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Higher Education Funding, Welfare and Inequality in Equilibrium

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Abstract

This paper analyses theoretically and quantitatively the effect that different higher education funding policies have on welfare (on aggregate and at the individual level) and wealth inequality. A heterogeneous agent model in continuous time, which has uninsurable income risk and endogenous educational choice is used to evaluate five different higher education financing schemes. Educational investments can be self financed, supported by government guaranteed student loans - that may come with or without income contingent support - or be covered by the public sector. When educational costs are small, differences in outcomes amongst systems are negligible. On the other hand, when these costs rise to realistic levels we see that there can be large gains in welfare and significant drops in inequality by moving to a system with more public sector support. This support can come in the form of tuition subsidies and/or income contingent student loans. However, as the cost of education and the share of debtors in society gets larger, it is preferable to increase public support in the form of tuition subsidies. The reason is that there is a pecuniary externality of debt that gets magnified when student loans become excessive. While I identify large steady state welfare gains from more public sector financing, I show that transition costs can be large enough to justify the status quo.

JEL codes: D52, D58, E24, I22, I23
Keywords: Incomplete markets, Higher education funding, Human capital

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Introduction

Student debt is now the second largest type of household debt in the United States, recently surpassing 1.6 trillon dollars. As shown in Figure (1), the average student at an American university is graduating with over \$34,000 of debt and the stock of student debt, which continues to grow, recently reached 8% of all personal disposable income. While the United States is usually held as a basket case, the United Kingdom is not fairing any better. According to the Institute of Fiscal Studies and the Sutton Trust, the average UK student graduates with over £44,000 worth of debt - Kirby (2016). The rising costs of higher education, student debt defaults in the US and recent modifications of the UK government accounting of student loans have continued to exacerbate calls from the left in favour of either student loan debt forgiveness and/or free tuition at public universities. Those opposing such policies argue that they are regressive. Since the benefits of higher education accrue to the individual pursuing a college degree, while the costs are shared amongst tax payers, many of whom who do not enjoy such benefit, these policies might actually make matters worse (for instance, by reinforcing inequality).

Income contingent student loans have been proposed as an efficient solution for financing tertiary education. They increase access to higher education for low income households by reducing the capital market imperfection in educational investments and lessening income uncertainty with protections against bad shocks. The leading proponents for financing higher education with income contingent student loans argue that such a system is the best suited at balancing equity and efficiency trade-offs, is the 'most efficient' and that 'tax funding (of higher education) is unfair' - Barr and Crawford (2000)[1].

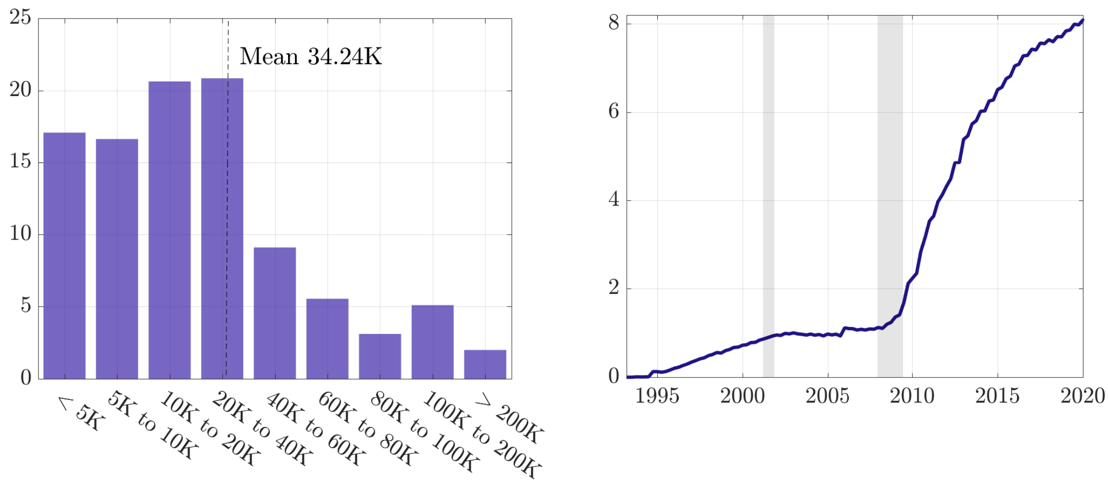

Figure 1: Left: % of borrowers by student loan balances at the 2nd quarter of 2020. Source: U.S. Department of Education. Right: Federal student debt as a percentage of disposable personal income. Source: BEA and Board of Governors

There are considerable reasons to ask if this should be the preferred way to finance tertiary education. First, while there seems to be a consensus, undisputed in some policy circles, on financing higher education with income contingent loans, there is no unique and preferred policy for financing higher education in the OECD[2]. In fact there is plenty of variability, as depicted

[1] *'(income contingent student debt) is efficient, in that it addresses the major capital market imperfection... It is fair, because people with low earnings make low repayments and people with low lifetime earnings do not repay their loan in full... tax funding (of higher education) is unfair'.*

[2] The same can be said of economists working on this field of research. There is no consensus on which is the best way to finance higher education. Even in the small subset of the literature cited further below we may find that either graduate taxes, tuition subsidies, merit grants or income contingent student loans can be found to be the preferred policy recommendation.

in Figure (2). The bars represent the share of GDP allocated to tertiary education. The red (blue) part captures the share of that expenditure coming from the public (private) sector. South Korea, Japan and anglophone countries fund tertiary education with a relatively higher participation of the private sector. Continental European countries, especially Nordic ones, have the state playing a larger role in financing higher education. Second, contrary to popular perceptions of generous tax financed tertiary education, it appears that larger public spending in higher education, relative to GDP, is associated with lower income inequality in the OECD (see Figure (27) in the appendix). Finally, a set of papers in heterogeneous agent macroeconomics have shown that agents' savings behaviour may generate pecuniary externalities that can steer away the economy from efficiency - Aiyagari (1994), Obiols-Homs (2011), Dávila et al. (2012), Nuño and Moll (2017) and Angelopoulos et al. (2017). It is not clear a priori if a system of higher education relying on student loans, tuition subsidies or on private self-financing may exacerbate the aforementioned externalities by pushing society to under/over accumulate human and physical capital.

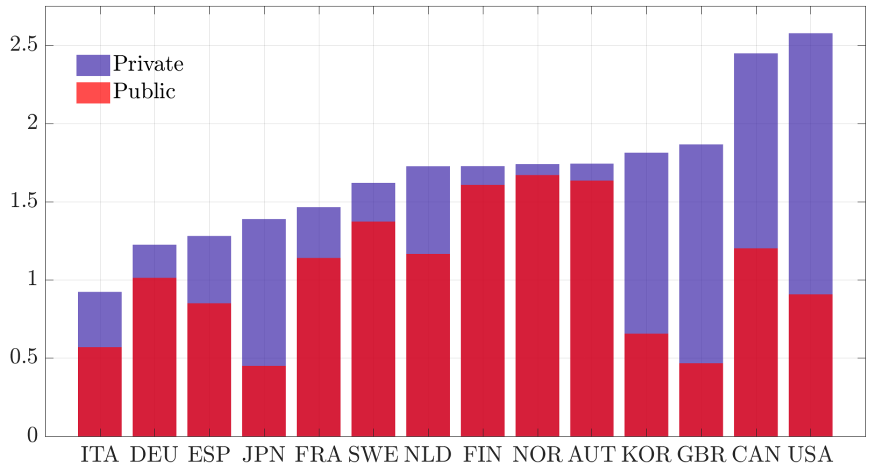

Figure 2: Public and private expenditure on tertiary education relative to GDP in 2015. Source: OECD

In this paper I propose a framework to evaluate the welfare and wealth inequality outcomes of five different higher education financing schemes. I use a heterogeneous agent production economy in continuous time, following Nuño and Moll (2017), extended to allow endogenous educational choices. In the first scheme, called self financing, there is no access to student loans nor tuition subsidies from the government. Only agents with sufficient wealth can afford education. In the next regime the government provides a student loan facility without income contingency features, i.e. agents must pay back their student loans regardless of their income. I then introduce two variants that offer income contingency protections; this is done to highlight how small tweaks in the design of the income contingent student loan program can generate significantly different outcomes. Finally, in a fifth regime the government provides support in the form of tuition subsidies.

This paper highlights the importance of assessing the macroeconomic impact of higher education financing under the light of the price effects of debt described in Obiols-Homs (2011)[3]; thus making the link between borrowing limits and welfare with higher education financing.

[3]Obiols-Homs (2011), shows that too lax borrowing constraints may drag down aggregate welfare. When society has a large fraction of net debtors, the beneficial *quantity effect* of large debt limits (because individuals can continue to optimise and smooth consumption with debt), can be overwhelmed by the *price effect* of more debtors putting upward pressure on the interest rate.

The main finding of this paper is that there is a pecuniary externality of debt that manifests itself through the student loan system and becomes more patent as the cost of education rises. When education is relatively easy to achieve, the capital market failures associated with educational investments do not matter enough to warrant government intervention. When the costs of education are calibrated to realistic values, government guaranteed income contingent loans and tuition subsidies provide the best alternatives to finance tertiary education, with the latter yielding the largest welfare gains and drops in wealth inequality. However, as the cost of education and the share of debtors in society gets larger, it becomes much more preferable to increase public support in the form of tuition subsidies. This is particularly important, since tuition costs have been rising in many countries. For instance, these costs have grown consistently faster than CPI, housing and healthcare in the United States - see Figure (28) in the appendix.

By using partial/general equilibrium comparisons, aggregate and individual measures of welfare and a large sensitivity analysis I show that results are affected by two forces: 1) *the shape of the endogenous distribution of income and wealth* and 2) *the price and quantity effects of debt* described in Obiols-Homs (2011). With regards to the former, I show that subsidies, as opposed to loans, generate wealth distributions with smaller amounts of the population as net debtors. Additionally, the equilibrium interest rate ends up being higher, which rewards a society with relatively more lenders. Moreover, equilibria with higher net debtor shares tend to be associated with larger wealth inequality. These distributional impacts have an influence on the public cost of higher education. For instance, I show that depending on the design of the student loan system, the fiscal burden generated by the loan program may turn out to be higher than that of tuition subsidies. While the price and quantity effects of debt are intricately linked to the distributional outcomes of each higher education financing scheme, I isolate the effect of prices by evaluating aggregate and individual welfare of each regime before and after markets clear. Welfare gains of policy changes in higher education financing can either double or halve depending on whether we let markets clear. This stresses the need to evaluate policy changes in general equilibrium.

This article emphasises the importance of evaluating the transitional dynamics of policy changes. While I show substantial steady state gains in terms of consumption equivalent variation of different higher education systems, large transition costs from one regime to another may justify the status-quo. Moving from self financing to a system of public funding (the system yielding the largest welfare gains in the baseline calibration) or to one relying on income contingent loans can be costly enough to eat up more than 70 % of the steady state welfare gains. As a consequence, just comparing steady states may be misleading for policy.

Related literature: There is a large literature at the cross-roads of macroeconomics, education financing and its distributional impact - García-Peñalosa and Wälde (2000), Bénabou (2002), Hanushek et al. (2003), Bovenberg and Jacobs (2005), Dearden et al. (2008), Johnson (2013), Herrington (2015), Cai and Heathcote (2018) and Luo and Mongey (2019). The closest studies to the one presented here are Ionescu and Simpson (2016), Krueger and Ludwig (2016), Abbott et al. (2013) and Hanushek et al. (2014). In the first article the authors arrive at similar findings as in this paper using a life-cycle environment: tax financed grants can have a larger impact in improving welfare than increasing student loan limits, especially if these are too lax. The present study seeks to expand on their results in two ways: endogenising the risk free equilibrium interest rate and factoring transition costs. As shown in this paper, a fixed interest rate dampens one of the major forces driving welfare. Hence, welfare and inequality are computed before and after general equilibrium effects kick in. While the model presented here fails to capture important aspects of lifetime earnings by abstracting from age, it allows us to go beyond steady state comparisons and consider transitional dynamics at a relatively lower computational cost. As will be shown, it is not enough to demonstrate that one regime is better than another, the costs of transition must also be taken into account as they can be large enough to significantly lessen the desirability of changing to another higher education system.

