]

The paper proceeds as follows. Section 2 describes the baseline model and the current and revised preference specifications. Section 3 analytically shows why asset prices depend so dramatically on the way valuation risk enters the Epstein-Zin utility function. Section 4 describes the data and estimation methodology. Section 5 quantifies the effects of the valuation risk specification in our baseline model with *i.i.d.* cash-flow risk. Section 6 estimates the standard long-run risk model with and without valuation risk. Section 7 extends the long-run risk model to include valuation risk shocks to cash-flow growth and stochastic volatility on cash-flow risk. Section 8 concludes.

2 BASELINE ASSET-PRICING MODEL

We begin by describing our baseline model. Each period t denotes 1 month. There are two assets: an endowment share, $s_{1,t}$, that pays income, y_t, and is in fixed unit supply, and an equity share, $s_{2,t}$, that pays dividends, d_t, and is in zero net supply. The agent chooses $\{c_t, s_{1,t}, s_{2,t}\}_{t=0}^{\infty}$ to maximize

$$U_t^C = [(1-\beta)c_t^{(1-\gamma)/\theta} + a_t^C \beta (E_t[(U_{t+1}^C)^{1-\gamma}])^{1/\theta}]^{\theta/(1-\gamma)}, \quad 1 \neq \psi > 0, \tag{3}$$

[2] Two other strands of the literature have interesting connections to our work. One, disaster risk (see Barro, 2009 and Gourio, 2012) can generate variation in the stochastic discount factor analogous to valuation risk. Two, Bansal et al. (2014), identify "discount rate risk" as a component of risk premia distinct from cash-flow and volatility risks.

as used in the current (C) asset pricing literature, or

$$U_t^R = \begin{cases} [(1 - a_t^R \beta) c_t^{(1-\gamma)/\theta} + a_t^R \beta (E_t[(U_{t+1}^R)^{1-\gamma}])^{1/\theta}]^{\theta/(1-\gamma)}, & \text{for } 1 \neq \psi > 0, \\ c_t^{1-a_t^R \beta} (E_t[(U_{t+1}^R)^{1-\gamma}])^{a_t^R \beta/(1-\gamma)}, & \text{for } \psi = 1, \end{cases} \tag{4}$$

as in the revised (R) specification of de Groot et al. (2018), where E_t is the mathematical expectation operator conditional on information available in period t. The time-preference shocks are denoted $a_t^C > 0$ and $0 < a_t^R < 1/\beta$.[3,4] The key difference between the preferences is as follows:

> *The time-varying weights of the time-aggregator in* (3), $(1-\beta)$ *and* $a_t^C \beta$*, do not sum to* 1*, whereas the weights in* (4), $(1 - a_t^R \beta)$ *and* $a_t^R \beta$*, do sum to* 1.

The representative agent's choices are constrained by the flow budget constraint given by

$$c_t + p_{y,t} s_{1,t} + p_{d,t} s_{2,t} = (p_{y,t} + y_t) s_{1,t-1} + (p_{d,t} + d_t) s_{2,t-1}, \tag{5}$$

where $p_{y,t}$ and $p_{d,t}$ are the endowment and dividend claim prices. The optimality conditions imply

$$E_t[m_{t+1}^j r_{y,t+1}] = 1, \quad r_{y,t+1} \equiv (p_{y,t+1} + y_{t+1})/p_{y,t}, \tag{6}$$

$$E_t[m_{t+1}^j r_{d,t+1}] = 1, \quad r_{d,t+1} \equiv (p_{d,t+1} + d_{t+1})/p_{d,t}, \tag{7}$$

where $j \in \{C, R\}$, $r_{y,t+1}$ and $r_{d,t+1}$ are the gross returns on the endowment and dividend claims,

$$m_{t+1}^C \equiv a_t^C \beta \left(\frac{c_{t+1}}{c_t}\right)^{-1/\psi} \left(\frac{(V_{t+1}^C)^{1-\gamma}}{E_t[(V_{t+1}^C)^{1-\gamma}]}\right)^{1-\frac{1}{\theta}}, \tag{8}$$

$$m_{t+1}^R \equiv a_t^R \beta \left(\frac{1 - a_{t+1}^R \beta}{1 - a_t^R \beta}\right) \left(\frac{c_{t+1}}{c_t}\right)^{-1/\psi} \left(\frac{(V_{t+1}^R)^{1-\gamma}}{E_t[(V_{t+1}^R)^{1-\gamma}]}\right)^{1-\frac{1}{\theta}}, \tag{9}$$

and V_t^j is the value function that solves the agent's constrained optimization problem.

To permit an approximate analytical solution, we rewrite (6) and (7) as follows

$$E_t[\exp(\hat{m}_{t+1}^j + \hat{r}_{y,t+1})] = 1, \tag{10}$$

$$E_t[\exp(\hat{m}_{t+1}^j + \hat{r}_{d,t+1})] = 1, \tag{11}$$

[3] Kraft and Seifried (2014) prove the continuous-time analog of recursive preferences—known as stochastic differential utility (Duffie and Epstein, 1992)—is the continuous-time limit of recursive utility if the weights of the time-aggregator sum to 1. Kollmann (2016) introduces a time-varying discount factor in an Epstein-Zin setting similar to our revised specification. In that setup, the discount factor is a function of endogenously determined consumption.

[4] In the literature, a_t^C typically hits current utility, rather than the risk aggregator. However, with a small change in the timing convention of the preference shock, (3) is isomorphic to the specification used in the literature. We use the specification in (3) because it better facilitates a comparison with the revised preferences. See Appendix A for details.

where $\hat{m}^j_{t+1}$ is defined in (1) and $\hat{a}_t \equiv \hat{a}^C_t \approx \hat{a}^R_t/(1-\beta)$ so the shocks in the current and revised models are directly comparable. The common time preference shock, $\hat{a}_{t+1}$, evolves according to

$$\hat{a}_{t+1} = \rho_a \hat{a}_t + \sigma_a \varepsilon_{a,t+1},\ \varepsilon_{a,t+1} \sim \mathbb{N}(0,1), \tag{12}$$

where $0 \leq \rho_a < 1$ is the persistence of the process, $\sigma_a \geq 0$ is the shock standard deviation, and a hat denotes a log variable. We then apply a Campbell and Shiller (1988) approximation to obtain

$$\hat{r}_{y,t+1} = \kappa_{y0} + \kappa_{y1}\hat{z}_{y,t+1} - \hat{z}_{y,t} + \Delta\hat{y}_{t+1}, \tag{13}$$

$$\hat{r}_{d,t+1} = \kappa_{d0} + \kappa_{d1}\hat{z}_{d,t+1} - \hat{z}_{d,t} + \Delta\hat{d}_{t+1}, \tag{14}$$

where $\hat{z}_{y,t+1}$ is the log price-endowment ratio, $\hat{z}_{d,t+1}$ is the log price-dividend ratio, and

$$\kappa_{y0} \equiv \log(1+\exp(\hat{z}_y)) - \kappa_{y1}\hat{z}_y, \quad \kappa_{y1} \equiv \exp(\hat{z}_y)/(1+\exp(\hat{z}_y)), \tag{15}$$

$$\kappa_{d0} \equiv \log(1+\exp(\hat{z}_d)) - \kappa_{d1}\hat{z}_d, \quad \kappa_{d1} \equiv \exp(\hat{z}_d)/(1+\exp(\hat{z}_d)), \tag{16}$$

are constants that are functions of the steady-state price-endowment and price-dividend ratios.

To close the model, the processes for log-endowment and log-dividend growth are given by

$$\Delta\hat{y}_{t+1} = \mu_y + \sigma_y \varepsilon_{y,t+1},\ \varepsilon_{y,t+1} \sim \mathbb{N}(0,1), \tag{17}$$

$$\Delta\hat{d}_{t+1} = \mu_d + \pi_{dy}\sigma_y\varepsilon_{y,t+1} + \psi_d\sigma_y\varepsilon_{d,t+1},\ \varepsilon_{d,t+1} \sim \mathbb{N}(0,1), \tag{18}$$

where μ_y and μ_d are the steady-state growth rates, $\sigma_y \geq 0$ and $\psi_d\sigma_y \geq 0$ are the shock standard deviations, and π_{dy} determines the covariance between consumption and dividend growth. Asset market clearing implies $s_{1,t} = 1$ and $s_{2,t} = 0$, so the aggregate resource constraint is $\hat{c}_t = \hat{y}_t$.

Equilibrium includes sequences of quantities $\{\hat{c}_t\}_{t=0}^\infty$, prices $\{\hat{m}_{t+1}, \hat{z}_{y,t}, \hat{z}_{d,t}, \hat{r}_{y,t+1}, \hat{r}_{d,t+1}\}_{t=0}^\infty$ and exogenous variables $\{\Delta\hat{y}_{t+1}, \Delta\hat{d}_{t+1}, \hat{a}_{t+1}\}_{t=0}^\infty$ that satisfy (1), (10)-(14), (17), (18), and the resource constraint, given the state of the economy, $\{\hat{a}_t\}$, and sequences of shocks, $\{\varepsilon_{y,t}, \varepsilon_{d,t}, \varepsilon_{a,t}\}_{t=1}^\infty$.

We posit the following solutions for the price-endowment and price-dividend ratios:

$$\hat{z}_{y,t} = \eta_{y0} + \eta_{y1}\hat{a}_t, \quad \hat{z}_{d,t} = \eta_{d0} + \eta_{d1}\hat{a}_t, \tag{19}$$

where $\hat{z}_y = \eta_{y0}$ and $\hat{z}_d = \eta_{d0}$. We solve the model with the method of undetermined coefficients. Appendix B derives the SDF, a Campbell-Shiller approximation, the solution, and key asset prices.

3 INTUITION

This section develops intuition for why the valuation risk specification has such large effects on the model predictions. To simplify the exposition, we consider different stylized shock processes.

3.1 CONVENTIONAL MODEL First, it is useful to review the role of Epstein-Zin preferences and the separation of the RA and IES parameters in matching the risk-free rate and equity premium. For simplicity, we remove valuation risk ($\sigma_a = 0$) and assume endowment/dividend risk is perfectly correlated ($\psi_d = 0$; $\pi_{dy} = 1$). The average risk-free rate and average equity premium are given by

$$E[\hat{r}_f] = -\log\beta + \mu_y/\psi + ((1/\psi - \gamma)(1-\gamma) - \gamma^2)\sigma_y^2/2, \tag{20}$$

$$E[ep] = \gamma\sigma_y^2, \tag{21}$$

where the first term in (20) is the subjective discount factor, the second term accounts for endowment growth, and the third term accounts for precautionary savings. Endowment growth creates an incentive for agents to borrow in order to smooth consumption. Since both assets are in fixed supply, the risk-free rate must be elevated to deter borrowing. When the IES, ψ, is high, agents are willing to accept higher consumption growth so the interest rate required to dissuade borrowing is lower. Therefore, the model requires a fairly high IES to match the low risk-free rate in the data.

With CRRA preferences, higher RA lowers the IES and pushes up the risk-free rate. With Epstein-Zin preferences, these parameters are independent, so a high IES can lower the risk-free rate without lowering RA. Notice the equity premium only depends on RA. Therefore, the model generates a low risk-free rate and modest equity premium with sufficiently high RA and IES parameter values. Of course, there is an upper bound on what constitute reasonable RA and IES values, which is the source of the risk-free rate and equity premium puzzles. Other prominent features such as long-run risk and stochastic volatility à la Bansal and Yaron (2004) help resolve these puzzles.

3.2 VALUATION RISK MODEL Now consider an example where we remove cash flow risk ($\sigma_y = 0$; $\mu_y = \mu_d$) and also assume the time preference shocks are *i.i.d.* ($\rho_a = 0$). Under these assumptions, the assets are identical so $(\kappa_{y0}, \kappa_{y1}, \eta_{y0}, \eta_{y1}) = (\kappa_{d0}, \kappa_{d1}, \eta_{d0}, \eta_{d1}) \equiv (\kappa_0, \kappa_1, \eta_0, \eta_1)$.

Current Specification We first solve the model with the current preferences, so the SDF is given by (1) with $\omega = 0$. In this case, the average risk-free rate and average equity premium are given by

$$E[\hat{r}_f] = -\log\beta + \mu_y/\psi + (\theta - 1)\kappa_1^2\eta_1^2\sigma_a^2/2, \tag{22}$$

$$E[ep] = (1-\theta)\kappa_1^2\eta_1^2\sigma_a^2. \tag{23}$$

It is also straightforward to show the log-price-dividend ratio is given by $\hat{z}_t = \eta_0 + \hat{a}_t$ (i.e., the loading on the preference shock, η_1, is 1). Therefore, when the agent becomes more patient and $\hat{a}_t$ rises, the price-dividend ratio rises one-for-one on impact and returns to the stationary equilibrium in the next period. Since η_1 is independent of the IES, there is no endogenous mechanism that prevents the asymptote in θ from influencing the risk-free rate or equity premium. It is easy to see from (16) that $0 < \kappa_1 < 1$. Therefore, θ dominates the average risk-free rate and average equity premium when the IES is near 1. The following result describes the comparative statics with the IES:

> *As ψ approaches 1 from above, θ tends to $-\infty$. As a result, the average risk-free rate tends to $-\infty$ while the average equity premium tends to $+\infty$.*

This key finding illustrates why valuation risk seems like such an attractive feature for resolving the risk-free rate and equity premium puzzles. As the IES tends to 1 from above, θ becomes increasingly negative, which dominates other determinants of the risk-free rate and equity premium. In particular, with an IES slightly above 1, the asymptote in θ causes the average risk-free rate to become arbitrarily small, while making the average equity premium arbitrarily large. Bizarrely, an IES marginally below 1 (a popular value in the macro literature), generates the opposite predictions. As the IES approaches infinity, $\theta - 1$ tends to γ. Therefore, even when the IES is far above 1, the last term in (22) and (23) is scaled by γ and can still have a meaningful effect on asset prices.

An IES equal to 1 is a key value in the asset pricing literature. For example, it is the basis of the "risk-sensitive" preferences in Hansen and Sargent (2008, section 14.3). Therefore, it is a desirable property for small perturbations around an IES of 1 to not materially alter the predictions of the model. A well-known example of where this property holds is the standard Epstein-Zin asset pricing model without valuation risk. Even though the log-SDF as written in (2) is undefined when the IES equals 1, both the risk-free rate and the equity premium in (20) and (21) are well-defined.

Revised Specification Next we solve the model with the revised preferences, so the SDF is given by (1) with $\omega = \beta$. In this case, the average risk-free rate and average equity risk premium become

$$E[\hat{r}_f] = -\log\beta + \mu_y/\psi + ((\theta-1)\kappa_1^2\eta_1^2 - \theta\beta^2)\sigma_a^2/2, \tag{24}$$

$$E[ep] = ((1-\theta)\kappa_1\eta_1 + \theta\beta)\kappa_1\eta_1\sigma_a^2. \tag{25}$$

Relative to the current specification, the preference shock loading, η_1, is unchanged. However, both asset prices include a new term that captures the direct effect of valuation risk on current utility, so a rise in a_t that makes the agent more patient raises the value of future certainty equivalent consumption and lowers the value of present consumption. The asymptote occurs with the current specification because it does not account for the effect of valuation risk on current consumption.

With the revised preferences, $\kappa_1 = \beta$ when $\psi = 1$, so the terms involving θ cancel out and the asymptote disappears.[5] Valuation risk lowers the average risk-free rate by $\beta^2\sigma_a^2/2$ and raises the average equity return by the same amount. Therefore, the average equity premium equals $\beta^2\sigma_a^2$, which is invariant to the level of RA. When $\psi > 1$, $\kappa_1 > \beta$, so an increase in RA lowers the risk-free rate and raises the equity return. As $\psi \to \infty$, the equity premium with the revised specification

[5] Notice κ_1 is a function of the steady-state price-dividend ratio, z_d. When the IES is 1, $z_d = \beta/(1-\beta)$, which is equivalent to its value absent any risk. Therefore, when the IES is 1, valuation risk has no effect on the price-dividend ratio. This result points to a connection with income and substitution effects, which usually cancel when the IES is 1.

relative to the current specification equals $1+\beta(1-\gamma)/(\gamma\kappa_1)$. This means the disparity between the predictions of the two models grows as RA increases. As a consequence, the revised preferences would require much larger RA to generate the same equity premium as the current preferences.