The paper by Krueger and Ludwig (2016) considers transitions, amongst concerns of optimal taxation and education finance. The paper, however, did not introduce income contingent student loans. This paper abstracts from optimal taxation, on purpose, so that we can see how the results go through even with a flat tax and no public externality in education. One of the most popular arguments against tax financing of higher education is that it is regressive and that in turn, it may reinforce inequality. In this paper I show that even with a tax schedule that is not progressive, we may still find that public financing can be welfare improving for all segments, or at least the vast majority, of society. Abbott et al. (2013) cast similar questions as in this study with a detail-rich life-cycle environment. They find that merit based grants and the current student loan system in the U.S. provides substantial increases in welfare. As the study focused on aspects of the U.S. student loan programs it did not expand on income contingent schemes. Hanushek et al. (2014) compare different higher education funding schemes, as in this paper, with an overlapping generations model. Their findings are somewhat similar to those herein and I contribute to their results by looking at disaggregated measures of welfare and a large sensitivity analysis of the effects of borrowing constraints. Whilst the papers mentioned above focus on the U.S. (controlling for variables such as ability, college quality, gender and elasticity of substitution between educated and non-educated workers), I propose a simpler framework that is general enough to allow for comparisons of different higher education systems. This allows us to evaluate the impact that the most salient features of each educational system (American, British and Continental European) have on welfare and inequality.

Finally, the model developed herein contributes to the literature on debt limits and welfare, confirming the presence of price and quantity effects in environments with two types of debt and the simultaneous presence of physical and human capital. For instance, this paper expands on Angelopoulos et al. (2017), who show the pecuniary externalities arising from agents' different savings policies, which vary by education and income profiles. Whereas Angelopoulos et al. (2017) fix exogenously the agent types and restricts flows between groups, this paper endogenises such flows through optimal education choice and evaluates how different higher education systems affect the composition of types in society. Additionally, this work complements findings in Caucutt and Lochner (2020), deepening our understanding of how borrowing constraints affect educational investments not only through the dynamic complementarity of early and late life investments in education - but through price and distributional effects as well.

This paper is structured as follows. In the first section I describe the model. In the next section I show steady state comparisons of the different higher education regimes with a large sensitivity analysis on various parameters and perform partial/general equilibrium welfare comparisons. In the third section I analyse if it is worth transitioning from one higher education system to another, specifically from a benchmark towards either of the two top alternatives. The fourth section concludes.

1 Model

The framework developed herein is based on Aiyagari (1994), Achdou et al. (2017) and Nuño and Moll (2017). Time is continuous. There is a continuum of unit measure of agents that are ex-ante identical but ex-post heterogeneous in their wealth, education status and employment state. The main difference is that there is now an endogenous choice of attending university. Getting a college degree increases the labour efficiency of agents. The production side of the economy follows Nuño and Moll (2017); a representative firm hires labour and rents capital to produce output. The labour input is in efficiency units and it's distribution is determined endogenously.

I will use this framework to rank five different higher education (HE) systems. The first regime, called 'self financing (SF)', depicts a system where there is no government funding of tertiary education nor government guaranteed student loans. Only agents that can cover P, the cost of a college degree, are allowed to go to university. The second regime then introduces government guaranteed student loans, referred to as NICL (non income contingent loan). The next system makes student loans income-contingent. Only those above a certain earnings threshold repay their student loans, and after 30 years the remaining balance of student debt gets cancelled. The ICL variant has two versions; one is closer to the NICL (ICL1) while the other relies on the repayment scheme that is in place in Britain (ICL2). This will shed light on how the design of loan repayments can affect outcomes. Finally, a fifth regime introduces a government subsidy for tuition fees, reducing the cost agents face to $P(1-s)$. This system does not have government guaranteed student loans and is called 'TS' for tuition subsidies[4]. I will first give a brief overview of agents in the economy. Each type of agent will broadly face the same problem regardless of the HE system. Nonetheless, each regime will have peculiarities affecting agents' budget constraints. Finally, I will then go into more detail of how the objective and constraints of each type of agent is mathematically formalised for each HE system.

1.1 Agents

Besides students, there are two broad groups of agents in the economy: those without a college degree and those with one. Each of these groups is subdivided into two categories: employed and unemployed. Figure (3) illustrates how agents move between the five types, denoted by θ_i and $i = 1, 2, 3, 4, 5$. The usual flows into and out of employment are written with subscripts denoting the origin and destination (λ_{12} is the flow of non college grads from the unemployed to employed state).

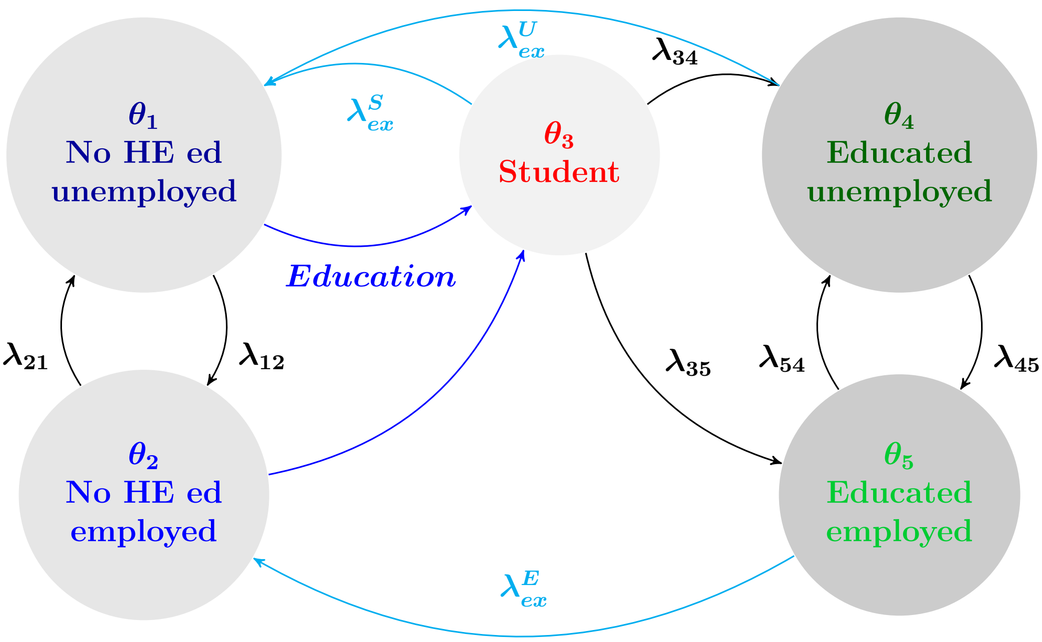

Figure 3: Agent flows

The novelty is the endogenous flow from no HE education to students to HE education.

[4]The model formulation is meant to capture the most salient features of the American (NICL), British (ICL) and Continental/European (TS) systems. In reality, these countries have a mix of the ingredients presented in this study.

Additionally, a distinctive feature are the flows in the opposite direction λ_{ex}^U and λ_{ex}^E. They represent the rate at which a college degree depreciates. There are three reasons why I introduced such flows. The first reason is that in an infinitely lived agents environment, education becomes an absorbing state if we shut off λ_{ex}^U and λ_{ex}^E. The second reason is that these flows can capture how technological advances make redundant some careers that required tertiary education qualifications. This opens the door to study policy in an environment of increasing automation. The approach is not different from Ben-Porath models, where skills can depreciate through time - Ben-Porath (1967) and Manuelli et al. (2012). Furthermore, given that $\lambda_{ex}^U > \lambda_{ex}^E$ we may capture how unemployment spells can have an impact on skills and labour market outcomes - Arrazola et al. (2005) and Hugonnier et al. (2019). Third, while I consider natural to account for HE degrees depreciating in an infinitely lived environment, as in Hugonnier et al. (2019), these flows can be re-interpreted as mortality rates with minor tweaks to the model[5].

The transitions between unemployment and employment will be calibrated so as to capture that people with a HE degree tend to face a better job market (higher transition rate into employment and a lower one into unemployment, relative to those without a college degree). Finally, in order to capture uncertainty at the student stage I introduce a college drop out rate λ_{ex}^S. All agents face a standard consumption-savings problem with a debt limit on b, the amount on money they have in a checking account. The debt limit $\underline{b}$ is tighter than the natural borrowing limit, i.e. $\underline{b} > -z_1/r$, where z_1 is the lowest possible income. Agents pay (receive) interest r if they are net debtors (savers) and r is the risk free rate. When we introduce government guaranteed student loans a, agents will be able to finance the cost of higher education with both a and b. There is also a finite debt limit on student loans $\underline{a}$. Common to all agents, preferences are determined by a strictly increasing and strictly concave utility function $u(c)$ and the subjective discount rate ρ. Agents have CRRA preferences described by $u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$. In the following subsection I formalise the different type of agents' problem.

1.1.1 Unemployed (θ_1) and employed (θ_2) and no higher education

As in Achdou et al. (2017) agents maximise utility subject to a flow budget constraint. The only idiosyncratic shock affects income z_i, $i = 1, 2$, which is a two point jump process, where λ_{12} and λ_{21} are the Poisson rates of jumps from unemployed to employed and vice-versa, respectively. Besides choosing consumption, the agent can now choose a time T where, if it has sufficient funds to cover the cost of education, it enrols in university and becomes a student. Since the problem will be solved in the state domain, we will essentially be looking for a free boundary in b (or in b and a in the systems with student loans). Let such boundaries in b and a be denoted by a $\dagger$ superscript and let b^* and a^* represent the target points where agents end up at after covering education costs. The general problem of a type 1 agent in any higher education regime is shown next.

$$V_i(b_t, a_t) = \max_{c,T} E_t \left[\int_t^T e^{-\rho(s-t)} u(c) \mathrm{d}s + e^{-\rho(T-t)} V_3(b^*, a^*) \right] \tag{1}$$

$$\text{s.t. } \mathrm{d}b = (z_i + rb - c + \phi_b(b, a, \theta_i))\, \mathrm{d}t, \quad b \geq \underline{b} > -\infty, \tag{2}$$

$$\mathrm{d}a = \phi_a(b, a, \theta_i)\mathrm{d}t - a\mathrm{d}q_j \qquad 0 \geq a \geq \underline{a} > -\infty, \tag{3}$$

$$\mathrm{d}z = [z_{i+1} - z_i]\mathrm{d}q_\nu - [z_{i+1} - z_i]\mathrm{d}q_\eta \qquad z_{i+1} > z_i \tag{4}$$

A type 1 agent receives unemployment benefits $z_1 = \mu w z_L$, where μ, w and z_L are the replacement rate, aggregate wage and uneducated labour efficiency, respectively. Type 2 agents receive after

[5] For instance, describing what happens to debt after death. I introduce a perpetual youth extension to this paper (in a separate article) where these rates capture stochastic lifetimes. The results, which can be reproduced upon request, are qualitatively similar to those described herein.

tax income $z_2 = (1-\tau)wz_L$ and supply labour inelastically. The Poisson process q_ν (q_η) counts when an agent leaves unemployment (loses employment)[6]. The Poisson process q_j counts when the student loan balance is cancelled (the arrival rate of this process is λ_{np}). Agents smooth consumption with b; they pay (receive) interest on b and must pay back student loans a (if they have any). If the agent has student loans, it pays them back according to $\phi_b(b, a, \theta)$ and $\phi_a(b, a, \theta)$. These functions depend on the peculiarities of each higher education system and they will be described further below. Following Moll (2016) we can show that the solution to this problem satisfies the Hamilton-Jacobi-Bellman (HJB) equation[7]

$$\rho V_i = \max_c u(c) + \frac{\partial V_i}{\partial b} S_b + \frac{\partial V_i}{\partial a} S_a + \lambda_{ij}\left[V_j - V_i\right] + \lambda_{np}\left[\tilde{V}_i - V_i\right], \tag{5}$$

where $\tilde{V}$ is the value function where the student loan balance is at zero andwhere the equation above satisfies the constraint (6)

$$V_i(b, a) > V_3(b, a) \quad i = 1, 2, \tag{6}$$

in the region where higher education is not chosen. We can express the problem as a variational inequality

$$\begin{aligned} &\min\left\{\rho V_i - \ u(c) - \boldsymbol{A} V_i, V_i - V_3(b^*, a^*)\right\} = 0, \\ &\text{where} \quad \boldsymbol{A} V_i = \frac{\partial V_i}{\partial b} S_b + \frac{\partial V_i}{\partial a} S_a + \lambda_{ij}\left[V_j - V_i\right] + \lambda_{np}\left[\tilde{V}_i - V_i\right] \quad i = 1, 2. \end{aligned} \tag{7}$$

When agents have access to student loans they will face a portfolio type problem[8], where they choose the combination of a and b that lets them achieve the highest $V_3(b, a)$ after covering the cost of education P. As mentioned earlier, instead of looking for the optimal stopping time T, we will be solving for the threshold values $b_i^\dagger$ and $a_i^\dagger$ where the agent optimally chooses to pay for education (if the agent does not have enough funds to pay, it cannot jump to type 3 and become a student). In systems such as SF and TS, we encounter single asset problems, e.g. there is no dependence on a, and as a consequence the third and fifth terms on the right hand side of equation (5) drop out. Additionally, there is no portfolio problem in the single asset case and as a consequence $V_3(b^*) = V_3(b^\dagger - (1-s)P)$. Equation (7) can be conveniently solved as a linear complementarity problem (LCP) - See Moll (2016) and Huang and Pang (1998).

Finally, in the no schooling region we have the standard first order condition in consumption given by

$$u'(c_i) = \frac{\partial V_i}{\partial b}. \tag{8}$$

1.1.2 θ_3 Students

Students are allowed to work a reduced number of hours[9]. As they supply labour inelastically, I scale their labour efficiency accordingly, $z_3 = wz_L z_s$. After spending, on average, $\frac{1}{\Delta_{ed}}$ years as

[6]Arrival rates depend on employment and educational status. See Figure (3) and further below.

[7]For notational convenience I will be denoting drifts as S_x instead of $\frac{\mathrm{d}x}{\mathrm{d}t}$.

[8]An earlier version of this model allowed for a fully fledged portfolio type problem, where the agent chooses the optimal combination of a^* and b^*. The results are equivalent to a less computationally demanding method akin to a so called finance pecking order model - for more on this see section (5.2) in the appendix.

[9]According to Sonnet (2010) the share of 'working students' varies substantially by country, from the low single digits to well over a third of students. According to Carnevale et al. (2015) around two thirds of tertiary students in the US are engaged in work. Of those that work, 30 hours per week is the average. Hence, z_s is set to 0.50625.

a student, the agent may graduate with (without) a job at rate λ_{35} (λ_{34}). There is a risk that the agent will not graduate and become unemployed without a college degree, captured as λ^S_{ex}. Students do not pay income taxes. The HJB equation of students is shown next.

$$\rho V_3 = \max_c \; u(c) + \frac{\partial V_3}{\partial b} S_b + \frac{\partial V_3}{\partial a} S_a + \lambda_{34} V_4 + \lambda_{35} V_5 + \lambda^S_{ex} V_1 - (\lambda_{34} + \lambda_{35} + \lambda^S_{ex}) V_3 + \lambda_{np} \left[\tilde{V_3} - V_3 \right] \quad (9)$$

1.1.3 θ_4 and θ_5 Unemployed and employed with higher education

Agents with a college degree face the standard consumption savings problem as in Huggett (1993) and Achdou et al. (2017). They have a higher labour efficiency $z_H > z_L$ and thus receive higher after tax income (or unemployment benefits, if unemployed). Agents gain (lose) jobs at a higher (lower) rate, when compared to agents without a university education. The two HJB equations for those with a college degree are given by

$$\rho V_i = \max_c \; u(c) + \frac{\partial V_i}{\partial b} S_b + \frac{\partial V_i}{\partial a} S_a + \lambda_{ij} \left[V_j - V_i \right] + \lambda^k_{ex} \left[V_{i-3} - V_i \right] + \lambda_{np} \left[\tilde{V_i} - V_i \right] \quad (10)$$

where $i = 4, 5$, $i \neq j$ and $k = E, U$. $\lambda^U_{ex} > \lambda^E_{ex}$ captures that skills gained by a college degree 'depreciate' faster when the agent is unemployed. The first order condition is analogous to that of (8). The next subsection elaborates on the peculiarities of each higher education system and specially on the student loan repayment function $\phi(b, a, \theta)$.

1.2 Higher education financing and agents' budget constraints

Self financing (SF) and tuition subsidies (TS): The main defining feature of self financing and tax financed systems is that they are single asset models, i.e. there are no student loans. The cost of education that the agent faces is $P(1 - s)$, where P and s are the price and subsidy rate from the state, respectively. Self financing is captured by setting $s = 0$ and not having access to student loans. In the SF and TS systems, if the agent decides to go to university the agent subtracts $P(1 - s)$ from its wealth stock and migrates to θ_3. As shown further below, the government covers the cost of tuition subsidies by adjusting the income tax rate.

Non income contingent student loans (NICL): Agents are now allowed to pay for higher education with student loans a (or combinations of b and a if the student loan debt limit is binding). The $\phi(b, a, \theta)$ functions describe the student loan repayment scheme.

$$\phi_b(b, a, \theta) = \begin{cases} (r_A + \delta) a & \text{for} \quad \theta_i = 1, 2, 4, 5 \\ 0 & \text{for} \quad \theta_3 \end{cases} \qquad \phi_a(b, a, \theta) = \begin{cases} -\delta a & \text{for} \quad \theta_i = 1, 2, 4, 5 \\ r_A a & \text{for} \quad \theta_3 \end{cases}$$

If the agent holds student loans, it pays $(r_A + \delta_A) a$, the interest and amortisation rates on student debt, regardless of its income state. The exception is for students, who accrue debt while at university. Debt forgiveness is not allowed, so the debt cancellation premium $\lambda_{np} = 0$ and thus $r_A = r$. This follows closely federal unsubsidised student loans in the U.S.

Income contingent loan with repayment subsidies (ICL1): The $\phi(b, a, \theta)$ functions describe the student loan repayment scheme.

$$\phi_b(b, a, \theta) = \begin{cases} (r_A + \delta) a & \text{for} \quad \theta_5 \\ 0 & \text{otherwise} \end{cases} \qquad \phi_a(b, a, \theta) = -\delta a \quad \forall \theta$$

In ICL1 the income contingency protection kicks in. Agents pay their student loans only when they reach a high enough income (they reach type 5, i.e. they become employed and educated). The government covers interest and amortisation otherwise. Agents are now allowed to receive debt forgiveness; loans are cancelled, on average, after $1/\lambda_{np}$ years. The government recovers such loses by charging a premium on student loans and thus $r_A = r + \lambda_{np}$.

Income contingent loan without repayment subsidies (ICL2): Agents pay a tax r_p on earnings above the threshold z_T. Earnings encompass labour and capital income, so any agent with earnings above the threshold will be subject to the tax as long as their student loan balance is not zero. That is, an uneducated agent carrying student loans (say because it suffered a college dropout or skill depreciation shock) that is wealthy in b can still be liable for student loan repayments. This charge draws down the student loan balance. High labour income earners pay an extra interest on their student debt, set to r_A to keep some comparability with ICL1. Students accumulate debt at rate r_A. This system follows closely that of the UK[10].

$$\phi_b(b,a,\theta) = \begin{cases} -r_p \mathbb{1}_{\{a<0\}} \max\{z_i + r\max\{b,0\} - z_T, 0\} & \text{for} \quad \theta_i = 1,2,4 \\ 0 & \text{for} \quad \theta_3 \\ -r_p \mathbb{1}_{\{a<0\}} \max\{z_i + r\max\{b,0\} - z_T, 0\} & \text{for} \quad \theta_5 \end{cases}$$

$$\phi_a(b,a,\theta) = \begin{cases} r_p \mathbb{1}_{\{a<0\}} \max\{z_i + r\max\{b,0\} - z_T, 0\} & \text{for} \quad \theta_i = 1,2,4 \\ r_A a & \text{for} \quad \theta_3 \\ r_A a + r_p \mathbb{1}_{\{a<0\}} \max\{z_i + r\max\{b,0\} - z_T, 0\} & \text{for} \quad \theta_5 \end{cases}$$

The remaining student loan balance is cancelled after a certain period (as in ICL1, after $1/\lambda_{np}$ years). Besides the repayment scheme, the main difference with ICL1 is that in ICL2 the government does not provide debt repayment subsidies for those receiving income contingency protections. In ICL1 the student loan balance is always decreasing regardless of the income state of the individual; in ICL2 the balance can increase if tax payments on earnings over the threshold z_T are not large enough to cover interest.

In NICL and ICL2 there is one additional subsidy from the state in the student loan program. Any agent with a negative drift at $\underline{a}$, will have interest payments on student debt covered by the government. This is done to prevent mass escaping the state space[11]. These costs are covered through the tax revenue raised from labour income. Additional variants of the ICL system are left as potential extensions of this paper. In the next subsection I describe how agents interact with the other sectors of the economy.

1.3 Firms, government, education and asset market

The rest of the economy is composed of a representative firm (as in Aiyagari (1994)), asset market and government. Figure (4) depicts the flows between the different players in the economy. Agents supply labour to a representative firm and receive wages net of taxes in return. Taxes go to fund unemployment insurance and education costs (if there is such support). Agents supply capital to the representative firm, through a financial market that is omitted from the figure since it acts as an invisible intermediary. In return, agents receive interest income. The simplified diagram in Figure (4) represents such flows as agents supplying labour and capital and

[10]As mentioned earlier, in the United Kingdom student loan interest rates are charged during studies and vary depending on income later in life. The rate charged to students and high income earners tends to be larger than the risk free rate.

[11]This rarely affects results for the calibrations considered in this paper.

receiving consumption goods and education in return. Figure (5) represents the additional flows in two asset economies (NICL, ICL1 and ICL2), mainly how the government acts as a financial intermediary by supporting the student loan program. For this purpose the channelling of funds through financial markets is made explicit.

Higher education has a fixed resource cost. This is an explicit modelling choice; this assumption is made so that we can evaluate the impact of P in the capital market imperfection of educational investments, and in turn, on the rankings between the different higher education regimes. It is important to highlight the role of P, especially when tuition costs have risen so dramatically in many countries. An alternative interpretation is to treat education as an import, which is not a far fetched assumption for small countries that educate their workforce abroad[12]. The economy invests $P \int_a \int_b g_1(b,a) \mathbb{1}_{\{V_3(b^*,a^*)>V_1(b,a)\}} \mathrm{d}b\mathrm{d}a$ in education.