Expected utility With CRRA preferences ($\gamma = 1/\psi$), the specifications in (3) and (4) reduce to

$$U_t^C = E_t \sum_{j=0}^{\infty}(1-\beta)(\prod_{i=1}^{j} a_{t+i-1}^C)\beta^j c_{t+j}^{1-\gamma}/(1-\gamma),$$
$$U_t^R = E_t \sum_{j=0}^{\infty}(1-\beta a_{t+j}^R)(\prod_{i=1}^{j} a_{t+i-1}^R)\beta^j c_{t+j}^{1-\gamma}/(1-\gamma).$$

There is no longer an asymptote with the current preferences because $\theta = 1$ with CRRA utility. The current and revised specifications also generate identical impulse responses to a time preference shock since $\eta_1 = 1$. However, the two specifications still have different asset pricing implications. Under the current specification, valuation risk has no effect on the risk-free rate and there is no equity premium. With the revised specification, the presence of valuation risk lowers the average risk-free rate by $\beta^2\sigma_a^2/2$ and the average equity premium equals $\beta^2\sigma_a^2$, just like when the IES equals unity with Epstein-Zin preferences. Therefore, the two expected utility specifications are not interchangeable, but the quantitative differences are insignificant. We can also conclude that the asymptote and stark differences in asset prices between the two Epstein-Zin preference specifications come through the continuation value, V_{t+1}, in the SDF, which drops out with expected utility.

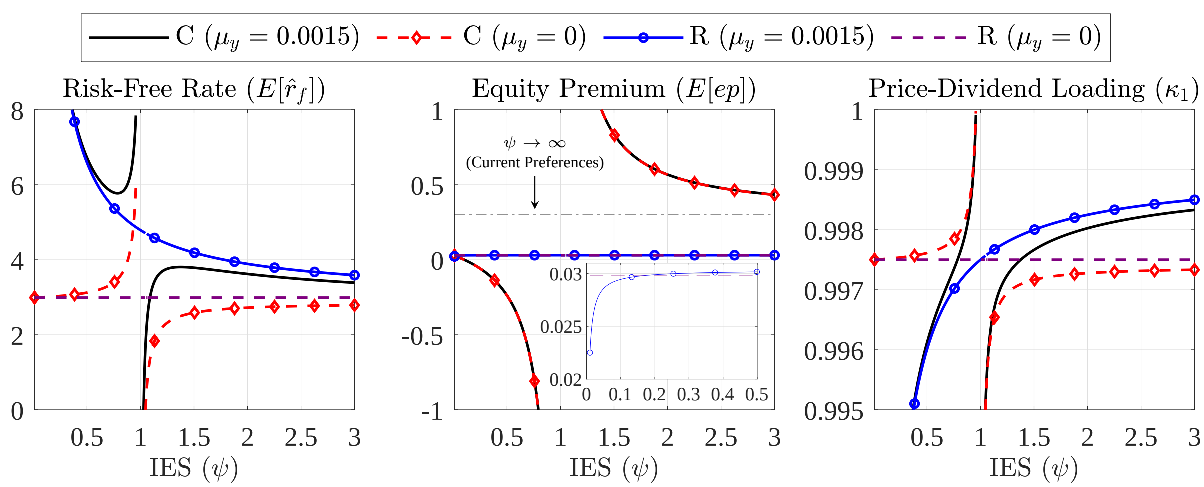

Figure 2: Equilibrium outcomes in the model without cash flow risk ($\sigma_y = 0$; $\mu_y = \mu_d$) and *i.i.d.* preference shocks ($\rho_a = 0$) under the current (C) and revised (R) preference specifications. We set $\beta = 0.9975$, $\gamma = 10$, and $\sigma_a = 0.005$.

3.3 ILLUSTRATION

Our analytical results show the way a time preference shock enters Epstein-Zin utility determines whether the asymptote in θ shows up in equilibrium outcomes. Figure 2 illustrates our results by plotting the average risk-free rate, the average equity premium, and κ_1 (i.e., the marginal response of the price-dividend ratio on the equity return). We focus on the setting in section 3.2 and plot the results under both preferences with and without endowment/dividend growth.

With the current preferences, the average risk-free rate and average equity premium exhibit a vertical asymptote when the IES is 1, regardless of whether μ_y is positive. As a result, the risk-free rate approaches positive infinity as the IES approaches 1 from below and negative infinity as the IES approaches 1 from above. The equity premium has the same comparative statics with the opposite sign, except there is a horizontal asymptote as the IES approaches infinity. These results occur because the current specification misses the direct effect of valuation risk on current consumption.[6]

In contrast, with the revised preferences the average risk-free rate and average equity premium are continuous in the IES, regardless of the value of μ_y. When $\mu_y = 0$, the endowment stream is constant. This means the agent is indifferent about the timing of when the preference uncertainty is resolved, so both κ_1 and the average equity premium are independent of the IES. When $\mu_y > 0$, the agent's incentive to smooth consumption interacts with uncertainty about how (s)he will value the higher future endowment stream.[7] When the IES is large, the agent has a stronger preference for an early resolution of uncertainty, so the equity premium rises as a result of the valuation risk (see the figure 2 inset). Therefore, the qualitative relationship between the IES and the equity premium has different signs under the current and revised specifications. However, the increase in the equity premium is quantitatively small and converges to a level significantly below the value with the current preferences. It is this difference in the sign and magnitude of the relationship between the IES and the equity premium that will explain many of the empirical results in subsequent sections.

4 Data and Estimation Methods

We construct our data using the procedure in Bansal and Yaron (2004), Beeler and Campbell (2012), Bansal et al. (2016), and Schorfheide et al. (2018). The moments are based on five time series from 1929 to 2017: real per capita consumption expenditures on nondurables and services, the real equity return, real dividends, the real risk-free rate, and the price-dividend ratio. Nominal equity returns are calculated with the CRSP value-weighted return on stocks. We obtain data with and without dividends to back out a time series for nominal dividends. Both series are converted to real series using the consumer price index (CPI). The nominal risk-free rate is based on the CRSP yield-to-maturity on 90-day Treasury bills. We first convert the nominal series to a real series using the CPI. Then we construct an *ex-ante* real rate by regressing the *ex-post* real rate on the nominal rate and inflation over the last year. The consumption data is annual. To match this frequency, the monthly asset pricing data are converted to annual time series using the last month of each year.

Using the annual time series, our target moments, $\hat{\Psi}_T^D$, are estimated with a two-step Generalized Method of Moments (GMM) estimator, where $T = 87$ is the sample size.[8] Given the GMM

[6]Pohl et al. (2018) find the errors from a Campbell-Shiller approximation of the nonlinear model can significantly affect equilibrium outcomes. Appendix C shows the undesirable asymptote also occurs in the fully nonlinear model.

[7]Andreasen and Jørgensen (2019) show how to decouple the agent's timing attitude from the RA and IES values.

[8]In total, there are 89 periods in our sample, but we lose one period for growth rates and one for serial correlations.

estimates, the model is estimated with Simulated Method of Moments (SMM). For parameterization θ and shocks $\mathcal{E}$, we solve the model and simulate it $R = 1{,}000$ times for T periods. The model-implied analogues of the target moments are the median moments across the R simulations, $\bar{\Psi}^M_{R,T}(\theta, \mathcal{E})$. The parameter estimates, $\hat{\theta}$, are obtained by minimizing the following loss function:

$$J(\theta, \mathcal{E}) = [\hat{\Psi}^D_T - \bar{\Psi}^M_{R,T}(\theta, \mathcal{E})]'[\hat{\Sigma}^D_T(1 + 1/R)]^{-1}[\hat{\Psi}^D_T - \bar{\Psi}^M_{R,T}(\theta, \mathcal{E})],$$

where $\hat{\Sigma}^D_T$ is the diagonal of the GMM estimate of the variance-covariance matrix.[9] We use Monte Carlo methods to calculate the standard errors on the parameter estimates. For different sequences of shocks, we re-estimate the structural model $N_s = 500$ times and report the mean and $(5, 95)$ percentiles. Appendix D and Appendix E provide more details about our data and estimation method.

The baseline model targets 15 moments: the means and standard deviations of consumption growth, dividend growth, equity returns, the risk-free rate, and the price-dividend ratio, the correlation between dividend growth and consumption growth, the autocorrelations of the price-dividend ratio and risk-free rate, and the cross-correlations of consumption growth, dividend growth, and equity returns. These targets are common in the literature and the same as Albuquerque et al. (2016), except we exclude 5- and 10-year correlations between equity returns and cash-flow growth. We omit the long-run correlations to allow a longer sample that includes the Great Depression period.

Many elements of our estimation procedure are common in the asset pricing literature. In particular, we use a limited information approach to match empirical targets. We use SMM to account for short-sample bias that can occur because asset pricing models often have very persistent processes. To improve on the current methodology, we repeat the estimation procedure for many different shock sequences. The estimations are run in parallel on a supercomputer. The literature typically estimates the model once based on a particular sequence of shocks and then uses the Delta method to compute standard errors. While our approach has a much higher computational burden, it makes our estimates independent of the seed and generates more precise standard errors.[10] The estimates allow us to numerically approximate the sampling distribution of our model's parameters and test whether our parameter estimates are significantly different across models. We also obtain a distribution of J values, which determine whether a model provides a significant improvement in fit over another model, and the corresponding p-values from a test of over-identifying restrictions.[11]

[9]For the revised preferences, we impose the restriction $\beta \exp(4(1-\beta)\sqrt{\sigma_a^2/(1-\rho_a^2)}) < 1$ when estimating the model parameters. This ensures the time-aggregator weights are positive in 99.997% of the simulated observations.

[10]We estimate 29 variants of our model. Since each variant is estimated 500 times, there are $14{,}500$ total estimations. The estimations are run in Fortran and the time per estimation ranges from 1-24 hours depending on model complexity.

[11]The test statistic is given by $\hat{J}^s = J(\hat{\theta}, \mathcal{E}^s)$, where $\mathcal{E}^s$ denotes a matrix of shocks given seed s. $J(\hat{\theta}, \mathcal{E})$ converges to a χ^2 distribution with $N_m - N_p$ degrees of freedom, where N_m is the number of empirical targets and N_p is the number of estimated parameters. The model passes the test if the loss, $\hat{J}^s$, is less than the desired significance level of the χ^2 distribution, or, in other words, if the corresponding p-value is greater than the desired value. We report the mean and $(5, 95)$ percentiles of the p-values across the seeds to determine whether a model reliably passes the test.

5 ESTIMATED BASELINE MODEL

This section takes the baseline model from section 2 and compares the estimates from the current and revised preference specifications. We fix the IES to 2.5, which is near the upper end of the plausible range of values in the literature.[12] This restriction helps us compare the estimates from the two preference specifications because the model fit, as measured by the J value, is insensitive to the value of the IES in the revised specification, but the unconstrained global minimum prefers an implausibly high IES. For example, the J value is only one decimal point lower with an IES equal to 10. Therefore, we are left with estimating nine parameters to match 15 empirical targets.

Table 1 shows the parameter estimates and moments. For each parameter, we report the average and $(5, 95)$ percentiles across 500 estimations of the model. For each moment, we report the mean and t-statistic for the null hypothesis that a model-implied moment equals its empirical counterpart.

In both specifications, the data prefers a very persistent valuation risk process with $\rho_a > 0.98$.[13] In the current specification, the risk aversion parameter, γ, is 1.55. In the revised specification $\gamma = 74.23$, which is well outside what is considered acceptable in the asset pricing literature.[14] Both specifications generate a sizable equity premium (the estimates are about 1% lower than the empirical equity premium) and a near zero risk-free rate. However, they significantly under-predict the standard deviation of dividend growth and over-predict the autocorrelation of the risk-free rate.

Using the analytical expressions for the average risk-free rate and equity premium (see (B.15) and (B.16) in Appendix B), it is possible to break down the fraction of each moment explained by cash-flow and valuation risk.[15] With the current specification valuation risk explains 98.9% and 99.2% of the risk-free rate and the equity premium, whereas with the revised preferences it explains only 63.1% and 79.0%. Since the estimate of the cash-flow shock standard deviation is unchanged, cash-flow risk has a bigger role in explaining the equity premium due to higher RA.

The revised specification has a significantly poorer fit than the current specification ($J = 48.0$ vs. $J = 29.3$), although both specifications fail the over-identifying restrictions test. The poorer fit is mostly due to the model significantly over-predicting the volatility of the risk-free rate and under-predicting the volatilities of the price-dividend ratio and equity return. The intuition is as follows.

[12] Estimation results with $\psi = 2$ and $\psi = 1.5$ for each specification considered below are included in Appendix F.

[13] The estimate of the valuation risk shock standard deviation, σ_a, is two orders of magnitude larger in the revised specification than the current specification. Recall that the valuation risk term in the SDF is given by $\hat{a}_t - \omega\hat{a}_{t+1}$. When the valuation risk shock is *i.i.d.*, the estimates of the shock standard deviation are very similar. However, as the persistence increases with the revised preferences, $SD_t[\hat{a}_t - \omega\hat{a}_{t+1}]$ shrinks, so σ_a rises to compensate for the extra term.

[14] Mehra and Prescott (1985, p. 154) state that "Any of the above cited studies. . . constitute an *a priori* justification for restricting the value of [RA] to be a maximum of ten, as we do in this study." Weil (1989, p. 411) describes $\gamma = 40$ as "implausibly" high. Swanson (2012) shows that γ does not equate to risk aversion when agents have a labor margin. Therefore, only in production economies can γ be reasonably above 10, where it is common to see values around 100.

[15] The mean risk-free rate is given by $E[\hat{r}_{f,t}] = \alpha_1 + \alpha_2\sigma_a^2 + \alpha_3\sigma_y^2$ and the mean equity premium is given by $E[ep_t] = \alpha_4\sigma_a^2 + \alpha_5\sigma_y^2$ for some function of model parameters α_i, $i \in \{1, \ldots, 5\}$. Therefore, the contribution of valuation risk to the risk-free rate and equity premium is given by $\alpha_2\sigma_a^2/(\alpha_2\sigma_a^2 + \alpha_3\sigma_y^2)$ and $\alpha_4\sigma_a^2/(\alpha_4\sigma_a^2 + \alpha_5\sigma_y^2)$.

Parameter	Current	Revised	Revised Max RA
γ	1.55 (1.52, 1.58)	74.23 (70.95, 77.47)	10.00 (10.00, 10.00)
β	0.9977 (0.9976, 0.9978)	0.9957 (0.9956, 0.9957)	0.9973 (0.9972, 0.9973)
ρ_a	0.9968 (0.9965, 0.9971)	0.9899 (0.9896, 0.9902)	0.9879 (0.9876, 0.9882)
σ_a	0.00031 (0.00030, 0.00033)	0.03547 (0.03491, 0.03596)	0.03880 (0.03832, 0.03927)
μ_y	0.0016 (0.0016, 0.0016)	0.0016 (0.0016, 0.0016)	0.0017 (0.0017, 0.0017)
μ_d	0.0015 (0.0015, 0.0016)	0.0021 (0.0020, 0.0021)	0.0010 (0.0009, 0.0010)
σ_y	0.0058 (0.0057, 0.0058)	0.0058 (0.0057, 0.0059)	0.0058 (0.0057, 0.0060)
ψ_d	1.54 (1.43, 1.64)	0.97 (0.87, 1.07)	1.09 (0.97, 1.19)
π_{dy}	0.815 (0.764, 0.872)	0.436 (0.400, 0.472)	0.617 (0.562, 0.674)
J	29.27 (28.62, 29.98)	47.98 (47.62, 48.35)	55.55 (54.93, 56.11)
pval	0.000 (0.000, 0.000)	0.000 (0.000, 0.000)	0.000 (0.000, 0.000)

(a) Average and $(5, 95)$ percentiles of the parameter estimates. The J-test has 6 degrees of freedom. The IES is 2.5.

Moment	Data	Current	Revised	Revised Max RA
$E[\Delta c]$	1.89	1.89 (0.00)	1.94 (0.18)	2.01 (0.49)
$E[\Delta d]$	1.47	1.84 (0.38)	2.47 (1.04)	1.17 (−0.32)
$E[r_d]$	6.51	5.46 (−0.66)	5.59 (−0.58)	4.06 (−1.53)
$E[r_f]$	0.25	0.25 (0.00)	0.36 (0.18)	1.06 (1.32)
$E[z_d]$	3.42	3.45 (0.18)	3.49 (0.47)	3.56 (1.02)
$SD[\Delta c]$	1.99	1.99 (0.00)	2.00 (0.01)	2.00 (0.02)
$SD[\Delta d]$	11.09	3.47 (−2.79)	2.13 (−3.28)	2.49 (−3.14)
$SD[r_d]$	19.15	18.41 (−0.39)	13.65 (−2.90)	13.44 (−3.01)
$SD[r_f]$	2.72	3.21 (0.96)	3.69 (1.92)	3.86 (2.25)
$SD[z_d]$	0.45	0.46 (0.22)	0.25 (−3.16)	0.23 (−3.49)
$AC[r_f]$	0.68	0.95 (4.12)	0.90 (3.36)	0.88 (3.14)
$AC[z_d]$	0.89	0.92 (0.64)	0.85 (−0.85)	0.83 (−1.30)
$Corr[\Delta c, \Delta d]$	0.54	0.47 (−0.32)	0.41 (−0.59)	0.50 (−0.19)
$Corr[\Delta c, r_d]$	0.05	0.09 (0.57)	0.06 (0.23)	0.09 (0.61)
$Corr[\Delta d, r_d]$	0.07	0.19 (1.41)	0.15 (1.03)	0.18 (1.38)

(b) Data and average model-implied moments. The t-statistics are shown in parentheses.

Table 1: Baseline model. Revised Max RA imposes that the risk aversion parameter, γ, cannot exceed 10.

In the revised specification, risk-free rate volatility is relatively more sensitive to valuation risk than equity return volatility. Since the volatility of equity returns is higher than the volatility of the risk-free rate in the data, valuation risk alone does not allow the model to match these moments. Dividend growth volatility, however, cannot rise to compensate for the lack of the equity return volatility because the target correlation between equity returns and dividend growth is near zero.

The revised preferences not only have a worse fit, but the risk aversion parameter is implausibly large. When we restrict γ to a maximum of 10—the upper end of the values used in the asset pricing literature—the fit deteriorates further ($J = 55.6$ vs. 48.0). The primary source of the poorer fit is the larger estimate of the risk-free rate (1.1% vs. 0.4%) and lower equity return (4.1% vs. 5.6%).

Overall, our results demonstrate that introducing valuation risk to the baseline model in its revised form does not resolve the equity premium and risk-free rate puzzles. The rest of the paper re-examines whether revised valuation risk has a significant role in richer asset pricing models.

6 ESTIMATED LONG-RUN RISK MODEL

Long-run risk provides a well-known resolution to many asset pricing puzzles. This section introduces this feature into our baseline model and re-examines the marginal contribution of valuation risk with the revised preferences. To introduce long-run risk, we modify (17) and (18) as follows:

$$\Delta\hat{y}_{t+1} = \mu_y + \hat{x}_t + \sigma_y\varepsilon_{y,t+1},\ \varepsilon_{y,t+1} \sim \mathbb{N}(0,1), \tag{26}$$

$$\Delta\hat{d}_{t+1} = \mu_d + \phi_d\hat{x}_t + \pi_{dy}\sigma_y\varepsilon_{y,t+1} + \psi_d\sigma_y\varepsilon_{d,t+1},\ \varepsilon_{d,t+1} \sim \mathbb{N}(0,1), \tag{27}$$

$$\hat{x}_{t+1} = \rho_x\hat{x}_t + \psi_x\sigma_y\varepsilon_{x,t+1},\ \varepsilon_{x,t+1} \sim \mathbb{N}(0,1), \tag{28}$$

where the specification of the persistent component, $\hat{x}_t$, follows Bansal and Yaron (2004). We apply the same estimation procedure as the baseline model, except there are three additional parameters, ϕ_d, ρ_x, and ψ_x. We also match up to five additional moments: the autocorrelations of consumption growth, dividend growth, and the equity return and two predictability moments—the correlations of consumption growth and the equity premium with the lagged price-dividend ratio.[16]

Once again, we find the model prefers an extremely high IES even though it does not significantly lower the J value. As a result, we continue to set the IES to 2.5 and estimate the remaining parameters. The parameter estimates are reported in table 2 and the moments are shown in table 3. The tables show the results for four variants of the same model: with and without targeting the standard deviation and autocorrelation of the risk-free rate and with and without valuation risk.

We begin with the long-run risk model without valuation risk and without the risk-free rate moments (column 1). This is a typical model estimated in the literature. The model fails to pass the over-identifying restrictions test at the 5% level, signalling that the standard long-run risk model

[16] Long-run risk adds one additional state variable, $\hat{x}_t$, so we use Mathematica to solve for unknown coefficients.

	Omits $SD[r_f]$ & $AC[r_f]$		All Moments	
Parameter	No VR	Revised	No VR	Revised
γ	2.58 $(2.31, 2.84)$	2.63 $(2.35, 2.93)$	2.70 $(2.41, 2.96)$	2.54 $(2.25, 2.83)$
β	0.9990 $(0.9989, 0.9991)$	0.9980 $(0.9979, 0.9982)$	0.9990 $(0.9988, 0.9991)$	0.9989 $(0.9987, 0.9990)$
ρ_a	–	0.9802 $(0.9800, 0.9835)$	–	0.9548 $(0.9531, 0.9565)$
σ_a	–	0.0476 $(0.0452, 0.0498)$	–	0.0167 $(0.0161, 0.0173)$
μ_y	0.0016 $(0.0014, 0.0017)$	0.0016 $(0.0014, 0.0018)$	0.0016 $(0.0015, 0.0017)$	0.0016 $(0.0014, 0.0017)$
μ_d	0.0013 $(0.0009, 0.0016)$	0.0013 $(0.0009, 0.0016)$	0.0014 $(0.0012, 0.0017)$	0.0013 $(0.0009, 0.0016)$
σ_y	0.0041 $(0.0040, 0.0042)$	0.0041 $(0.0039, 0.0043)$	0.0049 $(0.0048, 0.0050)$	0.0041 $(0.0040, 0.0042)$
ψ_d	3.25 $(3.02, 3.47)$	2.78 $(2.53, 3.02)$	3.05 $(2.83, 3.25)$	3.17 $(2.92, 3.41)$
π_{dy}	0.588 $(0.322, 0.868)$	0.813 $(0.547, 1.120)$	0.122 $(-0.200, 0.418)$	0.666 $(0.416, 0.916)$
ϕ_d	2.30 $(2.07, 2.51)$	1.55 $(1.44, 1.68)$	2.15 $(1.94, 2.34)$	2.19 $(1.97, 2.43)$
ρ_x	0.9988 $(0.9983, 0.9992)$	0.9994 $(0.9992, 0.9995)$	0.9977 $(0.9969, 0.9985)$	0.9990 $(0.9985, 0.9994)$
ψ_x	0.0260 $(0.0247, 0.0274)$	0.0261 $(0.0248, 0.0274)$	0.0314 $(0.0292, 0.0335)$	0.0255 $(0.0242, 0.0269)$
J	20.55 $(19.80, 21.30)$	14.29 $(13.86, 14.72)$	56.48 $(55.64, 57.39)$	19.59 $(18.96, 20.27)$
pval	0.009 $(0.006, 0.011)$	0.027 $(0.023, 0.031)$	0.000 $(0.000, 0.000)$	0.012 $(0.009, 0.015)$
df	8	6	10	8

Table 2: Long-run risk model. Average and $(5, 95)$ percentiles of the parameter estimates. The IES is 2.5.

is insufficient to adequately describe the behavior of asset prices and cash flows. The parameter estimates are similar to the estimates in the literature. In particular, the data requires a small but very persistent shock that generates risk in long-run cash-flow growth ($\rho_x = .9988$; $\psi_x = 0.0260$).

The literature typically excludes the standard deviation and autocorrelation of the risk-free rate when estimating the long-run risk model because the model does not generate sufficient volatility (a standard deviation of 0.51 vs. 2.72 in the data) and over-predicts the autocorrelation (0.96 vs. 0.68 in the data). Even when these two moments are targeted, as shown in column 3, long-run cash-flow risk is unable to significantly improve on these moments (the standard deviation rises to 0.68 and the autocorrelation falls to 0.95). The standard long-run risk model also fairs poorly on three additional moments: (1) the standard deviation of dividend growth (too low), (2) the correlation between dividend growth and the return on equity (too high), and (3) the predictability of consumption growth (too high). All of them are significantly different from their empirical targets.

Adding valuation risk (columns 2 and 4) significantly improves the fit of the model. With the restricted set of moments, the J value declines from 20.6 to 14.3. More importantly, the p-value from the over-identifying restrictions test rises from 0.01 to 0.03, even though the valuation risk

		Omits $SD[r_f]$ & $AC[r_f]$		All Moments	
Moment	Data	No VR	Revised	No VR	Revised
$E[\Delta c]$	1.89	1.89 (0.00)	1.89 (0.03)	1.89 (−0.01)	1.89 (0.00)
$E[\Delta d]$	1.47	1.53 (0.06)	1.54 (0.07)	1.71 (0.25)	1.50 (0.03)
$E[r_d]$	6.51	6.33 (−0.11)	6.44 (−0.05)	5.82 (−0.43)	6.43 (−0.05)
$E[r_f]$	0.25	0.26 (0.01)	0.26 (0.01)	0.26 (0.01)	0.25 (0.00)
$E[z_d]$	3.42	3.42 (0.00)	3.40 (−0.18)	3.41 (−0.07)	3.42 (0.00)
$SD[\Delta c]$	1.99	1.92 (−0.14)	1.96 (−0.07)	2.40 (0.84)	1.91 (−0.16)
$SD[\Delta d]$	11.09	5.59 (−2.01)	4.64 (−2.36)	6.38 (−1.72)	5.42 (−2.07)
$SD[r_d]$	19.15	18.15 (−0.53)	19.75 (0.32)	18.92 (−0.12)	18.21 (−0.50)
$SD[r_f]$	2.72	0.51 (−4.36)	5.48 (5.43)	0.68 (−4.03)	2.82 (0.19)
$SD[z_d]$	0.45	0.53 (1.29)	0.46 (0.10)	0.51 (0.98)	0.52 (1.14)
$AC[\Delta c]$	0.53	0.43 (−1.07)	0.46 (−0.74)	0.48 (−0.59)	0.43 (−1.07)
$AC[\Delta d]$	0.19	0.27 (0.76)	0.20 (0.12)	0.31 (1.16)	0.26 (0.65)
$AC[r_d]$	−0.01	0.00 (0.17)	−0.05 (−0.44)	0.00 (0.08)	−0.01 (0.02)
$AC[r_f]$	0.68	0.96 (4.33)	0.84 (2.47)	0.95 (4.21)	0.69 (0.14)
$AC[z_d]$	0.89	0.94 (1.05)	0.90 (0.29)	0.93 (0.83)	0.94 (1.00)
$Corr[\Delta c, \Delta d]$	0.54	0.48 (−0.28)	0.51 (−0.14)	0.44 (−0.46)	0.49 (−0.23)
$Corr[\Delta c, r_d]$	0.05	0.07 (0.32)	0.06 (0.18)	0.08 (0.50)	0.07 (0.29)
$Corr[\Delta d, r_d]$	0.07	0.24 (2.07)	0.19 (1.44)	0.28 (2.53)	0.23 (1.96)
$Corr[ep, z_{d,-1}]$	−0.16	−0.17 (−0.04)	−0.13 (0.38)	−0.14 (0.25)	−0.17 (−0.01)
$Corr[\Delta c, z_{d,-1}]$	0.19	0.66 (2.67)	0.59 (2.30)	0.69 (2.85)	0.65 (2.64)

Table 3: Long-run risk model. Data and average model-implied moments. The t-statistic is in parentheses.

model contains two more parameters than the standard model (6 degrees of freedom instead of 8).

Unlike cash-flow risk, valuation risk directly affects the time-series properties of the risk-free rate, which makes it important to target these moments in the estimation. In column 2, the model includes valuation risk but targets neither the standard deviation nor the autocorrelation of the risk-free rate. As a result, the estimated model significantly over-predicts both moments (the standard deviation is 5.48 vs. 2.72 in the data and the autocorrelation is 0.84 vs. 0.68 in the data). However, once these moments are targeted in the estimation (column 4), the standard deviation of the risk-free rate is 2.82 and the autocorrelation of the risk-free rate is 0.69, consistent with the data.

In both columns 2 and 4, the model closely matches the mean risk-free rate and equity return.

However, the contribution of valuation risk is quite different across the different sets of moments. Recall that in the baseline model, valuation risk explains a sizable majority of the risk-free rate and equity premium. In column 2, valuation risk has a smaller but still meaningful contribution (48.2% of the risk-free rate and 38.9% of the equity premium). In column 4, however, it explains very little of these moments (8.8% and 5.1%) because the model requires smaller and less persistent valuation risk shocks in order to match the dynamics of the risk-free rate. These results show that valuation risk does not unilaterally resolve the risk-free rate and equity premium puzzles, but the overall fit of the model indicates that it has a meaningful role in matching asset pricing moments.

Despite the improvements in fit, the long-run risk model with valuation risk still performs poorly on the three moments listed above. Furthermore, all four specifications fail to pass the over-identifying restrictions test at the 5% level. The next section tries to address these shortcomings.

7 ESTIMATED EXTENDED LONG-RUN RISK MODEL

We consider two extensions to the long-run risk model. First, we allow valuation risk shocks to directly affect cash-flow growth, in addition to their effect on asset prices through the SDF (henceforth, the "Demand" shock model). This feature is similar to a discount factor shock in a production economy model. For example, in the workhorse New Keynesian model, an increase in the discount factor looks like a negative demand shock that lowers interest rates, inflation, and consumption. Therefore, it provides another potential mechanism for valuation risk to help fit the data, especially the correlation moments. Following Albuquerque et al. (2016), we modify (26) and (27) as follows:

$$\Delta\hat{y}_{t+1} = \mu_y + \hat{x}_t + \sigma_y\varepsilon_{y,t+1} + \pi_{ya}\sigma_a\varepsilon_{a,t+1}, \tag{29}$$

$$\Delta\hat{d}_{t+1} = \mu_d + \phi_d\hat{x}_t + \pi_{da}\sigma_a\varepsilon_{a,t+1}, \tag{30}$$

where π_{ya} and π_{da} control the covariances between valuation risk shocks and cash-flow growth.

Second, we add stochastic volatility to cash-flow risk following Bansal and Yaron (2004) (henceforth, the "SV" model). SV introduces time-varying uncertainty. Bansal et al. (2016) show SV leads to a significant improvement in fit. An important question is therefore whether the presence of SV will affect the role of valuation risk. To introduce SV, we modify (26)-(28) as follows:

$$\Delta\hat{y}_{t+1} = \mu_y + \hat{x}_t + \sigma_{y,t}\varepsilon_{y,t+1}, \tag{31}$$

$$\Delta\hat{d}_{t+1} = \mu_d + \phi_d\hat{x}_t + \pi_{dy}\sigma_{y,t}\varepsilon_{y,t+1} + \psi_d\sigma_{y,t}\varepsilon_{d,t+1}, \tag{32}$$

$$\hat{x}_{t+1} = \rho_x\hat{x}_t + \psi_x\sigma_{y,t}\varepsilon_{x,t+1}, \tag{33}$$

$$\sigma^2_{y,t+1} = \sigma^2_y + \rho_{\sigma_y}(\sigma^2_{y,t} - \sigma^2_y) + \nu_y\varepsilon_{\sigma_y,t+1}, \tag{34}$$

where ρ_{σ_y} is the persistence of the SV process and ν_y is the standard deviation of the SV shock.