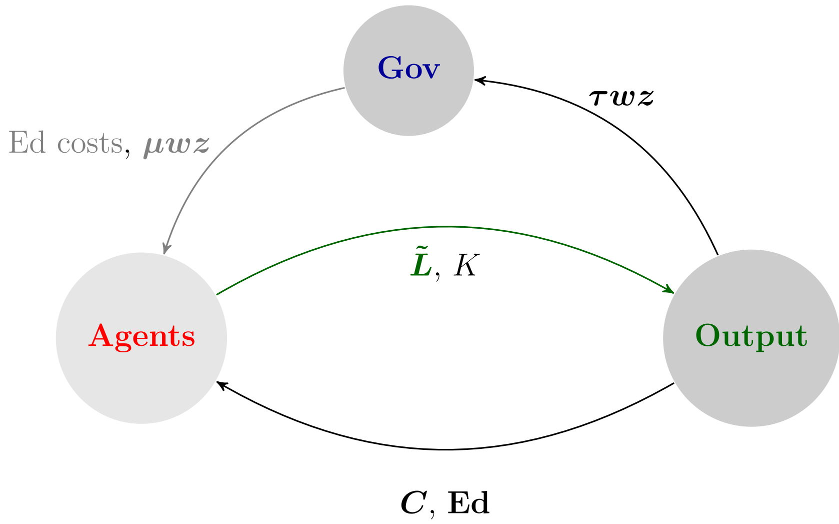

Figure 4: Common flows in all HE systems

1.3.1 Representative firm

As in Aiyagari (1994), there is a representative firm with Cobb-Douglas technology. The firm rents capital, which depreciates at rate δ, from agents and hires labour. Labour differs in productivity; agents with a university degree have a higher efficiency. The production function is given by

$$Y = AK^{\alpha}\tilde{L}^{1-\alpha}, \tag{11}$$

where A is a positive constant and α is the capital share. The effective labour supply is given by adding the efficiencies of the employed with and without college degree and students. This embeds an assumption of perfect labour substitutability between educated and non-educated workers and of production externalities[13]. Remark that students' effective labour supply scales z_L by z_s to capture their working hours.

$$\tilde{L} = z_L(\theta_2 + z_s\theta_3) + z_H\theta_5 \tag{12}$$

[12] The perpetual youth extension of this model endogenises P as in Hanushek et al. (2003). The conclusions are broadly similar to the ones presented here under plausible parametrisations, although in that setting we cannot evaluate how P magnifies differences between systems directly.

[13] In the perpetual youth extension of this article I consider a more general framework, using a CES aggregator of educated and non educated workers as in Hanushek et al. (2003) and Abbott et al. (2013). Once again, the results are broadly similar to those presented here under calibrations commonly found in the literature.

Factor prices are given by the next two expressions.

$$r = \alpha \frac{Y}{K} - \delta, \tag{13}$$

$$w = (1-\alpha)\frac{Y}{\tilde{L}}. \tag{14}$$

Equation (13) gives us capital demand.

1.3.2 Government and tertiary education

The government has a balanced budget constraint and raises revenue from labour income with a flat tax applied to workers τ. In all versions of the model we have unemployment insurance. Additional tax revenue may be raised to cover subsidies to P (in TS), to interest or contingency of student loans (in ICL1 and ICL2). Hence, the income tax rate τ is shown next.

$$\tau = \overbrace{\frac{\mu[z_L\theta_1 + z_H\theta_4]}{z_L\theta_2 + z_H\theta_5}}^{\tau_{UI}} + \frac{\text{Education costs}}{w[z_L\theta_2 + z_H\theta_5]} \tag{15}$$

The first term, τ_{UI}, is the tax rate needed to cover unemployment benefits. The unemployment benefit system is common in all the five regimes being considered. The second term captures the public cost of financing the higher education system. As mentioned previously, in the ICL regimes, the government raises extra revenue with premiums on student loans, so as to cover debt cancellation.

$$r_A = r + \lambda_{np} \tag{16}$$

The student loan balance of an individual gets cancelled after a period of length $\frac{1}{\lambda_{np}}$ has elapsed, hence this cost is covered by the risk premium λ_{np}. If we denote the total amount of newly issued student loans as A^{new}, the aggregate stock of student debt as A and agents' aggregated net savings as B, we can represent the government's role as an intermediary in student loan programs as follows.

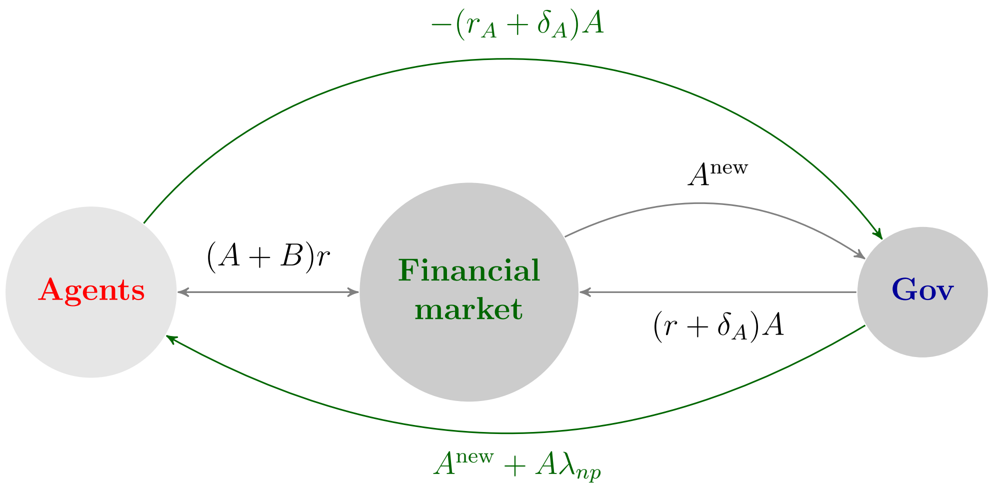

Figure 5: Government intermediation in student loans

The government acts as an intermediary, raising funds in the financial market, issuing student loans to agents and acting as guarantor in case of debt forgiveness. Any loses in the student loan program are covered by the state either though tax revenue and/or risk premiums. In the next subsection I define what is an equilibrium in the economy and the welfare ranking.

1.4 General equilibrium

The stationary equilibrium in this model is defined by a set of policy functions in consumption and educational investment (given by the HJB equations shown above) for each agent type, a joint income and wealth distribution that is ergodic, a government balanced budget and a risk free rate that clears the asset market. During transitions, the asset market clears at every instant. The income and wealth distribution is governed by the following Kolmogorov Forward Equations (KFE). Let g represent the density, ∂_k denote the partial derivative w.r.t. k and the subscripts in g depict the agent type. The KFEs are shown below

$$\partial_t g_1 = -\partial_a[S_{a,1}g_1] - \partial_b[S_{b,1}g_1] + \lambda_{21}g_2 - \lambda_{12}g_1 + \lambda_{ex}^S g_3 + \lambda_{ex}^U g_4 - \chi_1 - \lambda_{np}g_1 + \lambda_{np}\delta_1(a), \tag{17}$$
$$\partial_t g_2 = -\partial_a[S_{a,2}g_2] - \partial_b[S_{b,2}g_2] + \lambda_{12}g_1 - \lambda_{21}g_2 + \lambda_{ex}^E g_5 - \chi_2 - \lambda_{np}g_2 + \lambda_{np}\delta_2(a), \tag{18}$$
$$\partial_t g_3 = -\partial_a[S_{a,3}g_3] - \partial_b[S_{b,3}g_3] - [\lambda_{ex}^S + \lambda_{34} + \lambda_{35}]g_3 + \chi_1 + \chi_2 - \lambda_{np}g_3 + \lambda_{np}\delta_3(a), \tag{19}$$
$$\partial_t g_4 = -\partial_a[S_{a,4}g_4] - \partial_b[S_{b,4}g_4] + \lambda_{54}g_5 - [\lambda_{ex}^U + \lambda_{45}]g_4 + \lambda_{34}g_3 - \lambda_{np}g_4 + \lambda_{np}\delta_4(a), \tag{20}$$
$$\partial_t g_5 = -\partial_a[S_{a,5}g_5] - \partial_b[S_{b,5}g_5] + \lambda_{35}g_3 + \lambda_{45}g_4 - [\lambda_{ex}^E + \lambda_{54}]g_5 - \lambda_{np}g_5 + \lambda_{np}\delta_5(a), \tag{21}$$

where χ_i captures the flows[14], at or beyond $(b_i^\dagger, a_i^\dagger)$, of no HE agents into students and where b_i^*, a_i^* are the targets and $i = 1, 2$. The Dirac deltas δ_i track student loan cancellations. In single asset regimes we drop the dependence on a. In steady state $\dot{g} = 0 \ \forall \ i, a, b$. Market clearing requires $K_S - K_D = 0$, where

$$K_S = \sum_{i=1}^{5} \int_{\underline{a}}^{0} \int_{\underline{b}}^{\infty} (b + a) g_i \mathrm{d}b\mathrm{d}a.$$

Capital demand being equal to capital supply implies the national accounting identity[15] $Y = C + I +$ Education costs. There is no proof of existence and uniqueness of equilibrium for the model with educational choice. The downward sloping and continuous demand of capital remains the same as in Aiyagari (1994). Nevertheless, capital supply is affected by the different education types - Angelopoulos et al. (2017) - and by the educational choice. Quantitative evaluations for a large parameter space show that it is the case that the aggregate capital supply K_S is monotonically upward sloping, approaching ρ from below, continuous and that there is a single crossing of capital demand and supply. I evaluate aggregate and individual welfare via consumption equivalent loss (CEL), as shown next. Let V_0 and V_c denote the steady state value function in the benchmark and alternative regimes, respectively.

$$\tilde{c} = \left[\left(\frac{V_c + \frac{1}{\rho(1-\sigma)}}{V_o + \frac{1}{\rho(1-\sigma)}} \right)^{\frac{1}{1-\sigma}} - 1 \right] * 100 \tag{22}$$

Remark that CEL will be presented in percentage terms. V_c and V_o are computed as follows.

[14]Note that $(b^\dagger, a^\dagger) = (b^\dagger(a), a^\dagger)$ defines a boundary that generates a region of (b, a) pairs, beyond which becoming a student is optimal and feasible. Also, inflows to g_1 and g_2 beyond their respective boundaries are immediately redirected to g_3. I expand on χ in the appendix; it is shorthand to indicate that there is no mass in g_1 and g_2 beyond $(b_i^\dagger, a_i^\dagger), i = 1, 2$.

[15]A heuristic proof is left in the appendix.

$$\sum_{i=1}^{5} \int_{\underline{a}}^{0} \int_{\underline{b}}^{\infty} V_i(b,a) g_i(b,a) \mathrm{d}b \mathrm{d}a. \tag{23}$$

This measure will be computed as an average for the whole economy (as in (23) above) and also for each point in the state space (using $V_i(b,a)$ only), giving us a disaggregated view of which groups in society favour/are against policy changes relative to a common benchmark. Given that each regime yields a different distribution of income and wealth, the disaggregate CEL comparisons will be unweighed comparison of raw value functions. Hence, this analysis will be complemented by comparing the income and wealth distributions of HE systems. The benchmark V_o will be set to welfare in the self financing regime. Positive values of $\tilde{c}$ mean that agents in the SF system would be as well off as in the alternative system if their lifetime consumption is increased by $\tilde{c}$ per cent. Negative values mean that we would have to subtract $\tilde{c}$ per cent of the life time consumption of agents in the SF regime, in order to make them as worse off as in the alternative higher education system. The numerical method used is the finite differences approach presented in Achdou et al. (2017). The agent's decision to become a student is computed with an LCP solver as in Moll (2016) on non-uniform grids.

1.5 Calibration

The baseline calibration of the model is shown in Table (1). The model economy has 24 parameters. These are discussed below in separate categories. The baseline is set based on studies focused in the U.S[16]. The goal is not construct a model that matches U.S. data, but to show how different higher education schemes have an impact on welfare and inequality while using a reasonable calibration.

Preferences: Preferences are described by a constant relative risk aversion utility function with risk aversion coefficient σ and a subjective discount rate ρ. These are set to standard values found in the literature. All parameters are calibrated so that everything is understood in annual terms.