	Omits $SD[r_f]$ & $AC[r_f]$			All Moments		
Ptr	No VR+SV	Demand	Demand+SV	No VR+SV	Demand	Demand+SV
γ	2.47 (2.29, 2.65)	4.44 (4.08, 4.86)	7.27 (4.02, 11.81)	2.58 (2.41, 2.74)	3.22 (2.99, 3.42)	6.51 (5.14, 8.05)
β	0.9989 (0.9987, 0.9990)	0.9989 (0.9989, 0.9990)	0.9981 (0.9974, 0.9987)	0.9982 (0.9981, 0.9983)	0.9991 (0.9990, 0.9991)	0.9980 (0.9977, 0.9983)
ρ_a	–	0.9873 (0.9863, 0.9899)	0.9894 (0.9845, 0.9923)	–	0.9594 (0.9576, 0.9614)	0.9930 (0.9921, 0.9936)
σ_a	–	0.0375 (0.0351, 0.0398)	0.0339 (0.0306, 0.0367)	–	0.0185 (0.0179, 0.0193)	0.0288 (0.0275, 0.0296)
μ_y	0.0016 (0.0015, 0.0016)	0.0016 (0.0015, 0.0016)	0.0016 (0.0015, 0.0016)	0.0016 (0.0014, 0.0017)	0.0016 (0.0015, 0.0016)	0.0016 (0.0015, 0.0016)
μ_d	0.0015 (0.0014, 0.0017)	0.0015 (0.0013, 0.0017)	0.0015 (0.0013, 0.0017)	0.0013 (0.0010, 0.0016)	0.0015 (0.0012, 0.0016)	0.0015 (0.0014, 0.0017)
σ_y	0.0026 (0.0009, 0.0041)	0.0039 (0.0037, 0.0041)	0.0014 (0.0002, 0.0029)	0.0008 (0.0004, 0.0014)	0.0041 (0.0039, 0.0042)	0.0006 (0.0001, 0.0013)
ψ_d	3.41 (3.09, 3.79)	–	–	2.99 (2.79, 3.19)	–	–
π_{dy}	0.658 (0.280, 1.066)	–	–	0.771 (0.503, 1.049)	–	–
ϕ_d	2.19 (1.97, 2.42)	3.36 (3.14, 3.61)	3.17 (2.66, 3.52)	1.90 (1.81, 2.00)	2.69 (2.54, 2.85)	2.84 (2.65, 2.99)
ρ_x	0.9973 (0.9966, 0.9980)	0.9947 (0.9937, 0.9954)	0.9949 (0.9938, 0.9962)	0.9992 (0.9989, 0.9994)	0.9975 (0.9971, 0.9980)	0.9958 (0.9952, 0.9965)
ψ_x	0.0308 (0.0280, 0.0337)	0.0393 (0.0358, 0.0416)	0.0382 (0.0346, 0.0418)	0.0255 (0.0241, 0.0269)	0.0306 (0.0285, 0.0313)	0.0358 (0.0334, 0.0385)
π_{ya}	–	−0.041 (−0.050, −0.031)	−0.044 (−0.059, −0.030)	–	−0.055 (−0.074, −0.038)	−0.049 (−0.064, −0.033)
π_{da}	–	−0.738 (−0.767, −0.713)	−0.817 (−0.874, −0.771)	–	−1.036 (−1.068, −1.003)	−0.877 (−0.905, −0.852)
ρ_{σ_y}	0.9971 (0.9952, 0.9985)	–	0.7011 (0.1619, 0.9739)	0.9630 (0.9589, 0.9668)	–	0.7708 (0.5997, 0.8794)
ν_y	3.0e−6 (1.4e−6, 4.6e−6)	–	2.2e−5 (7.1e−6, 3.6e−5)	1.2e−5 (1.1e−5, 1.4e−5)	–	2.7e−5 (2.0e−5, 3.5e−5)
J	15.48 (14.88, 16.05)	9.47 (9.11, 9.85)	8.71 (7.82, 9.27)	18.09 (17.38, 18.81)	13.52 (12.98, 14.04)	9.25 (8.85, 9.66)
pval	0.017 (0.014, 0.021)	0.149 (0.131, 0.168)	0.070 (0.055, 0.098)	0.021 (0.016, 0.026)	0.096 (0.081, 0.113)	0.161 (0.140, 0.182)
df	6	6	4	8	8	6

Table 4: Extended long-run risk models. Average and $(5, 95)$ percentiles of the parameter estimates. The IES is 2.5.

Tables 4 and 5 present the estimates from three versions of the extended long-run risk model: (1) the SV model without valuation risk (columns 1 and 4), (2) the demand shock model (columns 2 and 5), and (3) the combination of the demand shock and SV models (columns 3 and 6). In each case, we report the results including and excluding the standard deviation and autocorrelation of the risk-free rate as targeted moments, but we focus on the estimates from the full set of moments.[17]

A key finding is that all three extensions improve on the p-values from the simpler long-run risk models in the previous section. Adding SV to the model without valuation risk increases the p-value from near zero (table 2, column 3) to 0.02 (table 4, column 4). The estimated SV process is very persistent ($\rho_{\sigma_y} = 0.9630$) and the shock is statistically significant, consistent with the

[17] With the inclusion of π_{ya} and π_{da}, π_{dy} and ψ_d are redundant so we exclude them from the Demand specifications.

		Omits $SD[r_f]$ & $AC[r_f]$			All Moments		
Moment	Data	No VR+SV	Demand	Demand+SV	No VR+SV	Demand	Demand+SV
$E[\Delta c]$	1.89	1.88 (−0.04)	1.89 (0.00)	1.89 (0.01)	1.90 (0.05)	1.87 (−0.08)	1.89 (0.02)
$E[\Delta d]$	1.47	1.84 (0.38)	1.78 (0.32)	1.79 (0.33)	1.58 (0.11)	1.74 (0.28)	1.83 (0.38)
$E[r_d]$	6.51	6.11 (−0.25)	5.61 (−0.56)	5.67 (−0.53)	6.68 (0.10)	5.81 (−0.44)	5.78 (−0.46)
$E[r_f]$	0.25	0.23 (−0.04)	0.26 (0.01)	0.26 (0.00)	0.13 (−0.21)	0.36 (0.17)	0.19 (−0.11)
$E[z_d]$	3.42	3.41 (−0.12)	3.39 (−0.22)	3.39 (−0.24)	3.41 (−0.07)	3.40 (−0.16)	3.39 (−0.21)
$SD[\Delta c]$	1.99	1.91 (−0.16)	1.98 (−0.03)	1.99 (0.00)	2.01 (0.03)	1.98 (−0.04)	2.09 (0.21)
$SD[\Delta d]$	11.09	5.65 (−1.99)	10.62 (−0.17)	10.54 (−0.20)	5.28 (−2.12)	7.60 (−1.28)	9.68 (−0.51)
$SD[r_d]$	19.15	19.54 (0.21)	18.93 (−0.11)	19.07 (−0.04)	18.71 (−0.23)	18.31 (−0.44)	18.69 (−0.24)
$SD[r_f]$	2.72	1.00 (−3.39)	3.37 (1.28)	3.25 (1.05)	2.54 (−0.36)	2.97 (0.49)	2.69 (−0.07)
$SD[z_d]$	0.45	0.47 (0.36)	0.47 (0.32)	0.46 (0.21)	0.51 (0.91)	0.50 (0.81)	0.48 (0.46)
$AC[\Delta c]$	0.53	0.46 (−0.78)	0.43 (−1.04)	0.44 (−0.99)	0.44 (−0.97)	0.43 (−1.07)	0.45 (−0.92)
$AC[\Delta d]$	0.19	0.26 (0.64)	0.17 (−0.22)	0.16 (−0.35)	0.24 (0.45)	0.21 (0.20)	0.17 (−0.24)
$AC[r_d]$	−0.01	−0.01 (0.04)	0.02 (0.32)	−0.02 (−0.07)	−0.03 (−0.26)	0.02 (0.32)	−0.03 (−0.20)
$AC[r_f]$	0.68	0.93 (3.93)	0.88 (3.13)	0.82 (2.13)	0.69 (0.08)	0.71 (0.49)	0.70 (0.25)
$AC[z_d]$	0.89	0.92 (0.53)	0.90 (0.24)	0.90 (0.15)	0.93 (0.87)	0.93 (0.81)	0.91 (0.41)
$Corr[\Delta c, \Delta d]$	0.54	0.49 (−0.21)	0.54 (0.02)	0.53 (−0.05)	0.51 (−0.13)	0.49 (−0.24)	0.51 (−0.11)
$Corr[\Delta c, r_d]$	0.05	0.07 (0.30)	0.11 (0.88)	0.10 (0.81)	0.06 (0.18)	0.09 (0.59)	0.10 (0.79)
$Corr[\Delta d, r_d]$	0.07	0.22 (1.88)	0.03 (−0.42)	0.04 (−0.37)	0.21 (1.72)	0.13 (0.79)	0.06 (−0.06)
$Corr[ep, z_{d,-1}]$	−0.16	−0.27 (−1.05)	−0.10 (0.72)	−0.12 (0.45)	−0.23 (−0.65)	−0.14 (0.26)	−0.12 (0.42)
$Corr[\Delta c, z_{d,-1}]$	0.19	0.57 (2.19)	0.62 (2.45)	0.61 (2.39)	0.65 (2.63)	0.66 (2.66)	0.62 (2.47)

Table 5: Extended long-run risk model. Data and average model-implied moments. The t-statistic is in parentheses.

literature. The improved fit largely occurs because SV helps match the risk-free rate dynamics (the standard deviation is 2.54 vs. 2.72 in the data and the autocorrelation is 0.69 vs. 0.68 in the data).[18]

The Demand model increases the p-value from 0.012 (table 2, column 4) to 0.096 (table 4, column 5). Thus, the Demand model easily passes the over-identifying restrictions test at the 5% level. Consistent with the predictions of a production economy model, π_{ya} and π_{da} are negative in the estimation. More specifically, a positive valuation risk shock, which makes households more

[18]The No VR+SV model is the same model BKY estimate. In that paper, the model passes the over-identifying restrictions test at the 5% level, while in our case it does not. The key difference is that BKY do not target the correlations between cash-flows and the equity return. When we exclude these moments, our p-value jumps to 0.15.

patient, reduces consumption and dividend growth. In a direct horse race between the SV model and the Demand model, which have the same number of parameters, the Demand model wins. The superior fit of the Demand model comes from the fact that it better matches the high volatility of dividend growth and the low correlation between dividend growth and equity returns. The model is better able to match these moments because the volatility of dividend growth increases with π_{da} while partially offsetting the positive relationship between valuation risk and the return on equity.

The Demand+SV model (column 6) raises the p-value to 0.161, passing the over-identifying restrictions test at the 10% level. This result reveals that the two extensions to the long-run risk model are complements, rather than substitutes, which is not obvious *a priori* because both features help match risk-free rate dynamics. It also occurs even though the two additional parameters in the model reduce the degrees of freedom and the critical value for the over-identifying restrictions test.

The model continues to fail on one key moment: the predictability of consumption growth given the price dividend ratio (i.e., $Corr[\Delta c, z_{d,-1}]$) remains too high (0.62 vs. 0.19 in the data). The overall improvement in fit occurs because the Demand+SV model does a much better job matching dividend growth dynamics. Specifically, it better matches the standard deviation of dividend growth (9.68 vs. 11.09 in the data) and the weak correlation between dividend growth and equity returns (0.06 vs. 0.07 in the data). In this model, valuation risk has a bigger role than in the Demand model ($\rho_a = 0.993$ vs. $\rho_a = 0.959$; $\sigma_a = 0.0288$ vs. $\sigma_a = 0.0185$), while the SV process is not as persistent ($\rho_{\sigma_y} = 0.771$ vs. $\rho_{\sigma_y} = 0.963$) as in the No VR+SV model. Also, σ_y is significantly smaller, so the contribution of consumption growth volatility from pure endowment risk is smaller when compared to the Demand model. The Demand model has trouble matching dividend growth dynamics while simultaneously matching risk-free rate dynamics. An expanded role of valuation risk is crucial for matching dividend growth dynamics. Without SV, this is not possible because it would cause the model to miss on the risk-free rate dynamics. Introducing SV, however, permits a lower σ_y, which helps offset the effect of valuation risk on the risk-free rate dynamics.

8 CONCLUSION

The way valuation risk enters Epstein-Zin recursive utility has important implications. Under the current specification in the literature, an undesirable asymptote in the parameter space permeates equilibrium outcomes. The asymptote occurs as the IES approaches unity, but it profoundly affects asset prices even when the IES is well above one. As a consequence, the asymptote perversely allows valuation risk alone to explain the historically low risk-free rate and high equity premium.

Once we revise the preferences to remove the undesirable asymptote, valuation risk has a much smaller role in resolving the equity premium and risk-free rate puzzles. However, we find valuation risk still plays an important role in matching the standard deviation and autocorrelation of the risk-free rate. Furthermore, allowing valuation risk to directly affect cash-flow growth, similar to a production economy model, adds a source of volatility that significantly improves the empirical fit and helps match the standard deviation of dividend growth and its correlation with equity returns.

Despite the importance of valuation risk, our paper and the literature is silent on its structural foundations. As a consequence, there are several open research questions. For example, what does it mean for a representative agent to have a time-varying time-preference? Is there an economy with multiple (heterogenous) agents that supports these preferences? Is there a decision-theoretic explanation and is it possible to back out the dynamics of a time-varying time-preference directly from experiments or other data? We believe these questions are important avenues for future research.

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A Isomorphic Representations of the Current Specification

In the current literature, the preference shock typically hits current utility. If, for simplicity, we abstract from Epstein-Zin preferences, then the value function and Euler equation are given by

$$V_t = \alpha_t u(c_t) + \beta E_t[V_{t+1}], \tag{A.1}$$

$$\beta E_t[(\alpha_{t+1}/\alpha_t) u'(c_{t+1})/u'(c_t) r_{y,t+1}] = 1. \tag{A.2}$$

The shock follows $\Delta\hat{\alpha}_{t+1} = \rho\Delta\hat{\alpha}_t + \sigma_\alpha\varepsilon_t$, so the change in α_t is known at time t. Alternatively, if the preference shock hits future consumption, the value function and Euler equation are given by

$$V_t = u(c_t) + a_t\beta E_t[V_{t+1}], \tag{A.3}$$

$$a_t\beta E_t[u'(c_{t+1})/u'(c_t) r_{y,t+1}] = 1. \tag{A.4}$$

If the shock follows $\hat{a}_t = \rho\hat{a}_{t-1} + \sigma_a\varepsilon_t$, the two specifications are isomorphic because setting $a_t \equiv \alpha_{t+1}/\alpha_t$ in (A.4) yields (A.2). We use the second specification because it is easier to compare the current and revised preferences when the shock always shows up in the Euler equation in levels.

B Analytical Derivations

Stochastic Discount Factor The value function for specification $j \in \{C, R\}$ is given by

$$\begin{aligned} V_t^j = \max[& w_{1,t}^j c_t^{(1-\gamma)/\theta} + w_{2,t}^j (E_t[(V_{t+1}^j)^{1-\gamma}])^{1/\theta}]^{\theta/(1-\gamma)} \\ & - \lambda_t(c_t + p_{y,t}s_{1,t} + p_{d,t}s_{2,t} - (p_{y,t} + y_t)s_{1,t-1} - (p_{d,t} + d_t)s_{2,t-1}), \end{aligned}$$

where $w_{1,t}^C = 1-\beta$, $w_{1,t}^R = 1-a_t^R\beta$, $w_{2,t}^C = a_t^C\beta$, and $w_{2,t}^R = a_t^R\beta$. The optimality conditions imply

$$w_{1,t}^j (V_t^j)^{1/\psi} c_t^{-1/\psi} = \lambda_t, \tag{B.1}$$

$$w_{2,t}^j (V_t^j)^{1/\psi} (E_t[(V_{t+1}^j)^{1-\gamma}])^{1/\theta-1} E_t[(V_{t+1}^j)^{-\gamma} (\partial V_{t+1}^j/\partial s_{1,t})] = \lambda_t p_{y,t}, \tag{B.2}$$

$$w_{2,t}^j (V_t^j)^{1/\psi} (E_t[(V_{t+1}^j)^{1-\gamma}])^{1/\theta-1} E_t[(V_{t+1}^j)^{-\gamma} (\partial V_{t+1}^j/\partial s_{2,t})] = \lambda_t p_{d,t}, \tag{B.3}$$

where $\partial V_t^j/\partial s_{1,t-1} = \lambda_t(p_{y,t} + y_t)$ and $\partial V_t^j/\partial s_{2,t-1} = \lambda_t(p_{d,t} + d_t)$ by the envelope theorem. Updating the envelope conditions and combining (B.1)-(B.3) generates (8) and (9) in the main text.

Following Epstein and Zin (1991), we posit the following minimum state variable solution:

$$V_t^j = \xi_{1,t} s_{1,t-1} + \xi_{2,t} s_{2,t-1} \quad \text{and} \quad c_t = \xi_{3,t} s_{1,t-1} + \xi_{4,t} s_{2,t-1}. \tag{B.4}$$

where ξ is a vector of unknown coefficients. The envelope conditions combined with (B.1) imply

$$\xi_{1,t} = w_{1,t}^j (V_t^j)^{1/\psi} c_t^{-1/\psi} (p_{y,t} + y_t), \tag{B.5}$$
$$\xi_{2,t} = w_{1,t}^j (V_t^j)^{1/\psi} c_t^{-1/\psi} (p_{d,t} + d_t). \tag{B.6}$$

Multiplying the respective conditions by $s_{1,t-1}$ and $s_{2,t-1}$ and then adding yields

$$V_t^j = w_{1,t}^j (V_t^j)^{1/\psi} c_t^{-1/\psi} ((p_{y,t} + y_t) s_{1,t-1} + (p_{d,t} + d_t) s_{2,t-1}), \tag{B.7}$$

which after plugging in the budget constraint, (5), and imposing equilibrium can be written as

$$(V_t^j)^{(1-\gamma)/\theta} = w_{1,t}^j c_t^{-1/\psi} (c_t + p_{y,t} s_{1,t} + p_{d,t} s_{2,t}) = w_{1,t}^j c_t^{-1/\psi} (c_t + p_{y,t}). \tag{B.8}$$

Therefore, the optimal value function is given by

$$w_{1,t}^j c_t^{-1/\psi} p_{y,t} = w_{2,t}^j (E_t[(V_{t+1}^j)^{1-\gamma}])^{1/\theta}. \tag{B.9}$$

Solving (B.8) for V_t^j and (B.9) for $E_t[(V_{t+1}^j)^{1-\gamma}]$ and then plugging into (8) and (9) implies

$$m_{t+1}^j = (x_t^j)^\theta (c_{t+1}/c_t)^{-\theta/\psi} r_{y,t+1}^{\theta-1}, \tag{B.10}$$

where $x_t^j \equiv w_{2t}^j w_{1t+1}^j / w_{1t}^j$. Taking logs of (B.10) yields (1), given the following definitions:

$$\hat{x}_t^C = \hat{\beta} + \hat{a}_t^C,$$
$$\hat{x}_t^R = \hat{\beta} + \hat{a}_t^R + \log(1 - \beta \exp(\hat{a}_{t+1}^R)) - \log(1 - \beta \exp(\hat{a}_t^R)) \approx \hat{\beta} + (\hat{a}_t^R - \beta \hat{a}_{t+1}^R)/(1-\beta),$$

and $\hat{a}_t \equiv \hat{a}_t^C = \hat{a}_t^R/(1-\beta)$ so the preference shocks in the current and revised models are directly comparable. It immediately follows that $\hat{x}_t^j = \hat{\beta} + \hat{a}_t - \omega^j \hat{a}_{t+1}$ as in (1), where $\omega^C = 0$ and $\omega^R = \beta$.

Campbell-Shiller Approximation The return on the endowment is approximated by

$$\begin{aligned}
\hat{r}_{y,t+1} &= \log(y_{t+1}(p_{y,t+1}/y_{t+1}) + y_{t+1}) - \log(y_t(p_{y,t}/y_t)) \\
&= \log(\exp(\hat{z}_{y,t+1}) + 1) - \hat{z}_{y,t} + \Delta\hat{y}_{t+1} \\
&\approx \log(\exp(\hat{z}_y) + 1) + \exp(\hat{z}_y)(\hat{z}_{y,t+1} - \hat{z}_y)/(1 + \exp(\hat{z}_y)) - \hat{z}_{y,t} + \Delta\hat{y}_{t+1} \\
&= \kappa_{y0} + \kappa_{y1}\hat{z}_{y,t+1} - \hat{z}_{y,t} + \Delta\hat{y}_{t+1}.
\end{aligned}$$

The derivation for the equity return, $\hat{r}_{d,t+1}$, is analogous to the return on the endowment.

Model Solution We use a guess and verify method. For the endowment claim, we obtain

$$\begin{aligned}
0 &= \log(E_t[\exp(\hat{m}_{t+1} + \hat{r}_{y,t+1})]) \\
&= \log(E_t[\exp(\theta\hat{\beta} + \theta(\hat{a}_t - \omega^j \hat{a}_{t+1}) + \theta(1-1/\psi)\Delta\hat{y}_{t+1} + \theta(\kappa_{y0} + \kappa_{y1}\hat{z}_{y,t+1} - \hat{z}_{y,t}))]) \\
&= \log\left(E_t\left[\exp\left(\begin{array}{c} \theta\hat{\beta} + \theta(\hat{a}_t - \omega^j \hat{a}_{t+1}) + \theta(1-1/\psi)(\mu_y + \sigma_y \varepsilon_{y,t+1}) \\ +\theta\kappa_{y0} + \theta\kappa_{y1}(\eta_{y0} + \eta_{y1}\hat{a}_{t+1}) - \theta(\eta_{y0} + \eta_{y1}\hat{a}_t) \end{array}\right)\right]\right) \\
&= \log\left(E_t\left[\exp\left(\begin{array}{c} \theta\hat{\beta} + \theta(1-1/\psi)\mu_y + \theta(\kappa_{y0} + \eta_{y0}(\kappa_{y1} - 1)) \\ +\theta(1 - \omega^j\rho_a + \eta_{y1}(\kappa_{y1}\rho_a - 1))\hat{a}_t \\ +\theta(1-1/\psi)\sigma_y\varepsilon_{y,t+1} + \theta(\kappa_{y1}\eta_{y1} - \omega^j)\sigma_a\varepsilon_{a,t+1} \end{array}\right)\right]\right) \\
&= \theta\hat{\beta} + \theta(1-1/\psi)\mu_y + \theta(\kappa_{y0} + \eta_{y0}(\kappa_{y1} - 1)) + \tfrac{\theta^2}{2}(1-1/\psi)^2\sigma_y^2 \\
&\quad + \tfrac{\theta^2}{2}(\kappa_{y1}\eta_{y1} - \omega^j)^2\sigma_a^2 + \theta(1 - \omega^j\rho_a + \eta_{y1}(\kappa_{y1}\rho_a - 1))\hat{a}_t,
\end{aligned}$$

where the last equality follows from the log-normality of $\exp(\varepsilon_{y,t+1})$ and $\exp(\varepsilon_{a,t+1})$.

After equating coefficients, we obtain the following exclusion restrictions:

$$\hat{\beta} + (1-1/\psi)\mu_y + (\kappa_{y0} + \eta_{y0}(\kappa_{y1} - 1)) + \tfrac{\theta}{2}((1-1/\psi)^2\sigma_y^2 + (\kappa_{y1}\eta_{y1} - \omega^j)^2\sigma_a^2) = 0, \quad \text{(B.11)}$$

$$1 - \omega^j\rho_a + \eta_{y1}(\kappa_{y1}\rho_a - 1) = 0. \quad \text{(B.12)}$$

For the dividend claim, we obtain

$$\begin{aligned}
0 &= \log(E_t[\exp(\hat{m}_{t+1} + \hat{r}_{d,t+1})]) \\
&= \log\left(E_t\left[\exp\left(\begin{array}{c} \theta\hat{\beta} + \theta(\hat{a}_t - \omega^j\hat{a}_{t+1}) + (\theta(1-1/\psi) - 1)\Delta\hat{y}_{t+1} + \Delta\hat{d}_{t+1} \\ +(\theta - 1)(\kappa_{y0} + \kappa_{y1}\hat{z}_{y,t+1} - \hat{z}_{y,t}) + (\kappa_{d0} + \kappa_{d1}\hat{z}_{d,t+1} - \hat{z}_{d,t}) \end{array}\right)\right]\right) \\
&= \log\left(E_t\left[\exp\left(\begin{array}{c} \theta\hat{\beta} + (\theta(1-1/\psi) - 1)\mu_y + \mu_d \\ +(\theta-1)(\kappa_{y0} + \eta_{y0}(\kappa_{y1} - 1)) + (\kappa_{d0} + \eta_{d0}(\kappa_{d1} - 1)) \\ +(\theta(1-\omega^j\rho_a) + (\theta-1)\eta_{y1}(\kappa_{y1}\rho_a - 1) + \eta_{d1}(\kappa_{d1}\rho_a - 1))\hat{a}_t \\ (\pi_{dy} - \gamma)\sigma_y\varepsilon_{y,t+1} + ((\theta-1)\kappa_{y1}\eta_{y1} + \kappa_{d1}\eta_{d1} - \theta\omega^j)\sigma_a\varepsilon_{a,t+1} + \psi_d\sigma_y\varepsilon_{d,t+1} \end{array}\right)\right]\right) \\
&= \theta\hat{\beta} + (\theta(1-1/\psi) - 1)\mu_y + \mu_d + (\theta-1)(\kappa_{y0} + \eta_{y0}(\kappa_{y1} - 1)) + (\kappa_{d0} + \eta_{d0}(\kappa_{d1} - 1)) \\
&\quad + (\theta(1-\omega^j\rho_a) + (\theta-1)\eta_{y1}(\kappa_{y1}\rho_a - 1) + \eta_{d1}(\kappa_{d1}\rho_a - 1))\hat{a}_t \\
&\quad + \tfrac{1}{2}((\pi_{dy} - \gamma)^2\sigma_y^2 + ((\theta-1)\kappa_{y1}\eta_{y1} + \kappa_{d1}\eta_{d1} - \theta\omega^j)^2\sigma_a^2 + \psi_d^2\sigma_y^2).
\end{aligned}$$

Once again, equating coefficients implies the following exclusion restrictions:

$$\begin{aligned}
&\theta\hat{\beta} + (\theta(1-1/\psi) - 1)\mu_y + \mu_d + (\theta-1)(\kappa_{y0} + \eta_{y0}(\kappa_{y1} - 1)) + (\kappa_{d0} + \eta_{d0}(\kappa_{d1} - 1)) \\
&\quad + \tfrac{1}{2}((\pi_{dy} - \gamma)^2\sigma_y^2 + ((\theta-1)\kappa_{y1}\eta_{y1} + \kappa_{d1}\eta_{d1} - \theta\omega^j)^2\sigma_a^2 + \psi_d^2\sigma_y^2) = 0, \quad \text{(B.13)}
\end{aligned}$$

$$\theta(1-\omega^j\rho_a) + (\theta-1)\eta_{y1}(\kappa_{y1}\rho_a - 1) + \eta_{d1}(\kappa_{d1}\rho_a - 1) = 0. \quad \text{(B.14)}$$

Equations (B.11)-(B.14), along with (15) and (16), form a system of 8 equations and 8 unknowns.

Asset Prices Given the coefficients, we can solve for the risk free rate. The Euler equation implies

$$\hat{r}_{f,t} = -\log(E_t[\exp(\hat{m}_{t+1})]) = -E_t[\hat{m}_{t+1}] - \tfrac{1}{2}\operatorname{Var}_t[\hat{m}_{t+1}],$$

since the risk-free rate is known at time-t. The pricing kernel is given by

$$\begin{aligned}
\hat{m}_{t+1} &= \theta\hat{\beta} + \theta(\hat{a}_t - \omega^j\hat{a}_{t+1}) - (\theta/\psi)\Delta\hat{y}_{t+1} + (\theta-1)\hat{r}_{y,t+1} \\
&= \theta\hat{\beta} + \theta(\hat{a}_t - \omega^j\hat{a}_{t+1}) - \gamma\Delta\hat{y}_{t+1} + (\theta-1)(\kappa_{y0} + \kappa_{y1}\hat{z}_{y,t+1} - \hat{z}_{y,t}) \\
&= \theta\hat{\beta} - \gamma\mu_y + (\theta-1)(\kappa_{y0} + \eta_{y0}(\kappa_{y1}-1)) + (\theta(1-\omega^j) + (\theta-1)\eta_{y1}(\kappa_{y1}\rho_a - 1))\hat{a}_t \\
&\qquad + ((\theta-1)\kappa_{y1}\eta_{y1} - \theta\omega^j)\sigma_a\varepsilon_{a,t+1} - \gamma\sigma_y\varepsilon_{y,t+1} \\
&= \theta\hat{\beta} - \gamma\mu_y + (\theta-1)(\kappa_{y0} + \eta_{y0}(\kappa_{y1}-1)) + (1-\omega^j\rho_a)\hat{a}_t \\
&\qquad + ((\theta-1)\kappa_{y1}\eta_{y1} - \theta\omega^j)\sigma_a\varepsilon_{a,t+1}) - \gamma\sigma_y\varepsilon_{y,t+1},
\end{aligned}$$

where the last line follows from imposing (B.12). Therefore, the risk-free rate is given by

$$\begin{aligned}
\hat{r}_{f,t} = \gamma\mu_y - \theta\hat{\beta} - (\theta-1)(\kappa_{y0} + \eta_{y0}(\kappa_{y1}-1)) - (1-\omega^j\rho_a)\hat{a}_t \\
- \tfrac{1}{2}\gamma^2\sigma_y^2 - \tfrac{1}{2}((\theta-1)\kappa_{y1}\eta_{y1} - \theta\omega^j)^2\sigma_a^2.
\end{aligned}$$

Note that $\hat{r}_{f,t} = \log(E_t[\exp(\hat{r}_{f,t})])$. After plugging in (B.11), we obtain

$$\hat{r}_{f,t} = \mu_y/\psi - \hat{\beta} - (1-\omega^j\rho_a)\hat{a}_t + \tfrac{1}{2}((\theta-1)\kappa_{y1}^2\eta_{y1}^2 - \theta(\omega^j)^2)\sigma_a^2 + \tfrac{1}{2}((1/\psi-\gamma)(1-\gamma) - \gamma^2)\sigma_y^2.$$

Therefore, the unconditional expected risk-free rate is given by

$$E[\hat{r}_f] = -\hat{\beta} + \mu_y/\psi + \tfrac{1}{2}((\theta-1)\kappa_{y1}^2\eta_{y1}^2 - \theta(\omega^j)^2)\sigma_a^2 + \tfrac{1}{2}((1/\psi-\gamma)(1-\gamma) - \gamma^2)\sigma_y^2. \quad \text{(B.15)}$$

We can also derive an expression for the equity premium, $E_t[ep_{t+1}]$, which given by

$$\log(E_t[\exp(\hat{r}_{d,t+1} - \hat{r}_{f,t})]) = E_t[\hat{r}_{d,t+1}] - \hat{r}_{f,t} + \tfrac{1}{2}\operatorname{Var}_t[\hat{r}_{d,t+1}] = -\operatorname{Cov}_t[\hat{m}_{t+1}, \hat{r}_{d,t+1}],$$

where the last equality stems from the Euler equation, $E_t[\hat{m}_{t+1}+\hat{r}_{d,t+1}]+\frac{1}{2}\operatorname{Var}_t[\hat{m}_{t+1}+\hat{r}_{d,t+1}] = 0$. We already solved for the SDF, so the last step is to solve for the equity return, which given by

$$\begin{aligned}
\hat{r}_{d,t+1} &= \kappa_{d0} + \kappa_{d1}\hat{z}_{d,t+1} - \hat{z}_{d,t} + \Delta\hat{d}_{t+1} \\
&= \kappa_{d0} + \kappa_{d1}(\eta_{d0} + \eta_{d1}\hat{a}_{t+1}) - (\eta_{d0} + \eta_{d1}\hat{a}_t) + \Delta\hat{d}_{t+1} \\
&= \mu_d + \kappa_{d0} + \eta_{d0}(\kappa_{d1}-1) + \eta_{d1}(\kappa_{d1}\rho_a - 1)\hat{a}_t + \kappa_{d1}\eta_{d1}\sigma_a\varepsilon_{a,t+1} + \pi_{dy}\sigma_y\varepsilon_{y,t+1} + \psi_d\sigma_y\varepsilon_{d,t+1}.
\end{aligned}$$

Therefore, the unconditional equity premium can be written as

$$E[ep] = \gamma\pi_{dy}\sigma_y^2 + (\theta\omega^j + (1-\theta)\kappa_{y1}\eta_{y1})\kappa_{d1}\eta_{d1}\sigma_a^2. \quad \text{(B.16)}$$

B.1 SPECIAL CASE 1 ($\sigma_a = \psi_d = 0$ & $\pi_{dy} = 1$) In this case, there is no valuation risk ($\hat{a}_t = 0$) and cash flow risk is perfectly correlated ($\Delta\hat{y}_{t+1} = \mu_y + \sigma_y\varepsilon_{y,t+1}$; $\Delta\hat{d}_{t+1} = \mu_d + \sigma_y\varepsilon_{y,t+1}$). Under these assumptions, it is easy to see that (B.15) and (B.16) reduce to (20) and (21) in the main text.

B.2 SPECIAL CASE 2 ($\sigma_y = 0$, $\rho_a = 0$, & $\mu_y = \mu_d$) In this case, there is no cash flow risk ($\Delta\hat{y}_{t+1} = \Delta\hat{d}_{t+1} = \mu_y$) and the time preference shocks are *i.i.d.* ($\hat{a}_{t+1} = \sigma_a\varepsilon_{a,t+1}$). Under these two assumptions, the return on the endowment and dividend claims are identical, so $\{\kappa_{y0}, \kappa_{y1}, \eta_{y0}, \eta_{y1}\} = \{\kappa_{d0}, \kappa_{d1}, \eta_{d0}, \eta_{d1}\} \equiv \{\kappa_0, \kappa_1, \eta_0, \eta_1\}$. Therefore, (B.15) and (B.16) reduce to (22) and (23) for the current specification and (24) and (25) for the revised specification.