Labour market transitions: The labour market transition rates from unemployment to employment, and vice-versa, are taken from Lamadon et al. (2013). One can see in Table (1) how labour market outcomes are more favourable for graduates as they face a higher probability of being employed and lower probability of falling into unemployment. The transition rates from student to educated is set to $\Delta_{ed} = 0.25$, reflecting that on average it takes four years to complete a bachelor's degree in the U.S.[17]. The flow from student to educated Δ_{ed} is split into transitions to unemployed and educated (λ_{34}), and employed and educated (λ_{35}). According to the National Center for Educational Statistics (NCES), roughly two thirds of students find employment within the first 9 months after graduation - Staklis and Bentz (2016). This figure is roughly constant despite fluctuations over the business cycle. The skills depreciation rate is taken from Manuelli et al. (2012). The magnitude seems to be more or less the same among other papers using Ben-Porath type models, for instance Ionescu (2009). The doubling of this rate for those that are unemployed is inspired from evidence highlighted in Arrazola et al. (2005) and Hugonnier et al. (2019). The dropout rate for students λ_{ex}^{S} is taken from NCES (2019) while the replacement rate μ is taken from 2019 estimates from the U.S. Department of Labor.

Education and skills premium: The cost of education is difficult to pin down since it is not clear if we should include living expenses. For instance, what fraction of students stay at home

[16]Forthcoming work expands the model presented in this paper in a life-cycle environment based on a UK calibration.

[17]According to NCES, in the U.S., the most common is to graduate in 4 years. Results with further sensitivity analysis on Δ_{ed} can be reproduced upon request.

	Values	Description	Source
σ	2	CRRA	Nuño and Moll (2017)
λ_{12}	1.368	Poisson rate $z_1 \to z_2$	Lamadon et al. (2013)
λ_{21}	0.36	Poisson rate $z_2 \to z_1$	Lamadon et al. (2013)
λ_{45}	1.5	Poisson rate $z_4 \to z_5$	Lamadon et al. (2013)
λ_{54}	0.072	Poisson rate $z_5 \to z_4$	Lamadon et al. (2013)
λ_{34}	$\Delta_{ed}\frac{1}{3}$	Poisson rate $z_3 \to z_4$	Staklis and Bentz (2016)
λ_{35}	$\Delta_{ed}\frac{2}{3}$	Poisson rate $z_3 \to z_5$	Staklis and Bentz (2016)
ρ	0.05	Discount rate	Caucutt and Lochner (2020)
λ^E_{ex}	0.024	Poisson obsolescence	Manuelli et al. (2012)
λ^U_{ex}	0.048	Poisson obsolescence	Manuelli et al. (2012) and Arrazola et al. (2005)
λ^S_{ex}	0.148	Poisson dropout	USDE-NCES (2019)
s	[0,1]	Subsidy	-
μ	0.382	Replacement rate	USDL (2019)
Δ_{ed}	$\{1, 0.25\}$	Inverse years until grad	-
A	0.45	Productivity	Convenient
λ_{np}	1/30	Premium on student loans	UK
r_p	0.09	ICL2 graduate tax	UK
z_T	0.2811	ICL2 income thresh	UK
$\underline{b}$	$[0, -1.5]$	Exogenous b limit	0-268% avg US income
$\underline{a}$	$[0, -2.35]$	Exogenous a limit	0-150% avg ed costs US*
P	$[0.3, 2.1]$	Education cost	30-130% avg ed costs US*
ψ	1.7	HE premium	James (2012), Valletta (2018)
δ	0.05	Capital depreciation	Common
δ_A	1/30	Amortisation in NICL and ICL1	**

Table 1: *Average cost of a private non-profit university. The % may vary depending on including board.** Maximum maturity for *Standard Repayment* in the US. Values in red denote parameters where sensitivity analysis is performed.

or rent elsewhere during their studies? It is not clear what percentage of students move out of home when they enrol at university[18]. This is important since it gives us a better idea of whether student accommodation counts as an extra expense accounted by c or P. The benchmark P will be set to lie between %95 (including living expenses) and %129 (just tuition) of costs in the U.S., according to data from The College Board (2019). These percentages are found by matching the ratio of higher education costs to GDP per capita, with the latter rescaled to reflect only those in the labour force (this model only has employed and unemployed people, we exclude those not in the labour force). This ratio is then multiplied by the mean income in the economy. Whilst mean income is endogenous and varies with each higher education regime, it is broadly stable for the vast majority of cases considered here, lying in a range between 0.5 and 0.59. Hence, the baseline P is set to 1.1. This is a conservative calibration of education costs, nonetheless a sensitivity analysis with $P \in [0.3, 2.1]$ is performed. This is crucial, since as Figure (28) shows, tuition inflation has outstripped healthcare and housing costs and, as will be shown next, as P rises we magnify welfare differences amongst the different HE systems. The appendix goes into more detail on how the benchmark calibration of P is obtained. The higher education wage premium $\psi = z_H/z_L$ is set to 1.7, following[19] evidence from James (2012) and Valletta (2018).

Student loans and debt limits: The amortisation rate in NICL and ICL1 is set to 1/30. This corresponds to the maximum maturity in the *Standard Repayment* schedule for American

[18]According to NCES (2016) about a quarter of university students in the U.S. move out of state. This is not enough information to pin down the fraction of students that incur extra accommodation costs due to tertiary education.

[19]I emphasize a fixed premium as the last decades have seen a stable, if not rising, college wage premium in both the US and UK despite large increases in the supply of college educated workers. Belfield et al. (2018a) and Belfield et al. (2018b) have shown that the premium is driven by substantial heterogeneity. Introducing heterogeneity in ψ is left for an extension.

student loans. The reason I picked this number is twofold. First, high amortisation rates reduce the state space where consumption can remain positive when indebted and second, a 30 year loan allows some degree of comparability with other regimes where loans are forgiven after 30 years. Using the re-scaled GDP per capita method outlined in the appendix, we obtain the values for $\underline{b}$ and $\underline{a}$. The student debt limit (4 year degree cumulative) for Stafford loans is \$23000 while the average unsecured debt amount is at \$17000 according to the Survey of Consumer Finances (2016). Thus, following Athreya et al. (2019) this allows us to set the debt limits $\underline{b}$ and $\underline{a}$ to -0.174 and -0.24, respectively. Aggregate welfare is sensitive to debt limits, as pointed out by Obiols-Homs (2011). Therefore, a large sensitivity analysis on debt limits is carried out to illustrate how they affect the rankings of the higher education funding schemes discussed herein. The graduate tax in ICL1 is described by two parameters: z_T and r_p. The threshold z_T is set to mimic the UK's student loan system, matching the ratio of the taxable threshold to GDP per capita while the tax rate r_p on earnings above the threshold is set to 9 %. Following the methodology in the calibration for P, we translate z_T to a comparable figure in the US.

Production: Three parameters describe the productive technology in the economy. These are the capital elasticity α, depreciation δ and TFP A. The first two are set to commonly used values (0.33 and 0.05); the third is adjusted for convenience to keep the state space in wealth of a reasonable size.

2 Steady state results

All CEL computations set SF as the benchmark regime. Table (2) shows[20] that self financing is the worst out of all systems. It has the lowest CEL, share of employed workers with a college degree, capital stock, GDP, expected earnings, the highest wealth inequality and the highest net debtor population share. Debtors face an interest rate that is higher than most systems, so net borrowers (net lenders) suffer more (gain more). The income tax rate is higher than in the student loan systems and that of TS 50%. As we see more involvement of the public sector, be it with student loans or tuition subsidies, wealth inequality decreases, relative to self financing. Besides this, there are two striking results. First, while the system with income contingent loans brings substantial CEL gains, it can vary substantially depending on how it is designed. Second, tuition subsidies at 100 % of education costs bring the largest CEL gains vis-à-vis self financing; it attains the highest share of workers with a college degree - more than doubling the share in self financing. Moreover it reaches the largest interest rate *with the lowest share of the population in debt.* This last result points to a powerful force driving aggregate welfare: *the price and quantity effects of debt* described in Obiols-Homs (2011).

Welfare rankings coincide almost perfectly with the share of the population in net debt and with measures of wealth inequality. Systems that generate more net debtors and more inequality have lower aggregate CEL. If a relatively high net debtor share is compounded with a larger interest rate welfare will be depressed even further. Systems with student loans have a larger fraction of the population as net debtors. That is, student loan systems are more prone to the negative impact on welfare coming through the price effect described in Obiols-Homs (2011). Whilst the income contingent loan systems shields agents from the effect of interest rates, debt balances may potentially accumulate at a faster rate and for longer periods. This is because of the debt cancellation premium that is added on top of r and because agents accrue debt when they don't have a high enough income[21]. Even if the amount of debt is notional and may not

[20] In Table (2), the TS system columns have a percentage attached to denote what fraction of education costs are covered by the state.

[21] This is one of the main distinctions between ICL1 and ICL2. In the former the government covers student loan repayments when agents do not make contributions whereas in the latter debt continues to accrue. Changing

affect the individual, the government still has to raise revenue to cover interest payments and cancellations. This will become quantitatively more patent in the next subsection.

	SF	NICL	ICL1	ICL2	TS 50 %	TS 75 %	TS 100 %
CEL %	0.0000	17.7328	31.8064	29.7862	53.9028	57.3654	58.3104
K	2.1729	2.6736	2.9484	2.9266	3.1971	3.0623	2.9166
$\mathbb{E}[z]$	0.4269	0.5007	0.5435	0.5391	0.5909	0.5861	0.5777
r	0.0415	0.0389	0.0383	0.0381	0.0392	0.0425	0.0457
Y	0.6027	0.7201	0.7885	0.7814	0.8644	0.8580	0.8459
θ_1	0.1307	0.0757	0.0425	0.0466	0.0052	0.0013	0.0000
θ_2	0.5077	0.3047	0.1823	0.1960	0.0226	0.0048	0.0000
θ_3	0.0331	0.0567	0.0710	0.0693	0.0890	0.0910	0.0915
θ_4	0.0163	0.0279	0.0350	0.0342	0.0438	0.0448	0.0451
θ_5	0.3123	0.5349	0.6693	0.6540	0.8393	0.8581	0.8634
τ	0.0583	0.0399	0.0363	0.0349	0.0557	0.0738	0.0929
τ_{UI}	0.0583	0.0388	0.0295	0.0306	0.0210	0.0202	0.0199
Gini_w	0.6375	0.4994	0.4104	0.4261	0.2514	0.2304	0.2485
Net debtors	0.0391	0.0342	0.0301	0.0325	0.0161	0.0085	0.0061

Table 2: Inverse of years until graduation $\Delta_{ed} = 1/4$ and cost of education $P = 1.1$

Rankings between systems change depending on the cost of education, which is affected by P, the price of a college degree, and the time it takes to graduate ($1/\Delta_{ed}$). When P is low and Δ_{ed} is high, borrowing constraints and the riskiness of educational investment are relatively less important. In fact, result go in the opposite direction, as shown in Table (4) and Figure (25) in the appendix. Even though it is highly implausible that we could keep education quality constant with high Δ_{ed} and low P, it is worth considering these results. They illustrate under which conditions we may get that government intervention in education financing can reinforce inequality, a commonly repeated link by detractors of public financing. Thus, there seems to be a bit of truth in two popular perceptions higher education: financing tertiary education with too much debt or with too much government support can foster inequality and reduce the social gains from educational attainment. Nonetheless, as Tables (2) and (4) illustrate, results are sensitive to P and Δ_{ed}.

We know that it is not empirically plausible to have low P and high Δ_{ed}. It is also worth noting that larger public expenditure in tertiary education relative to GDP is associated with lower income inequality[22] in the OECD - see Figure (27) in the appendix. Table (2) shows a similar relationship between wealth inequality and the amount of public financial support in tertiary education. The results in Table (2) show that the income tax rate τ does not display a discernible relationship with welfare rankings. However, the gap between τ and τ_{UI} (the rate of income tax needed to cover the unemployment insurance program), which indicates the amount of tax that needs to be raised to fund the higher education system, moves in lockstep with the CEL and inequality rankings.