The exclusion restriction, (B.12), implies $\eta_1 = 1$ so (B.11) simplifies to

$$0 = \hat{\beta} + (1 - 1/\psi)\mu_y + \kappa_0 + \eta_0(\kappa_1 - 1) + \tfrac{\theta}{2}(\kappa_1 - \omega^j)^2\sigma_a^2. \tag{B.17}$$

First, recall that $0 < \kappa_1 < 1$. Therefore, the asymptote in θ will permeate the solution with the current preferences ($\omega^C = 0$). However, with the revised preferences ($\omega^R = \beta$), we guess and verify that $\kappa_1 = \beta$ when $\psi = 1$. In this case, (B.17) reduces to $\hat{\beta} + \kappa_0 + \eta_0(\beta - 1) = 0$. Combining with (15), this restriction implies that $\eta_0 = \log\beta - \log(1-\beta)$ and $\kappa_0 = -(1-\beta)\log(1-\beta) - \beta\log\beta$. Plugging the expressions for η_0, κ_0, and κ_1 back into (15) and (B.17) verifies our guess for κ_1.

C NONLINEAR MODEL ASYMPTOTE

Assuming $\mu_{t+1} \equiv y_{t+1}/y_t = d_{t+1}/d_t$, the (nonlinear) Euler equation is given by

$$z_t = \frac{a_t\beta}{1 - \chi^j a_t\beta}\big(E_t[\underbrace{((1 - \chi^j a_{t+1}\beta)\mu_{t+1}^{1-1/\psi}(1 + z_{t+1}))^{\theta}}_{x_{t+1}}]\big)^{1/\theta}, \tag{C.1}$$

where $\chi^C = 0$ and $\chi^R = 1$. Notice the asymptote disappears if $\mathrm{SD}(x_{t+1}) \to 0$ as $\psi \to 1$. The main text focuses on results from a Campbell and Shiller (1988) approximation of the model. In this appendix, we demonstrate three noteworthy results using the model's exact, nonlinear, form.

One, consider the case without valuation risk, so $a_t = 1$ for all t. The Euler equation reduces to

$$z_t = \beta(E_t[(\mu_{t+1}^{1-1/\psi}(1 + z_{t+1}))^{\theta}])^{1/\theta}. \tag{C.2}$$

When $\psi = 1$, we guess and verify that $z_t = \beta/(1-\beta)$, so the price-dividend ratio is constant. This is the well know result that when the IES is 1, the income and substitution effects of a change in endowment growth offset. Therefore, the price-dividend ratio does not respond to cash flow risk.

Two, consider the case when a_t is stochastic under the revised preferences ($\chi^R = 1$) and either $\psi = 1$ (CRRA preferences) or $\mu_t = 1$ for all t (no cash-flow growth). In both cases, we guess and verify that $z_t = a_t\beta/(1 - a_t\beta)$. The price dividend ratio is time-varying but independent of θ.

Therefore, an asymptote does not affect equilibrium outcomes. The agent is certainty-equivalent.

Three, consider what happens under the current preferences ($\chi^C = 0$), which do not account for the offsetting movements in $1 - a_t\beta$. To obtain a closed-form solution for any IES, we assume $\mu_t = \mu$ and the preference shock evolves according to $\log(1 + a_{t+1}\eta) = \sigma\varepsilon_{t+1}$, where ε_{t+1} is standard normal. Under these assumptions, we guess and verify that the price-dividend ratio is given by

$$z_t = a_t\eta = a_t\beta\mu^{1-1/\psi}\exp(\theta\sigma^2/2). \tag{C.3}$$

In this case, θ appears in the price-dividend ratio, so the asymptote affects equilibrium outcomes. These results prove that the asymptote is not due to a Campbell-Shiller approximation of the model.

D DATA SOURCES

We drew from the following data sources to estimate our models:

1. **[$RCONS$] Per Capita Real PCE (excluding durables)**: Annual, chained 2012 dollars. Source: Bureau of Economic Analysis, National Income and Product Accounts, Table 7.1.
2. **[$RETD$] Value-Weighted Return (including dividends)**: Monthly. Source: Wharton Research Data Services, CRSP Stock Market Indexes (CRSP ID: VWRETD).
3. **[$RETX$] Value-Weighted Return (excluding dividends)**: Monthly. Source: Wharton Research Data Services, CRSP Stock Market Indexes (CRSP ID: VWRETX).
4. **[CPI] Consumer Price Index for All Urban Consumers**: Monthly, not seasonally adjusted, index 1982-1984=100. Source: Bureau of Labor Statistics (FRED ID: CPIAUCNS).
5. **[RFR] Risk-free Rate**: Monthly, annualized yield calculated from nominal price. Source: Wharton Research Data Services, CRSP Treasuries, Risk-free Series (CRSP ID: TMYTM).

We applied the following transformations to the above data sources:

1. **Annual Per Capita Real Consumption Growth (annual frequency)**:

$$\Delta\hat{c}_t = 100\log(RCONS_t/RCONS_{t-1})$$

2. **Annual Real Dividend Growth (monthly frequency)**:

$$P_{1928M1} = 100, \quad P_t = P_{t-1}(1 + RETX_t), \quad D_t = (RETD_t - RETX_t)P_{t-1},$$
$$d_t = \textstyle\sum_{i=t-11}^{t} D_i/CPI_t, \quad \Delta\hat{d}_t = 100\log(d_t/d_{t-12})$$

3. **Annual Real Equity Return (monthly frequency)**:

$$\pi_t^m = \log(CPI_t/CPI_{t-1}), \quad \hat{r}_{d,t} = 100\textstyle\sum_{i=t-11}^{t}(\log(1 + RETD_i) - \pi_i^m)$$

4. **Annual Real Risk-free Rate (monthly frequency)**:

$$rfr_t = RFR_t - \log(CPI_{t+3}/CPI_t), \quad \pi_t^q = \log(CPI_t/CPI_{t-12})/4,$$
$$\hat{r}_{f,t} = 400(\hat{\beta}_0 + \hat{\beta}_1 RFR_t + \hat{\beta}_2 \pi_t^q),$$

   where $\hat{\beta}_j$ are the OLS estimates in a regression of the *ex-post* real rate, rfr, on the nominal rate, RFR, and lagged inflation, π^q. The fitted values are estimates of the *ex-ante* real rate.

5. **Price-Dividend Ratio (monthly frequency)**:

$$\hat{z}_{d,t} = \log(P_t/\textstyle\sum_{i=t-11}^{t} D_i)$$

We use December of each year to convert each of the monthly time series to an annual frequency.

E Estimation Method

The estimation procedure has two stages. The first stage estimates moments in the data using a 2-step Generalized Method of Moments (GMM) estimator with a Newey and West (1987) weighting matrix with 10 lags. The second stage is a Simulated Method of Moments (SMM) procedure that searches for a parameter vector that minimizes the distance between the GMM estimates in the data and short-sample predictions of the model, weighted by the diagonal of the GMM estimate of the variance-covariance matrix. The second stage is repeated for many different draws of shocks to obtain a sampling distribution for each parameter. The following steps outline the algorithm:

1. Use GMM to estimate the moments, $\hat{\Psi}_T^D$, and the diagonal of the covariance matrix, $\hat{\Sigma}_T^D$.
2. Use SMM to estimate the structural asset pricing model. Given a random seed, s, draw a T-period sequence of shocks for each shock in the model. Denote the shock matrix $\mathcal{E}_T^s$ (e.g., in the baseline model $\mathcal{E}_T^s = [\varepsilon_{y,t}^s, \varepsilon_{d,t}^s, \varepsilon_{a,t}^s]_{t=1}^T$). For $s \in \{1, \ldots, N_s\}$, run the following steps:
   (a) Specify a guess, $\hat{\theta}_0$, for the N_p estimated parameters and the parameter variance-covariance matrix, Σ_P, which is initialized as a diagonal matrix.
   (b) Use simulated annealing to minimize the loss function.
      i. For $i \in \{0, \ldots, N_d\}$, repeat the following steps:
         A. Draw a candidate vector of parameters, $\hat{\theta}_i^{cand}$, where

$$\hat{\theta}_i^{cand} \sim \begin{cases} \hat{\theta}_0 & \text{for } i = 0, \\ \mathbb{N}(\hat{\theta}_{i-1}, c_0 \Sigma_P) & \text{for } i > 0. \end{cases}$$

We set c to target an acceptance rate of 30%. For the revised preferences, we restrict $\hat{\theta}_i^{cand}$ so that $\beta\exp(4(1-\beta)\sqrt{\sigma_a^2/(1-\rho_a^2)}) < 1$. This ensures the utility function weights are positive in 99.997% of the simulated observations.

B. Solve the Campbell-Shiller approximation of the model given $\hat{\theta}_i^{cand}$.

C. Given $\mathcal{E}_T^s(r)$, simulate the monthly model R times for T periods. We draw initial states, $\hat{a}_0$, from $\mathbb{N}(0,\sigma_a^2/(1-\rho_a^2))$. For each repetition r, calculate the moments, $\Psi_T^M(\hat{\theta}_i^{cand},\mathcal{E}_T^s(r))$, the same way they are calculated in the data.

D. Calculate the median moments across the R simulations, $\bar{\Psi}_{R,T}^M(\hat{\theta}_i^{cand},\mathcal{E}_T^s) = \text{median}\{\Psi_T^M(\hat{\theta}_i^{cand},\mathcal{E}_T^s(r))\}_{r=1}^R$, and evaluate the loss function:

$$J_i^{s,cand} = [\hat{\Psi}_T^D - \bar{\Psi}_{R,T}^M(\hat{\theta}_i^{cand},\mathcal{E}_T^s)]'[\hat{\Sigma}_T^D(1+1/R)]^{-1}[\hat{\Psi}_T^D - \bar{\Psi}_{R,T}^M(\hat{\theta}_i^{cand},\mathcal{E}_T^s)].$$

E. Accept or reject the candidate draw according to

$$(\hat{\theta}_i^s, J_i^s) = \begin{cases} (\hat{\theta}_i^{cand}, J_i^{s,cand}) & \text{if } i = 0, \\ (\hat{\theta}_i^{cand}, J_i^{s,cand}) & \text{if } \min(1,\exp(J_{i-1}^s - J_i^{s,cand})/c_1) > \hat{u}, \\ (\hat{\theta}_{i-1}, J_{i-1}^s) & \text{otherwise,} \end{cases}$$

where c_1 is the temperature and $\hat{u}$ is a draw from a uniform distribution. The lower the temperature, the more likely it is that the candidate draw is rejected.

ii. Find the parameter draw $\hat{\theta}_{\min}^s$ that corresponds to $\min\{J_i^s\}_{i=1}^{N_d}$, and update Σ_P^s.

A. Discard the first $N_d/2$ draws. Stack the remaining draws in a $N_d/2 \times N_p$ matrix, $\hat{\Theta}^s$, and define $\tilde{\Theta}^s = \hat{\Theta}^s - \mathbf{1}_{N_d/2\times 1}\sum_{i=N_d/2}^{N_d}\hat{\theta}_i^s/(N_d/2)$.

B. Calculate $\Sigma_P^{s,up} = (\tilde{\Theta}^s)'\tilde{\Theta}^s/(N_d/2)$.

(c) Repeat the previous step N_{SMM} times, initializing at draw $\hat{\theta}_0 = \hat{\theta}_{\min}^s$ and covariance matrix $\Sigma_P = \Sigma_P^{s,up}$. Gradually decrease the temperature. Of all the draws, find the lowest N_J J values, denoted $\{J_{guess}^{s,j}\}_{j=1}^{N_J}$, and the corresponding draws, $\{\theta_{guess}^{s,j}\}_{j=1}^{N_J}$.

(d) For $j \in \{1,\ldots,N_J\}$, minimize the same loss function with MATLAB's `fminsearch` starting at $\theta_{guess}^{s,j}$. The resulting minimum is $\hat{\theta}_{\min}^{s,j}$ with a loss function value of $J_{\min}^{s,j}$. Repeat, each time updating the guess, until $J_{guess}^{s,j} - J_{\min}^{s,j} < 0.001$. The parameter estimates reported in the tables in the main paper, denoted $\hat{\theta}^s$, correspond to $\min\{J_{\min}^{s,j}\}_{j=1}^{N_J}$.

3. The set of SMM parameter estimates $\{\hat{\theta}^s\}_{s=1}^{N_s}$ approximate the joint sampling distribution of the parameters. We report its mean, $\bar{\theta} = \sum_{s=1}^{N_s}\hat{\theta}^s/N_s$, and $(5,95)$ percentiles.

For all model specifications, the results in the main paper are based on $N_s = 500$, $R = 1{,}000$, $N_d = 20{,}000$, $N_{SMM} = 5$, and $N_J = 50$. N_p, c_0, and the temperatures, c_1, are model-specific.

F ESTIMATION ROBUSTNESS

Baseline Model: $\psi = 2.0$

Parameter	Current	Revised	Revised Max RA
γ	1.46 (1.44, 1.48)	75.79 (72.61, 79.16)	10.00 (10.00, 10.00)
β	0.9978 (0.9977, 0.9980)	0.9957 (0.9956, 0.9958)	0.9974 (0.9974, 0.9975)
ρ_a	0.9968 (0.9965, 0.9971)	0.9899 (0.9896, 0.9902)	0.9877 (0.9874, 0.9880)
σ_a	0.00031 (0.00030, 0.00033)	0.03554 (0.03504, 0.03605)	0.03907 (0.03864, 0.03955)
μ_y	0.0016 (0.0016, 0.0016)	0.0016 (0.0016, 0.0016)	0.0017 (0.0017, 0.0017)
μ_d	0.0015 (0.0015, 0.0016)	0.0020 (0.0020, 0.0021)	0.0010 (0.0009, 0.0010)
σ_y	0.0058 (0.0057, 0.0058)	0.0058 (0.0057, 0.0059)	0.0058 (0.0057, 0.0060)
ψ_d	1.54 (1.43, 1.63)	0.97 (0.88, 1.07)	1.07 (0.96, 1.18)
π_{dy}	0.816 (0.765, 0.870)	0.438 (0.405, 0.475)	0.606 (0.550, 0.668)
J	29.27 (28.62, 29.98)	48.09 (47.73, 48.47)	56.08 (55.47, 56.67)
pval	0.000 (0.000, 0.000)	0.000 (0.000, 0.000)	0.000 (0.000, 0.000)

(a) Average and $(5, 95)$ percentiles of the parameter estimates. The J-test has 6 degrees of freedom. The IES is 2.0.

Moment	Data	Current	Revised	Revised Max RA
$E[\Delta c]$	1.89	1.89 (0.00)	1.94 (0.19)	2.00 (0.45)
$E[\Delta d]$	1.47	1.84 (0.38)	2.45 (1.02)	1.15 (−0.34)
$E[r_d]$	6.51	5.46 (−0.66)	5.57 (−0.59)	4.03 (−1.55)
$E[r_f]$	0.25	0.25 (0.00)	0.37 (0.19)	1.07 (1.34)
$E[z_d]$	3.42	3.45 (0.18)	3.49 (0.48)	3.56 (1.02)
$SD[\Delta c]$	1.99	1.99 (0.00)	1.99 (−0.01)	2.01 (0.04)
$SD[\Delta d]$	11.09	3.47 (−2.79)	2.12 (−3.28)	2.47 (−3.15)
$SD[r_d]$	19.15	18.41 (−0.39)	13.64 (−2.91)	13.39 (−3.04)
$SD[r_f]$	2.72	3.21 (0.96)	3.70 (1.92)	3.87 (2.27)
$SD[z_d]$	0.45	0.46 (0.22)	0.25 (−3.17)	0.23 (−3.52)
$AC[r_f]$	0.68	0.95 (4.12)	0.90 (3.35)	0.88 (3.12)
$AC[z_d]$	0.89	0.92 (0.64)	0.85 (−0.86)	0.83 (−1.33)
$Corr[\Delta c, \Delta d]$	0.54	0.47 (−0.31)	0.41 (−0.59)	0.50 (−0.19)
$Corr[\Delta c, r_d]$	0.05	0.09 (0.57)	0.06 (0.23)	0.09 (0.61)
$Corr[\Delta d, r_d]$	0.07	0.19 (1.41)	0.15 (1.03)	0.18 (1.37)

(b) Data and average model-implied moments.

Table F.1: Baseline model.

Baseline Model: $\psi = 1.5$

Parameter	Current	Revised	Revised Max RA
γ	1.31 (1.29, 1.32)	78.83 (75.37, 82.75)	10.00 (10.00, 10.00)
β	0.9981 (0.9980, 0.9982)	0.9958 (0.9957, 0.9958)	0.9977 (0.9976, 0.9977)
ρ_a	0.9968 (0.9965, 0.9971)	0.9898 (0.9895, 0.9901)	0.9875 (0.9871, 0.9878)
σ_a	0.00031 (0.00030, 0.00033)	0.03566 (0.03515, 0.03618)	0.03946 (0.03898, 0.04000)
μ_y	0.0016 (0.0016, 0.0016)	0.0016 (0.0016, 0.0016)	0.0017 (0.0016, 0.0017)
μ_d	0.0015 (0.0015, 0.0016)	0.0020 (0.0020, 0.0021)	0.0009 (0.0009, 0.0010)
σ_y	0.0058 (0.0057, 0.0058)	0.0057 (0.0056, 0.0058)	0.0059 (0.0057, 0.0060)
ψ_d	1.54 (1.44, 1.63)	0.98 (0.88, 1.09)	1.05 (0.95, 1.16)
π_{dy}	0.816 (0.763, 0.873)	0.443 (0.409, 0.477)	0.600 (0.548, 0.662)
J	29.27 (28.62, 29.98)	48.26 (47.90, 48.64)	57.00 (56.39, 57.59)
pval	0.000 (0.000, 0.000)	0.000 (0.000, 0.000)	0.000 (0.000, 0.000)

(a) Average and $(5, 95)$ percentiles of the parameter estimates. The J-test has 6 degrees of freedom. The IES is 1.5.

Moment	Data	Current	Revised	Revised Max RA
$E[\Delta c]$	1.89	1.89 (0.00)	1.94 (0.21)	1.99 (0.40)
$E[\Delta d]$	1.47	1.84 (0.38)	2.44 (1.01)	1.12 (−0.37)
$E[r_d]$	6.51	5.46 (−0.66)	5.55 (−0.60)	4.00 (−1.57)
$E[r_f]$	0.25	0.25 (0.00)	0.38 (0.20)	1.09 (1.38)
$E[z_d]$	3.42	3.45 (0.18)	3.49 (0.49)	3.56 (1.03)
$SD[\Delta c]$	1.99	1.99 (0.00)	1.97 (−0.05)	2.02 (0.06)
$SD[\Delta d]$	11.09	3.47 (−2.79)	2.12 (−3.28)	2.44 (−3.16)
$SD[r_d]$	19.15	18.41 (−0.39)	13.61 (−2.92)	13.28 (−3.09)
$SD[r_f]$	2.72	3.21 (0.96)	3.70 (1.93)	3.88 (2.29)
$SD[z_d]$	0.45	0.46 (0.22)	0.25 (−3.19)	0.23 (−3.58)
$AC[r_f]$	0.68	0.95 (4.12)	0.90 (3.34)	0.88 (3.09)
$AC[z_d]$	0.89	0.92 (0.64)	0.85 (−0.87)	0.82 (−1.39)
$Corr[\Delta c, \Delta d]$	0.54	0.47 (−0.31)	0.41 (−0.59)	0.50 (−0.18)
$Corr[\Delta c, r_d]$	0.05	0.09 (0.57)	0.06 (0.23)	0.09 (0.61)
$Corr[\Delta d, r_d]$	0.07	0.19 (1.41)	0.15 (1.03)	0.18 (1.37)

(b) Data and average model-implied moments.

Table F.2: Baseline model.

Long-Run Risk Model: $\psi = 2.0$

	Omits $SD[r_f]$ & $AC[r_f]$		All Moments	
Parameter	No VR	Revised	No VR	Revised
γ	2.40 $(2.18, 2.62)$	2.58 $(2.33, 2.87)$	2.49 $(2.20, 2.75)$	2.43 $(2.19, 2.68)$
β	0.9992 $(0.9991, 0.9993)$	0.9983 $(0.9982, 0.9984)$	0.9992 $(0.9990, 0.9993)$	0.9991 $(0.9990, 0.9992)$
ρ_a	–	0.9811 $(0.9793, 0.9829)$	–	0.9537 $(0.9519, 0.9555)$
σ_a	–	0.0483 $(0.0460, 0.0507)$	–	0.0165 $(0.0159, 0.0171)$
μ_y	0.0016 $(0.0014, 0.0017)$	0.0016 $(0.0014, 0.0018)$	0.0016 $(0.0014, 0.0017)$	0.0016 $(0.0014, 0.0017)$
μ_d	0.0012 $(0.0009, 0.0015)$	0.0013 $(0.0009, 0.0016)$	0.0014 $(0.0011, 0.0017)$	0.0012 $(0.0009, 0.0015)$
σ_y	0.0041 $(0.0040, 0.0043)$	0.0040 $(0.0038, 0.0043)$	0.0050 $(0.0049, 0.0051)$	0.0041 $(0.0039, 0.0042)$
ψ_d	3.26 $(3.05, 3.47)$	2.89 $(2.66, 3.13)$	3.01 $(2.81, 3.18)$	3.25 $(3.01, 3.49)$
π_{dy}	0.593 $(0.354, 0.834)$	0.782 $(0.487, 1.114)$	0.132 $(-0.184, 0.419)$	0.640 $(0.392, 0.885)$
ϕ_d	2.31 $(2.13, 2.51)$	1.65 $(1.53, 1.78)$	2.11 $(1.88, 2.30)$	2.27 $(2.06, 2.50)$
ρ_x	0.9990 $(0.9986, 0.9993)$	0.9994 $(0.9993, 0.9995)$	0.9981 $(0.9974, 0.9988)$	0.9990 $(0.9986, 0.9994)$
ψ_x	0.0255 $(0.0242, 0.0269)$	0.0260 $(0.0247, 0.0273)$	0.0306 $(0.0287, 0.0328)$	0.0252 $(0.0240, 0.0266)$
J	20.91 $(20.16, 21.71)$	14.36 $(13.93, 14.78)$	54.54 $(53.67, 55.47)$	19.91 $(19.25, 20.63)$
pval	0.007 $(0.005, 0.010)$	0.026 $(0.022, 0.030)$	0.000 $(0.000, 0.000)$	0.011 $(0.008, 0.014)$
df	8	6	10	8

Table F.3: Long-run risk model. Average and $(5, 95)$ percentiles of the parameter estimates. The IES is 2.0.

		Omits $SD[r_f]$ & $AC[r_f]$		All Moments	
Moment	Data	No VR	Revised	No VR	Revised
$E[\Delta c]$	1.89	1.88 (−0.03)	1.89 (0.02)	1.88 (−0.02)	1.89 (0.00)
$E[\Delta d]$	1.47	1.48 (0.01)	1.55 (0.08)	1.68 (0.21)	1.48 (0.01)
$E[r_d]$	6.51	6.47 (−0.02)	6.46 (−0.04)	5.93 (−0.37)	6.49 (−0.01)
$E[r_f]$	0.25	0.30 (0.07)	0.26 (0.01)	0.28 (0.05)	0.26 (0.01)
$E[z_d]$	3.42	3.43 (0.03)	3.40 (−0.17)	3.42 (−0.05)	3.43 (0.02)
$SD[\Delta c]$	1.99	1.92 (−0.14)	1.94 (−0.10)	2.45 (0.95)	1.89 (−0.21)
$SD[\Delta d]$	11.09	5.62 (−2.00)	4.79 (−2.30)	6.40 (−1.71)	5.50 (−2.04)
$SD[r_d]$	19.15	18.03 (−0.59)	19.79 (0.34)	18.74 (−0.21)	18.16 (−0.52)
$SD[r_f]$	2.72	0.64 (−4.11)	5.56 (5.60)	0.87 (−3.66)	2.83 (0.21)
$SD[z_d]$	0.45	0.53 (1.34)	0.46 (0.08)	0.52 (1.12)	0.52 (1.17)
$AC[\Delta c]$	0.53	0.43 (−1.07)	0.46 (−0.75)	0.48 (−0.55)	0.43 (−1.09)
$AC[\Delta d]$	0.19	0.27 (0.77)	0.21 (0.20)	0.31 (1.16)	0.26 (0.69)
$AC[r_d]$	−0.01	0.01 (0.21)	−0.05 (−0.45)	0.00 (0.12)	−0.01 (0.04)
$AC[r_f]$	0.68	0.96 (4.34)	0.84 (2.44)	0.96 (4.25)	0.69 (0.14)
$AC[z_d]$	0.89	0.94 (1.09)	0.90 (0.27)	0.93 (0.91)	0.94 (1.02)
$Corr[\Delta c, \Delta d]$	0.54	0.48 (−0.27)	0.50 (−0.18)	0.44 (−0.45)	0.48 (−0.26)
$Corr[\Delta c, r_d]$	0.05	0.07 (0.30)	0.06 (0.21)	0.08 (0.47)	0.07 (0.29)
$Corr[\Delta d, r_d]$	0.07	0.24 (2.09)	0.19 (1.51)	0.28 (2.54)	0.24 (2.01)
$Corr[ep, z_{d,-1}]$	−0.16	−0.17 (−0.10)	−0.13 (0.37)	−0.15 (0.16)	−0.17 (−0.04)
$Corr[\Delta c, z_{d,-1}]$	0.19	0.66 (2.67)	0.60 (2.31)	0.69 (2.87)	0.65 (2.64)

Table F.4: Long-run risk model. Data and average model-implied moments.

Long-Run Risk Model: $\psi = 1.5$

	Omits $SD[r_f]$ & $AC[r_f]$		All Moments	
Parameter	No VR	Revised	No VR	Revised
γ	2.05 $(1.92, 2.18)$	2.44 $(2.22, 2.68)$	2.08 $(1.86, 2.31)$	2.13 $(1.97, 2.34)$
β	0.9995 $(0.9994, 0.9995)$	0.9988 $(0.9987, 0.9990)$	0.9995 $(0.9994, 0.9995)$	0.9995 $(0.9994, 0.9995)$
ρ_a	–	0.9801 $(0.9781, 0.9820)$	–	0.9514 $(0.9490, 0.9538)$
σ_a	–	0.0497 $(0.0472, 0.0521)$	–	0.0160 $(0.0153, 0.0168)$
μ_y	0.0015 $(0.0014, 0.0017)$	0.0016 $(0.0014, 0.0018)$	0.0015 $(0.0014, 0.0017)$	0.0015 $(0.0014, 0.0017)$
μ_d	0.0011 $(0.0008, 0.0015)$	0.0013 $(0.0009, 0.0017)$	0.0013 $(0.0010, 0.0017)$	0.0012 $(0.0008, 0.0015)$
σ_y	0.0042 $(0.0040, 0.0044)$	0.0040 $(0.0037, 0.0043)$	0.0051 $(0.0050, 0.0052)$	0.0041 $(0.0039, 0.0043)$
ψ_d	3.22 $(3.01, 3.44)$	3.10 $(2.83, 3.38)$	2.94 $(2.78, 3.11)$	3.29 $(3.06, 3.53)$
π_{dy}	0.552 $(0.311, 0.798)$	0.740 $(0.414, 1.066)$	0.191 $(-0.091, 0.456)$	0.611 $(0.372, 0.895)$
ϕ_d	2.29 $(2.12, 2.44)$	1.84 $(1.71, 1.98)$	2.02 $(1.85, 2.21)$	2.33 $(2.15, 2.51)$
ρ_x	0.9993 $(0.9991, 0.9995)$	0.9995 $(0.9993, 0.9995)$	0.9988 $(0.9983, 0.9993)$	0.9993 $(0.9990, 0.9994)$
ψ_x	0.0250 $(0.0238, 0.0263)$	0.0258 $(0.0246, 0.0270)$	0.0290 $(0.0275, 0.0307)$	0.0248 $(0.0236, 0.0259)$
J	21.89 $(21.04, 22.74)$	14.61 $(14.15, 15.08)$	52.00 $(51.02, 53.11)$	20.68 $(19.95, 21.47)$
pval	0.005 $(0.004, 0.007)$	0.024 $(0.020, 0.028)$	0.000 $(0.000, 0.000)$	0.008 $(0.006, 0.011)$
df	8	6	10	8

Table F.5: Long-run risk model. Average and $(5, 95)$ percentiles of the parameter estimates. The IES is 1.5.

		Omits $SD[r_f]$ & $AC[r_f]$		All Moments	
Moment	Data	No VR	Revised	No VR	Revised
$E[\Delta c]$	1.89	1.85 (−0.16)	1.89 (0.03)	1.86 (−0.09)	1.86 (−0.11)
$E[\Delta d]$	1.47	1.38 (−0.10)	1.55 (0.07)	1.58 (0.11)	1.42 (−0.05)
$E[r_d]$	6.51	6.85 (0.21)	6.47 (−0.03)	6.34 (−0.11)	6.77 (0.16)
$E[r_f]$	0.25	0.53 (0.45)	0.27 (0.02)	0.40 (0.24)	0.43 (0.29)
$E[z_d]$	3.42	3.44 (0.09)	3.40 (−0.16)	3.43 (0.02)	3.43 (0.08)
$SD[\Delta c]$	1.99	1.98 (−0.03)	1.90 (−0.18)	2.51 (1.06)	1.90 (−0.19)
$SD[\Delta d]$	11.09	5.70 (−1.97)	5.04 (−2.21)	6.36 (−1.73)	5.60 (−2.01)
$SD[r_d]$	19.15	17.81 (−0.71)	19.88 (0.39)	18.32 (−0.44)	17.98 (−0.61)
$SD[r_f]$	2.72	0.88 (−3.63)	5.75 (5.97)	1.18 (−3.03)	2.86 (0.27)
$SD[z_d]$	0.45	0.54 (1.41)	0.46 (0.08)	0.54 (1.36)	0.53 (1.23)
$AC[\Delta c]$	0.53	0.44 (−1.03)	0.46 (−0.78)	0.49 (−0.49)	0.43 (−1.08)
$AC[\Delta d]$	0.19	0.28 (0.81)	0.23 (0.33)	0.31 (1.14)	0.27 (0.75)
$AC[r_d]$	−0.01	0.02 (0.32)	−0.05 (−0.47)	0.01 (0.23)	0.00 (0.12)
$AC[r_f]$	0.68	0.96 (4.37)	0.83 (2.36)	0.96 (4.33)	0.69 (0.17)
$AC[z_d]$	0.89	0.94 (1.15)	0.90 (0.26)	0.94 (1.05)	0.94 (1.07)
$Corr[\Delta c, \Delta d]$	0.54	0.48 (−0.29)	0.49 (−0.22)	0.45 (−0.40)	0.48 (−0.27)
$Corr[\Delta c, r_d]$	0.05	0.07 (0.26)	0.06 (0.26)	0.08 (0.40)	0.07 (0.27)
$Corr[\Delta d, r_d]$	0.07	0.25 (2.14)	0.20 (1.64)	0.28 (2.51)	0.24 (2.07)
$Corr[ep, z_{d,-1}]$	−0.16	−0.18 (−0.21)	−0.13 (0.37)	−0.17 (−0.05)	−0.18 (−0.12)
$Corr[\Delta c, z_{d,-1}]$	0.19	0.66 (2.68)	0.60 (2.32)	0.70 (2.89)	0.66 (2.65)

Table F.6: Long-run risk model. Data and average model-implied moments.