Table (2) also illustrates how the welfare ranking moves in lockstep with the net debtor population share and wealth inequality. This points to the welfare effects of borrowing limits described earlier. This is explored further in the next subsection. In order to verify that it is indeed the price and quantity effects outlined by Obiols-Homs (2011), I repeat the exercise with 2304 combinations of $\underline{b}$ and $\underline{a}$, and various values of P ranging from 27% to 190% of its benchmark calibration value.

z_T and r_p may accentuate or soften these effects.

[22]Given the lack of available data that is consistent for cross country comparisons of wealth inequality in the OECD, I could only estimate this relationship for measures of income inequality.

2.1 Borrowing limits and welfare

A natural question to ask is how much are the aggregate CEL results driven by the choice of debt limits $\underline{b}$ and $\underline{a}$. In a first round of experiments, we keep the SF benchmark at the baseline calibration and compute steady state results for all other higher education financing schemes with 2304 combinations of $\underline{b}$ and s ($\underline{b}$ and $\underline{a}$ in systems with student loans). Figure (6) depicts the case of SF and TS.

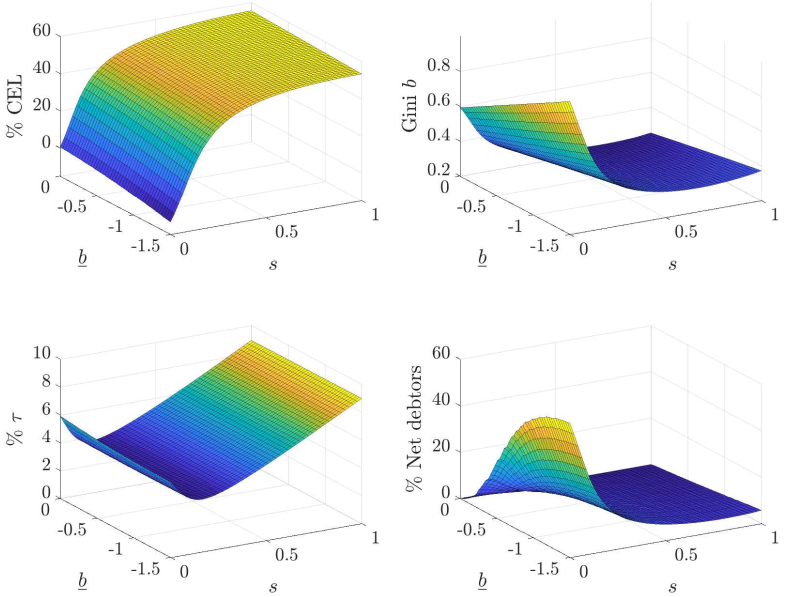

Figure 6: CEL, wealth Gini, tax rate and net debtor share in SF and TS

The patterns seen in Table (2) become clearer in this new experiment. Wealth inequality and the net debtor population share appear to be closely related to the CEL welfare ranking. This relationship becomes more apparent as we loosen the debt limit and lower the tuition subsidy rate. As seen above, there is no discernible relationship between aggregate welfare and the income tax rate. The U-shaped tax surface actually has a minimum at strictly positive values of the subsidy rate. Financing tuition subsidies with higher tax rates are not necessarily associated with lower aggregate welfare, even when we have a flat tax rate. Further below we verify that this stil holds when we look at dissaggregated welfare. However, the most striking result is how welfare is sensitive to debt limits. For each value of the subisidy rate, the optimal debt limit $\underline{b}$ is always located at zero. This is a natural result in an Aiyagari economy - see Obiols-Homs (2011) and Nuño and Moll (2017). The negative impact of laxer debt limits is diminished by increasing the subsidy rate. As we will see further below, a higher subsidy rate compresses the wealth distribution and shifts it to the right, generating an economy with relatively less borrowers, thus decreasing the impact of the price effect of debt.

In order to repeat the same exercise with the regimes that have student loans, steady states are computed for 2304 combinations of $\underline{b}$ and $\underline{a}$ and compared to the SF benchmark. The cases for ICL1 and NICL are displayed here. Since ICL1 consistently performs better than ICL2, the results are left in the appendix - see Figure (29). The choice of focusing on ICL1 is also motivated by the fact that the ICL2 results are qualitatively similar. Figure (7) below displays the outcomes in ICL1. The direction of results go in the same way as in the previous experiment with tuition subsidies; mainly that there is lower aggregate welfare when the economy generates a larger mass of debtors, and that higher debt goes hand in hand with larger wealth inequality.

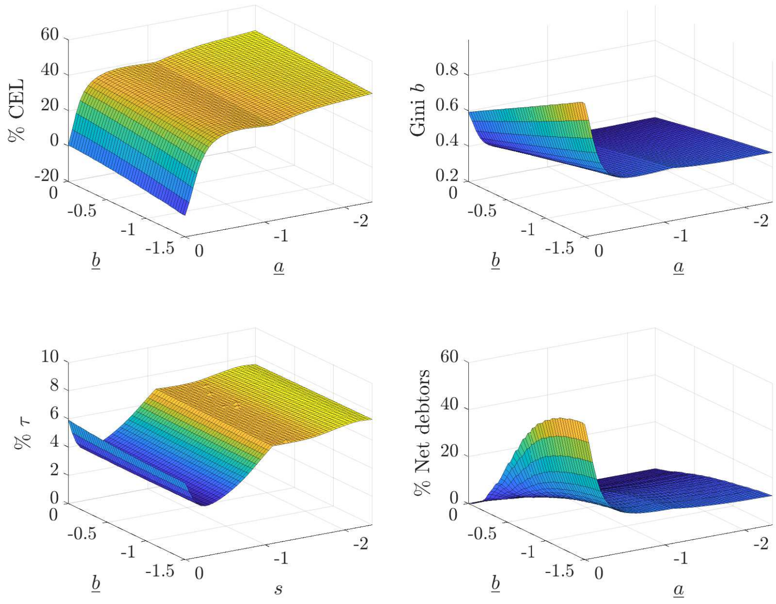

Figure 7: CEL, wealth Gini, tax rate and net debtor share in ICL1

The debt limit on student loans has a similar effect to that of the subsidy rate in Figure (6) with the twist that the relationship between aggregate welfare and $\underline{a}$ in ICL1 and ICL2 is not monotonic. The % CEL over SF initially peaks around average income, then dips and then starts to increase again when $\underline{a}$ reaches P, the monetary cost of education. Similar results have been found in the literature, Johnson (2013), Ionescu and Simpson (2016) and Abbott et al. (2013) identify a similar effect of student debt limits on welfare: laxer limits on a can provide diminishing gains or even drag down welfare - while more generous subsidies can generate stronger gains. The novelty of this paper is making the connection with Obiol-Homs' price and quantity effects of debt and comparing systems under this light. An additional remarkable result is the scale of the vertical axes on the top left panels of Figures (6) and (7): welfare ranks consistently higher in TS relative to ICL1 for a large area of the parameter space.

Another outcome that is worth considering is that the income tax rate can be higher in economies with income contingent loans, relative to that of a system relying on tuition subsidies; this is specially patent in the results for ICL2 - see Figure (29) in the appendix. It is surprising that this is more salient in ICL2. In ICL1 the government provides relatively more generous income contingency support: it covers the interest *and amortisation* of agents that do not earn a high enough income. This happens for two reasons. First, remember that in ICL2 agents accumulate student loan interest when they do not earn enough (when the income contingency protects low earners). Furthermore, for a vast area of the state space, the tax r_p of earnings over the threshold z_T contributes little, if at all, to pay down the student loan balance. This is probably putting a larger burden on the public sector. Higher taxes and lower income contingency protections in ICL2 explains why it delivers a smaller share of the population with a college degree. This takes us to the second point. Notice how ICL1 spends less in unemployment insurance, relative to ICL2 and NICL, in Table (2) and how the overall income tax rate is lower, especially when the student debt limit is lax, relative to ICL2 in Figures (7) and (29). As ICL1 delivers a larger share of college graduates, who have face a better labour market (less unemployment) and earn more than non-graduates, the cost of unemployment insurance drops, lowering the income tax rate. It seems that, on aggregate, it is a better deal for the government to help bring down student loan balances of those that do not earn enough. The case with student loans that do not have an income contingency protections (NICL) is shown next.

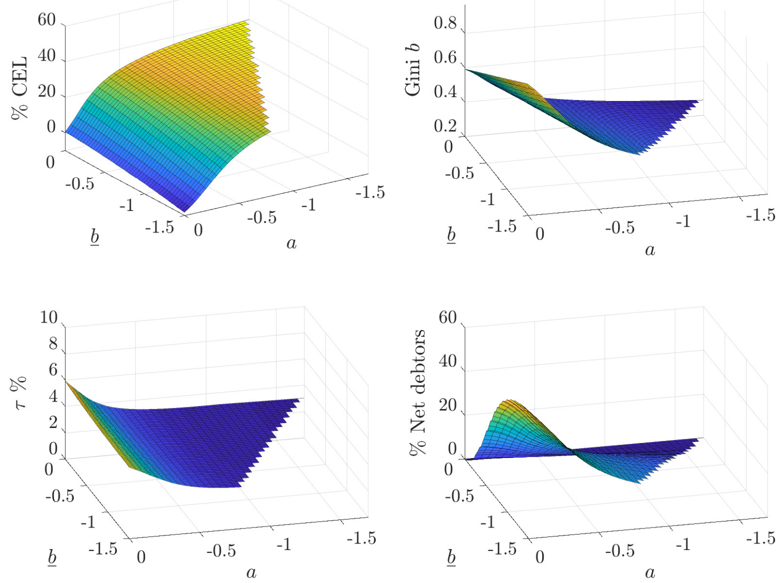

Figure 8: CEL, wealth Gini, tax rate and net debtor share in NICL

Figure (8) reveals two defining aspects of the NICL system. First, income taxes are almost always lower than in any other system, beating income contingent student loans, tuition subsidies and even self financing. Second, the regions where a steady state equilibrium can be achieved are reduced. This is because agents must pay amortisation and interest in student loans regardless of their income state. When a is large enough (in absolute value), interest and amortisation overtake all earnings, pushing towards zero consumption. These cases are ruled out since bankruptcy is not possible. CEL gains relative to self financing are much smaller than those of the income contingent loan programs, despite having lower income tax rates. NICL displays the same pattern of the other systems we have considered: aggregate welfare tends to be lower when the economy has a relatively larger fraction of net borrowers and high inequality. This pattern can be magnified even when the education system provides income contingency protections. For instance, as will be shown further below, NICL outperforms ICL2 when the student loan limit is large. This happens around the same region where ICL2 starts to generate a relatively larger mass of net debtors.

It is interesting to revisit the link between CEL, subsidy rate and debt limits. In the TS case, as the subsidy rate approaches zero, aggregate CEL decreases more sharply as the debt limit is relaxed. As the subsidy rate becomes positive we see that this relationship is weakened. This relates to the pecuniary externalities described in Nuño and Moll (2017), where higher wealth inequality was found to be associated with higher welfare[23]. The twist is that now the configuration yielding the highest CEL in TS is associated with the lowest wealth inequality. The same can be said of all student loan programs. While more lax student debt limits increase welfare relative to SF, the welfare gains drop as $\underline{b}$ is loosened.

The role of monetary and time costs of education

The relationship between government support for education and welfare is inverted when the monetary and time costs in educational investments are low. As shown in Figures (25) and (26) in the appendix, loosening $\underline{a}$ or providing tuition subsidies decreases welfare, regardless of the debt limit. This confirms what we saw in Tables (2) and (4): when education is relatively easy

[23]This opens an interesting extension for constrained efficiency considerations in Nuño and Moll (2017).

to achieve, the capital market failures associated with educational investments do not matter enough to warrant government intervention.

I repeat the exercises discussed above, although this time with different values of P for each system. Given that this adds an extra dimension to the analysis, these results are illustrated as animations[24]. These show how CEL, the tax rate, the wealth Gini coefficient and the net debtor share change with P. The animations illustrate how the results shown above are robust to sensible values of P. Larger values of P magnify the capital market frictions and riskiness in educational investments and welfare gains from government intervention. As P rises the marginal benefit from laxer $\underline{a}$ flattens out more markedly. As for the benchmark calibration of P (and higher values) and in most of the parameter configurations considered here, it is safe to say that TS often yields higher steady state consumption equivalent gains (and lower debtor shares and wealth inequality) than in the student loan systems. I confirm this with a new experiment in our baseline calibration.

Net debtor shares in ICLs and NICL

Figures (9) and (10) show the differences in CEL and net debtor share between the outcomes shown above. I first compare TS vs. ICL1 and TS vs. ICL2 in Figure (9). Then, in Figure (10) I do the same for ICL1 vs. NICL and ICL2 vs NICL[25]. Since the student loan systems have an extra dimension I compare single and two asset regimes by fixing $\underline{b}$ and picking the student loan debt limit that yields the highest CEL relative to SF. This is then used to compare with the results in TS under the same $\underline{b}$. I then vary the tuition subsidy rate s. Figures (9) reveals that when the subsidy rate is low, the student loan systems ICL1 and ICL2 tend to outperform TS. The figure clearly indicates that this coincides with whichever system has the larger share of the population as net debtors. TS starts to outperform ICLs at subsidy rates above %30.

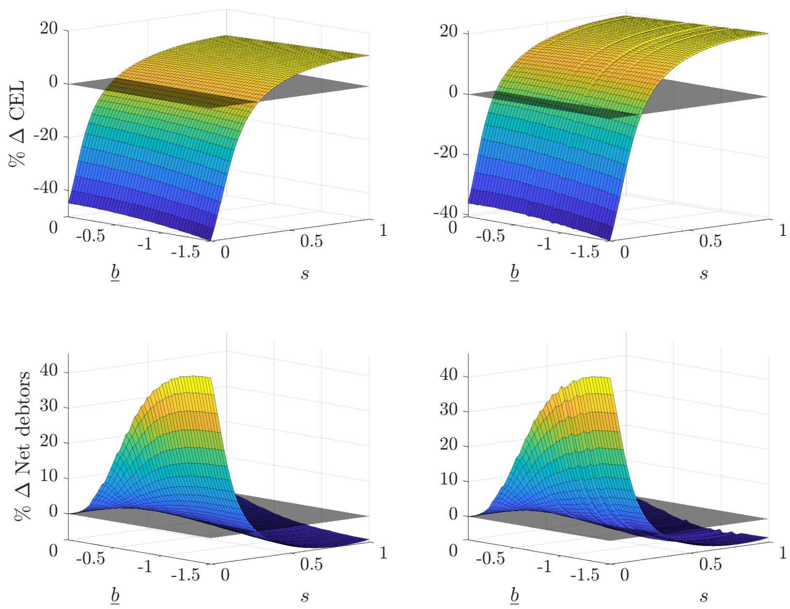

Figure 9: Left to right: $\text{CEL}_{\text{TS}} - \text{CEL}_{\text{ICL1}}$ and $\text{CEL}_{\text{TS}} - \text{CEL}_{\text{ICL2}}$ and difference in net debtor share

Comparing the student loan systems only, for each $(\underline{b}, \underline{a})$ combination, yields Figure (10) furter below. We can see once again how CEL is tightly linked to the debtor share. At laxer

[24] These animations can be found at http://gustavomellior.com/animations/.

[25] Comparisons between TS and NICL are omitted for brevity but can be reproduced upon request.

debt limits ICL2 and NICL are not that different in terms of aggregate welfare (and sometimes ICL2 generates more borrowers and lower CEL). As NICL forces agents to pay down student debt regardless of income state, ICL2 lets agents accumulate interest in student debt when earnings are low (this becomes less of an attractive outcome when $\underline{a}$ is too loose).

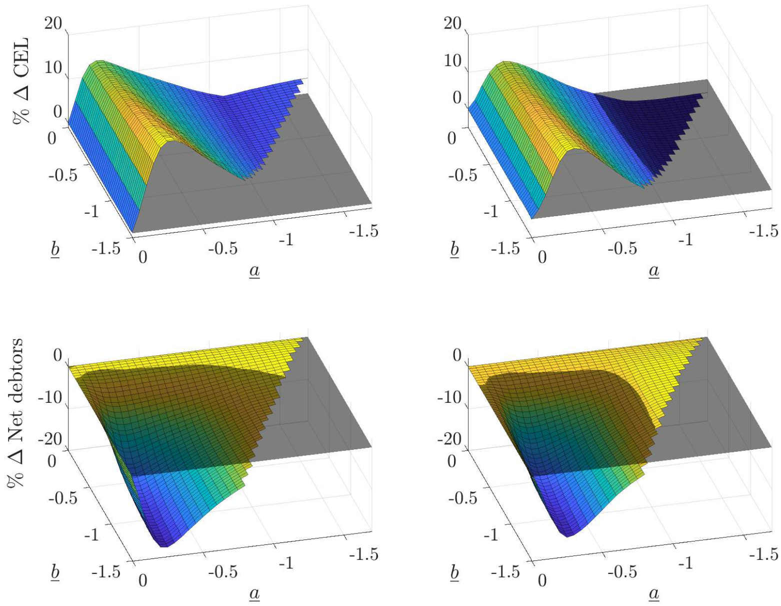

Figure 10: Left to right: $\text{CEL}_{\text{ICL1}} - \text{CEL}_{\text{NICL}}$ and $\text{CEL}_{\text{ICL2}} - \text{CEL}_{\text{NICL}}$ and difference in net debtor share

Having conducted a sensitivity analysis to illustrate how *aggregate* welfare is affected by debt limits and thus how higher education funding policies interact with the price and quantity effects of debt, the next section looks at *disaggregated* measures of welfare for each system under the baseline calibration described in Table (1).

2.2 Disaggregated CEL

A benefit of working with a heterogeneous agent model is that we can disaggregate the CEL and see how it fares at each point of the state space using equation (22). Due to the fact that the state space is not the same between systems without student loans (SF and TS) and those that do have them (NICL, ICL1 and ICL2), I compare single asset and student loan systems in two different ways. The first method, presented further below, sets SF as a benchmark $V_o(b, z)$ for each a grid point. In an separate appendix that can be given upon request, disaggregated comparisons are performed between systems with the same state space only[26]. In this section I concentrate on the two systems delivering the largest welfare gains - TS and ICL1 - and results for ICL2 and NICL are left in the appendix. Remark that each higher education system will generate a distinct density. In order to abstract from different general equilibrium densities, which may place more or less weight in different parts of the state space, I compare raw value functions following (22). I elaborate on densities further below.

Self financing and tuition subsidies: Figure (11) illustrates the unweighted and disaggregated CEL of the single asset regimes considered back in Table (2). The student type is omitted to save space and since it generally has small mass relative to the other types. Figure (11) reveals

[26]That is, SF vs. TS, NICL vs. ICLs, ICL1 vs ICL2. A disaggregated CEL of NICL vs. ICL1, the worst and best student loan programs, is nonetheless shown in Figure (30) in the appendix.

that the disaggregated CEL is not monotone in assets. Asset-poor non-college graduates see the biggest gains in welfare. These gains initially rise in wealth, reaching a peak before falling on larger values of wealth. This pattern is most salient for non-college grads, as they now have easier access to education. As agents' asset income dominates labour income they see less benefit in moving away from SF to TS. Nevertheless, agents almost anywhere in the state space favour the policy change from self financing towards high partial or full public financing of tertiary education. The exception are asset-rich agents in TS % 50. This is partly due to the relatively lower equilibrium interest rate ($r_{SF} > r_{TS50}$), as will be shown in the partial equilibrium exercises in the next subsection. However, once we weigh who finds themselves in this region post-reform it turns out that it is less than 0.01% of the population. It is clear that comparing raw value functions is not enough as each higher education financing scheme yields a different distribution.

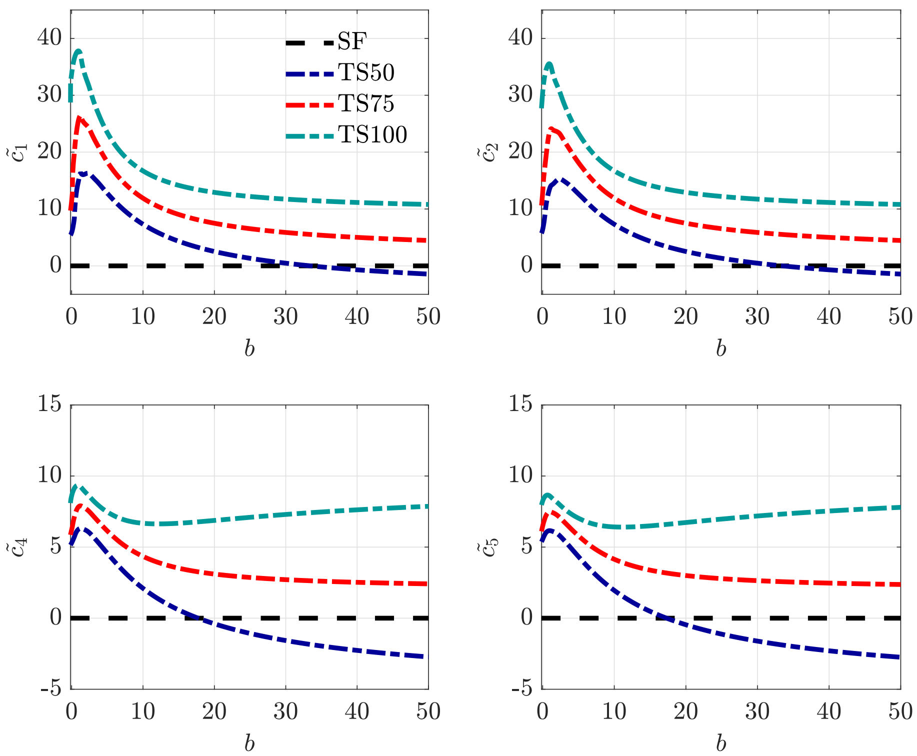

Figure 11: Disaggregated CEL (in % terms) in baseline calibration

In the next figure we compare distributions and see wether TS places more or less mass in the regions where dissaggregated welfare gains are highest. We compare distributions by taking the mass in each TS regime and then subtracting that of SF. This allows us to see how tuition subsidies changes the location and spread of the distribution. Values below (above) zero tell us that SF places more (less) mass, relative to TS, in that region. Figure (12) confirms what we saw in Table (2): TS generates less debtors, reduces wealth inequality and increases the capital stock. Partial and full subsidisation of tuition puts more mass in moderately high values of b and less on the high-low extremes[27]. As tuition subsidies increase, precautionary savings fall, labour income rises and the equilibrium interest rate goes up, as can be seen in Table (2). This becomes more patent as the subsidy rate rises, analogous to how results would look if employment and skill depreciation transition rates were more even more favourable for educated agents (if the risk of falling into bad states decreased). The disaggregated results for NICL vs ICLs are shown next.

[27] Angelopoulos et al. (2017) reach similar findings in an environment where education types are exogenously defined.