Extended Long-Run Risk Model: $\psi = 2.0$

	Omits $SD[r_f]$ & $AC[r_f]$			All Moments		
Ptr	No VR+SV	Demand	Demand+SV	No VR+SV	Demand	Demand+SV
γ	2.31 (2.15, 2.48)	4.35 (3.66, 4.85)	6.92 (3.58, 11.51)	2.43 (2.29, 2.56)	3.02 (2.79, 3.27)	5.85 (4.67, 6.93)
β	0.9990 (0.9989, 0.9992)	0.9992 (0.9990, 0.9992)	0.9984 (0.9978, 0.9989)	0.9985 (0.9984, 0.9986)	0.9992 (0.9992, 0.9993)	0.9984 (0.9982, 0.9986)
ρ_a	–	0.9813 (0.8971, 0.9892)	0.9877 (0.9825, 0.9918)	–	0.9586 (0.9565, 0.9606)	0.9925 (0.9908, 0.9934)
σ_a	–	0.0393 (0.0365, 0.0479)	0.0351 (0.0314, 0.0383)	–	0.0184 (0.0176, 0.0191)	0.0285 (0.0268, 0.0297)
μ_y	0.0016 (0.0015, 0.0016)	0.0016 (0.0015, 0.0016)	0.0016 (0.0015, 0.0016)	0.0016 (0.0014, 0.0018)	0.0015 (0.0015, 0.0016)	0.0016 (0.0015, 0.0016)
μ_d	0.0015 (0.0014, 0.0017)	0.0015 (0.0013, 0.0017)	0.0015 (0.0013, 0.0017)	0.0013 (0.0009, 0.0016)	0.0014 (0.0012, 0.0016)	0.0015 (0.0013, 0.0017)
σ_y	0.0021 (0.0007, 0.0037)	0.0039 (0.0037, 0.0041)	0.0014 (0.0002, 0.0031)	0.0007 (0.0003, 0.0014)	0.0041 (0.0040, 0.0043)	0.0006 (0.0001, 0.0014)
ψ_d	3.39 (3.06, 3.73)	–	–	3.00 (2.81, 3.21)	–	–
π_{dy}	0.654 (0.299, 0.990)	–	–	0.754 (0.482, 1.038)	–	–
ϕ_d	2.20 (1.98, 2.42)	3.41 (2.98, 3.71)	3.20 (2.59, 3.62)	1.93 (1.84, 2.03)	2.68 (2.51, 2.87)	2.76 (2.60, 2.89)
ρ_x	0.9978 (0.9971, 0.9984)	0.9949 (0.9938, 0.9963)	0.9951 (0.9938, 0.9965)	0.9993 (0.9991, 0.9995)	0.9978 (0.9973, 0.9983)	0.9965 (0.9958, 0.9971)
ψ_x	0.0292 (0.0268, 0.0319)	0.0378 (0.0341, 0.0411)	0.0375 (0.0336, 0.0422)	0.0253 (0.0241, 0.0266)	0.0290 (0.0276, 0.0305)	0.0342 (0.0319, 0.0367)
π_{ya}	–	-0.039 (-0.049, -0.030)	-0.042 (-0.057, -0.027)	–	-0.053 (-0.072, -0.035)	-0.051 (-0.067, -0.034)
π_{da}	–	-0.712 (-0.750, -0.552)	-0.792 (-0.848, -0.745)	–	-1.044 (-1.078, -1.008)	-0.866 (-0.897, -0.836)
ρ_{σ_y}	0.9964 (0.9938, 0.9982)	–	0.7231 (0.1739, 0.9787)	0.9608 (0.9559, 0.9651)	–	0.7758 (0.6417, 0.8724)
ν_y	3.6e−6 (2.1e−6, 5.1e−6)	–	2.1e−5 (7.0e−6, 3.6e−5)	1.3e−5 (1.2e−5, 1.5e−5)	–	2.7e−5 (2.1e−5, 3.5e−5)
J	15.76 (15.11, 16.44)	9.59 (9.20, 9.99)	8.79 (7.98, 9.39)	18.44 (17.74, 19.18)	13.99 (13.40, 14.54)	9.77 (9.32, 10.22)
pval	0.015 (0.012, 0.019)	0.143 (0.125, 0.163)	0.068 (0.052, 0.092)	0.018 (0.014, 0.023)	0.082 (0.069, 0.099)	0.135 (0.116, 0.157)
df	6	6	4	8	8	6

Table F.7: Extended long-run risk models. Average and $(5, 95)$ percentiles of the parameter estimates. The IES is 2.0.

		Omits $SD[r_f]$ & $AC[r_f]$			All Moments		
Moment	Data	No VR+SV	Demand	Demand+SV	No VR+SV	Demand	Demand+SV
$E[\Delta c]$	1.89	1.88 (−0.04)	1.89 (0.00)	1.89 (0.01)	1.91 (0.08)	1.85 (−0.15)	1.89 (0.03)
$E[\Delta d]$	1.47	1.81 (0.35)	1.78 (0.32)	1.79 (0.33)	1.53 (0.06)	1.73 (0.27)	1.84 (0.38)
$E[r_d]$	6.51	6.14 (−0.24)	5.62 (−0.56)	5.67 (−0.52)	6.77 (0.16)	5.90 (−0.38)	5.83 (−0.43)
$E[r_f]$	0.25	0.24 (−0.03)	0.28 (0.04)	0.26 (0.00)	0.09 (−0.27)	0.45 (0.33)	0.15 (−0.18)
$E[z_d]$	3.42	3.41 (−0.11)	3.39 (−0.23)	3.39 (−0.24)	3.42 (−0.03)	3.40 (−0.16)	3.39 (−0.20)
$SD[\Delta c]$	1.99	1.91 (−0.17)	1.98 (−0.03)	1.99 (0.00)	2.03 (0.09)	1.98 (−0.03)	2.12 (0.26)
$SD[\Delta d]$	11.09	5.64 (−1.99)	10.71 (−0.14)	10.58 (−0.19)	5.38 (−2.09)	7.58 (−1.28)	9.47 (−0.59)
$SD[r_d]$	19.15	19.51 (0.19)	18.94 (−0.11)	19.04 (−0.06)	18.67 (−0.25)	18.14 (−0.53)	18.53 (−0.33)
$SD[r_f]$	2.72	1.07 (−3.25)	3.98 (2.49)	3.42 (1.39)	2.47 (−0.50)	2.99 (0.53)	2.66 (−0.12)
$SD[z_d]$	0.45	0.48 (0.40)	0.47 (0.32)	0.46 (0.22)	0.51 (0.93)	0.51 (0.91)	0.49 (0.56)
$AC[\Delta c]$	0.53	0.45 (−0.83)	0.43 (−1.06)	0.44 (−1.01)	0.44 (−0.95)	0.43 (−1.06)	0.45 (−0.92)
$AC[\Delta d]$	0.19	0.26 (0.65)	0.17 (−0.20)	0.16 (−0.33)	0.24 (0.49)	0.21 (0.20)	0.17 (−0.21)
$AC[r_d]$	−0.01	−0.01 (0.02)	0.02 (0.35)	−0.01 (−0.06)	−0.03 (−0.25)	0.02 (0.36)	−0.03 (−0.20)
$AC[r_f]$	0.68	0.93 (3.89)	0.85 (2.68)	0.83 (2.29)	0.69 (0.07)	0.71 (0.50)	0.70 (0.35)
$AC[z_d]$	0.89	0.92 (0.57)	0.90 (0.26)	0.90 (0.17)	0.93 (0.89)	0.93 (0.88)	0.91 (0.51)
$Corr[\Delta c, \Delta d]$	0.54	0.49 (−0.22)	0.54 (0.02)	0.53 (−0.06)	0.51 (−0.13)	0.48 (−0.27)	0.52 (−0.10)
$Corr[\Delta c, r_d]$	0.05	0.07 (0.29)	0.11 (0.88)	0.10 (0.81)	0.06 (0.20)	0.09 (0.55)	0.10 (0.76)
$Corr[\Delta d, r_d]$	0.07	0.22 (1.87)	0.03 (−0.43)	0.04 (−0.37)	0.22 (1.77)	0.14 (0.80)	0.07 (0.04)
$Corr[ep, z_{d,-1}]$	−0.16	−0.27 (−1.06)	−0.10 (0.70)	−0.13 (0.41)	−0.23 (−0.67)	−0.15 (0.18)	−0.13 (0.33)
$Corr[\Delta c, z_{d,-1}]$	0.19	0.58 (2.23)	0.62 (2.46)	0.61 (2.40)	0.65 (2.64)	0.66 (2.66)	0.63 (2.52)

Table F.8: Extended long-run risk model. Data and average model-implied moments.

Extended Long-Run Risk Model: $\psi = 1.5$

	Omits $SD[r_f]$ & $AC[r_f]$			All Moments		
Ptr	No VR+SV	Demand	Demand+SV	No VR+SV	Demand	Demand+SV
γ	1.98 $(1.80, 2.16)$	3.71 $(3.09, 4.66)$	7.10 $(3.67, 11.70)$	2.17 $(2.06, 2.28)$	2.61 $(2.32, 2.96)$	4.57 $(3.67, 5.54)$
β	0.9994 $(0.9992, 0.9995)$	0.9993 $(0.9990, 0.9995)$	0.9989 $(0.9983, 0.9994)$	0.9990 $(0.9990, 0.9991)$	0.9995 $(0.9995, 0.9995)$	0.9991 $(0.9989, 0.9992)$
ρ_a	–	0.9159 $(0.8586, 0.9874)$	0.9792 $(0.9217, 0.9904)$	–	0.9564 $(0.9536, 0.9592)$	0.9898 $(0.9860, 0.9929)$
σ_a	–	0.0485 $(0.0394, 0.0566)$	0.0378 $(0.0338, 0.0439)$	–	0.0178 $(0.0168, 0.0188)$	0.0260 $(0.0231, 0.0287)$
μ_y	0.0016 $(0.0015, 0.0017)$	0.0016 $(0.0015, 0.0016)$	0.0016 $(0.0015, 0.0016)$	0.0016 $(0.0014, 0.0018)$	0.0015 $(0.0014, 0.0016)$	0.0016 $(0.0015, 0.0017)$
μ_d	0.0015 $(0.0012, 0.0016)$	0.0015 $(0.0013, 0.0017)$	0.0015 $(0.0013, 0.0017)$	0.0012 $(0.0009, 0.0016)$	0.0014 $(0.0011, 0.0016)$	0.0015 $(0.0013, 0.0017)$
σ_y	0.0012 $(0.0004, 0.0027)$	0.0039 $(0.0037, 0.0041)$	0.0012 $(0.0001, 0.0027)$	0.0007 $(0.0003, 0.0013)$	0.0041 $(0.0039, 0.0043)$	0.0006 $(0.0000, 0.0013)$
ψ_d	3.32 $(3.02, 3.66)$	–	–	3.07 $(2.88, 3.27)$	–	–
π_{dy}	0.728 $(0.371, 1.079)$	–	–	0.692 $(0.400, 0.970)$	–	–
ϕ_d	2.14 $(1.93, 2.35)$	3.17 $(2.71, 3.82)$	3.31 $(2.71, 3.75)$	2.02 $(1.91, 2.13)$	2.65 $(2.42, 2.92)$	2.62 $(2.44, 2.79)$
ρ_x	0.9987 $(0.9981, 0.9992)$	0.9962 $(0.9943, 0.9976)$	0.9954 $(0.9940, 0.9971)$	0.9994 $(0.9993, 0.9995)$	0.9984 $(0.9978, 0.9989)$	0.9978 $(0.9972, 0.9983)$
ψ_x	0.0268 $(0.0248, 0.0288)$	0.0345 $(0.0310, 0.0392)$	0.0364 $(0.0318, 0.0406)$	0.0253 $(0.0242, 0.0265)$	0.0276 $(0.0262, 0.0291)$	0.0305 $(0.0283, 0.0326)$
π_{ya}	–	-0.033 $(-0.042, -0.026)$	-0.039 $(-0.052, -0.025)$	–	-0.055 $(-0.078, -0.035)$	-0.053 $(-0.071, -0.033)$
π_{da}	–	-0.578 $(-0.718, -0.481)$	-0.742 $(-0.800, -0.620)$	–	-1.065 $(-1.111, -1.024)$	-0.891 $(-0.954, -0.833)$
ρ_{σ_y}	0.9949 $(0.9900, 0.9973)$	–	0.6669 $(0.1155, 0.9654)$	0.9545 $(0.9489, 0.9601)$	–	0.7629 $(0.6252, 0.8652)$
ν_y	4.5e−6 $(3.2e{-6}, 6.1e{-6})$	–	2.4e−5 $(8.8e{-6}, 3.8e{-5})$	1.4e−5 $(1.3e{-5}, 1.6e{-5})$	–	2.8e−5 $(2.2e{-5}, 3.6e{-5})$
J	16.16 $(15.52, 16.78)$	9.85 $(9.54, 10.12)$	9.03 $(8.34, 9.55)$	19.38 $(18.62, 20.24)$	15.22 $(14.53, 15.84)$	11.18 $(10.65, 11.77)$
pval	0.013 $(0.010, 0.017)$	0.131 $(0.120, 0.146)$	0.061 $(0.049, 0.080)$	0.013 $(0.009, 0.017)$	0.055 $(0.045, 0.069)$	0.084 $(0.067, 0.100)$
df	6	6	4	8	8	6

Table F.9: Extended long-run risk models. Average and $(5, 95)$ percentiles of the parameter estimates. The IES is 1.5.

		Omits $SD[r_f]$ & $AC[r_f]$			All Moments		
Moment	Data	No VR+SV	Demand	Demand+SV	No VR+SV	Demand	Demand+SV
$E[\Delta c]$	1.89	1.88 (−0.04)	1.88 (−0.01)	1.89 (0.01)	1.93 (0.15)	1.83 (−0.24)	1.90 (0.06)
$E[\Delta d]$	1.47	1.73 (0.27)	1.77 (0.31)	1.79 (0.33)	1.46 (−0.02)	1.68 (0.22)	1.78 (0.32)
$E[r_d]$	6.51	6.33 (−0.11)	5.65 (−0.54)	5.67 (−0.52)	6.83 (0.20)	6.15 (−0.23)	6.00 (−0.32)
$E[r_f]$	0.25	0.25 (−0.01)	0.30 (0.08)	0.25 (0.00)	0.01 (−0.40)	0.66 (0.67)	0.09 (−0.27)
$E[z_d]$	3.42	3.41 (−0.08)	3.39 (−0.26)	3.39 (−0.24)	3.43 (0.03)	3.40 (−0.13)	3.40 (−0.19)
$SD[\Delta c]$	1.99	1.90 (−0.20)	1.99 (−0.01)	1.99 (−0.01)	2.07 (0.16)	1.97 (−0.04)	2.11 (0.24)
$SD[\Delta d]$	11.09	5.51 (−2.04)	10.47 (−0.22)	10.67 (−0.15)	5.60 (−2.01)	7.49 (−1.32)	8.89 (−0.80)
$SD[r_d]$	19.15	19.34 (0.10)	19.24 (0.05)	19.03 (−0.06)	18.70 (−0.24)	17.81 (−0.71)	18.13 (−0.54)
$SD[r_f]$	2.72	1.20 (−2.99)	9.61 (13.58)	4.24 (2.99)	2.31 (−0.81)	3.01 (0.57)	2.66 (−0.12)
$SD[z_d]$	0.45	0.48 (0.50)	0.46 (0.18)	0.46 (0.23)	0.51 (0.97)	0.52 (1.08)	0.50 (0.84)
$AC[\Delta c]$	0.53	0.45 (−0.88)	0.43 (−1.07)	0.43 (−1.04)	0.45 (−0.91)	0.43 (−1.06)	0.44 (−0.95)
$AC[\Delta d]$	0.19	0.25 (0.57)	0.16 (−0.34)	0.16 (−0.27)	0.26 (0.61)	0.21 (0.20)	0.17 (−0.19)
$AC[r_d]$	−0.01	−0.01 (0.02)	0.01 (0.21)	−0.01 (0.03)	−0.03 (−0.27)	0.03 (0.43)	−0.02 (−0.15)
$AC[r_f]$	0.68	0.93 (3.87)	0.56 (−1.86)	0.81 (2.04)	0.69 (0.07)	0.71 (0.51)	0.71 (0.48)
$AC[z_d]$	0.89	0.92 (0.66)	0.91 (0.42)	0.90 (0.21)	0.93 (0.90)	0.94 (1.00)	0.93 (0.75)
$Corr[\Delta c, \Delta d]$	0.54	0.50 (−0.18)	0.54 (−0.01)	0.53 (−0.04)	0.50 (−0.16)	0.48 (−0.28)	0.50 (−0.16)
$Corr[\Delta c, r_d]$	0.05	0.06 (0.25)	0.10 (0.70)	0.11 (0.82)	0.06 (0.23)	0.08 (0.47)	0.09 (0.59)
$Corr[\Delta d, r_d]$	0.07	0.22 (1.81)	0.04 (−0.37)	0.04 (−0.38)	0.22 (1.89)	0.14 (0.83)	0.09 (0.24)
$Corr[ep, z_{d,-1}]$	−0.16	−0.27 (−1.09)	−0.11 (0.57)	−0.12 (0.42)	−0.23 (−0.67)	−0.16 (0.02)	−0.16 (0.09)
$Corr[\Delta c, z_{d,-1}]$	0.19	0.59 (2.30)	0.64 (2.56)	0.62 (2.43)	0.66 (2.66)	0.66 (2.66)	0.65 (2.59)

Table F.10: Long-run risk model. Data and average model-implied moments.