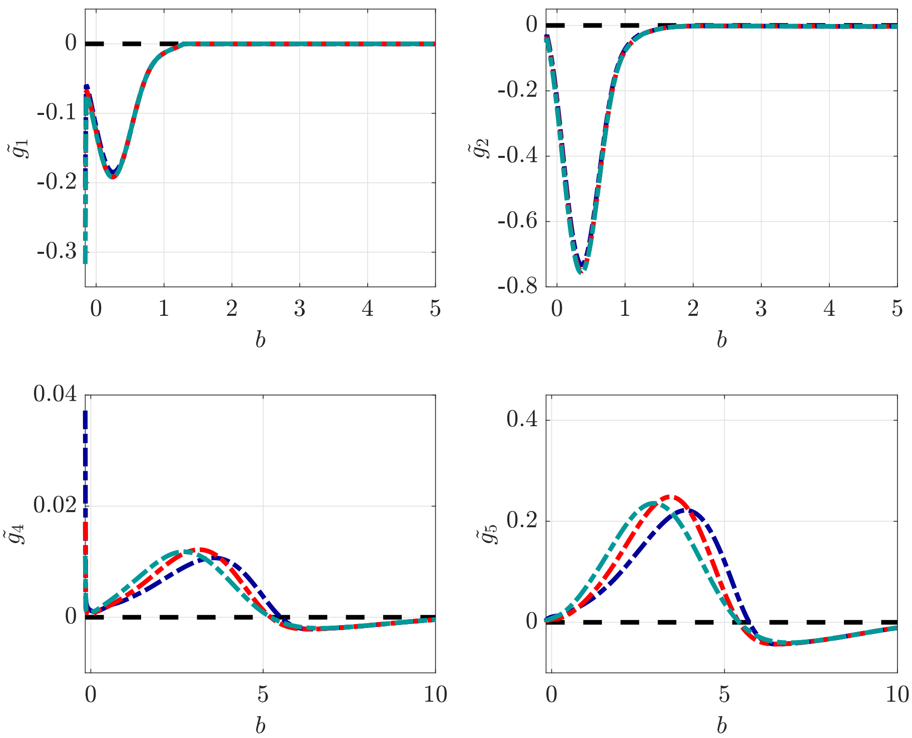

Figure 12: Density difference $\tilde{g} = g_{TS} - g_{SF}$

Government guaranteed student loans ICL1: The disaggregated CEL of ICL1 reveals the same forces outlined earlier. Relative to SF, ICL1 is more favourable for lower income and lower wealth agents. Besides the income contingency protections, ICL1 has a lower equilibrium interest rate. This benefits borrowers and not lenders. This is reflected in Figure (13); the point where SF performs better is when asset income dominates labour income. As the student loan balance increases, ICL1 becomes less desirable, and this is most patent in the lower right panel of the figure, depicting educated and employed agents. This is precisely where agents have to pay back student loans. We will see how these effects accentuate when we repeat this exercise in a partial equilibrium setting.

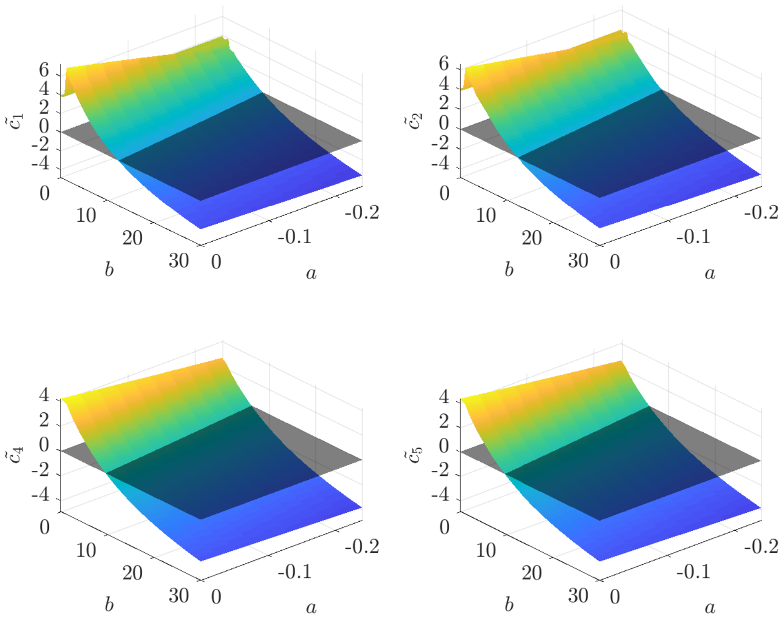

Figure 13: General equilibrium disaggregated CEL in ICL1

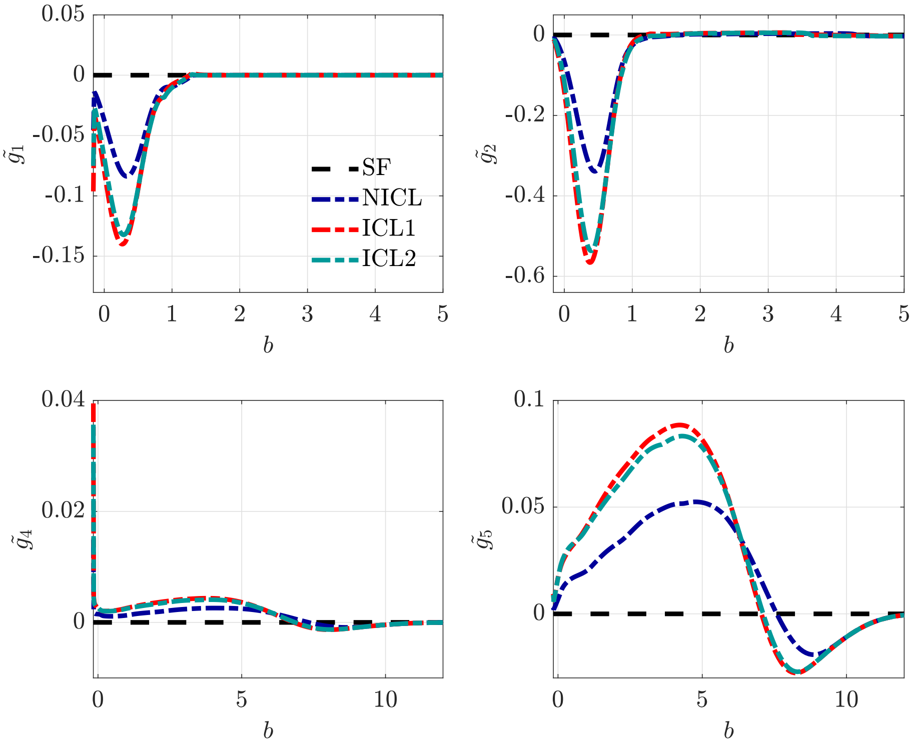

Figure 14: Density difference $\tilde{g} = g_x - g_{SF}$, where $x =$ NICL, ICL1, ICL2.

The density difference between student loan programs and SF is shown in[28] Figure (14). The density of NICL is the closest to that of SF and has a larger spread than those of the income contingent loan systems. As the student loan program becomes more generous, precautionary savings decrease and so does the spread of the distribution. Despite the fall in precautionary savings the capital stock is bigger. This happens as more mass is placed in the educated group (the lower row in the figure) as the loan program becomes more generous. Given the debt cancellation offered in the income contingent loan programs, more agents will have no student loans in the ICLs than in NICL; the distribution will place more mass in the regions where welfare gains are strongest.

2.3 Partial and general equilibrium comparisons

This section demonstrates how results look in partial, as opposed to general equilibrium. We will see how changes in welfare are once again tightly linked to the price effects of debt mentioned above and how welfare rankings can change depending on whether we solve for steady state equilibrium letting markets clear or keeping market prices fixed. This is illustrated by evaluating the CEL (relative to self financing) of changes in higher education financing before market forces respond, i.e. by holding the prices of labour and capital fixed[29]. I will first report aggregate results, shown in Figure (15) and Table (3). I then repeat the disaggregated exercise, comparing raw value functions and densities. Figure (15) and Table (3) reveal that imposing SF prices and tax rate leads substantial welfare improvements (relative to when we let markets clear) in the ICLs, moderate improvement in TS 50%, small loss of welfare gains in TS 100 % and moderate gains in NICL.

[28]Welfare and density comparison between NICL and ICL1 and NICL and ICL2 can be found in the appendix in Figures (30) - (31).

[29]The income tax rate is also held fixed at its equilibrium value prior to the enactment of the new higher education financing policy. Prices and taxes are fixed at the SF equilibrium values.

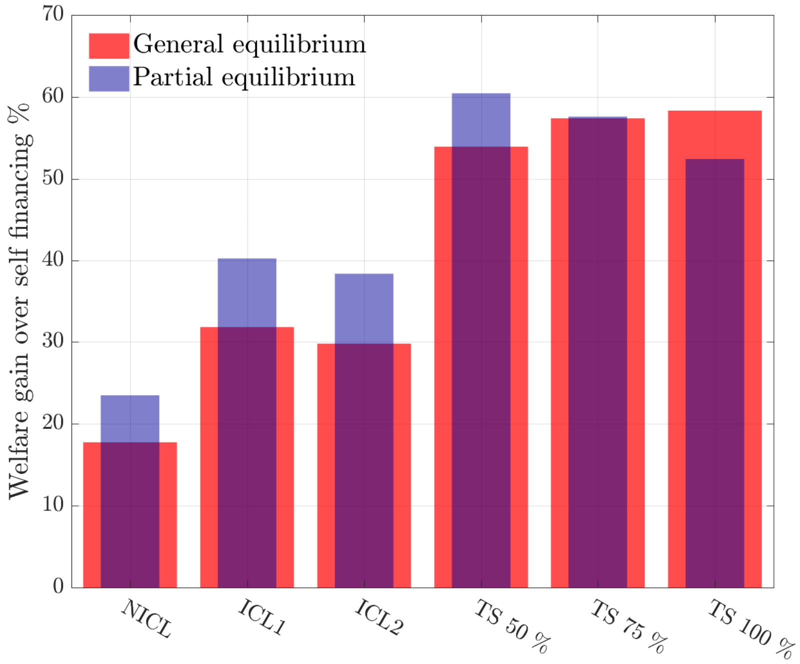

Figure 15: Partial vs general equilibrium steady state welfare gains

The change in rankings and in welfare gains broadly follow the price effects of debt and the relative change in the exposure to idiosyncratic shocks. With regards to the former it must be noted that the general equilibrium interest rate in SF is higher than in the student loan programs and TS 50 %, it is about the same in TS 75 % and lower than in TS 100 %. A higher r benefits lenders and harms borrowers so the direction of additional welfare gains in partial equilibrium will ultimately depend on where does the endogenous distribution of income and wealth places more mass. This takes us to the second point, how prices interact with the relative change to exposure to idiosyncratic shocks. Systems with income contingency protections such as ICL1 and ICL2 shield agents from movements in the interest rate during bad times (when unemployed or with low income), effectively reducing the risk of investing in education. A higher interest rate increases student loan payments; in NICL this happens in both good and bad states, elevating the riskiness of educational investments. In the next few paragraphs I elaborate on how these forces (price effects of debt) and exposure to risk compound in each system.

	SF	NICL	ICL1	ICL2	TS 50 %	TS 75 %	TS 100 %
CEL %	0.0000	23.4718	40.2025	38.3288	60.4175	57.5695	52.3949
$\mathbb{E}[z]$	0.4269	0.5061	0.5453	0.5418	0.5884	0.5890	0.5877
θ_1	0.1307	0.0656	0.0333	0.0366	0.0039	0.0014	0.0000
θ_2	0.5077	0.2675	0.1489	0.1586	0.0170	0.0055	0.0000
θ_3	0.0331	0.0610	0.0749	0.0737	0.0896	0.0909	0.0915
θ_4	0.0163	0.0301	0.0369	0.0363	0.0441	0.0448	0.0451
θ_5	0.3123	0.5758	0.7061	0.6948	0.8453	0.8574	0.8633
Gini_w	0.6375	0.4728	0.3846	0.3972	0.2461	0.2295	0.2392
Net debtors	0.0391	0.0253	0.0205	0.0222	0.0118	0.0096	0.0127

Table 3: $\Delta_{ed} = 1/4$ - $P = 1.1$

The largest gains relative to general equilibrium are is in ICL1 and ICL2. In these two cases the composition of the distribution improves; there are more students and more college educated agents (employed and unemployed). These systems now have a larger share of the population facing a better labour market (lower transitions to unemployment and higher ones to employment) and diminished uncertainty. Given that we are holding prices fixed at the SF level, wages do not respond to the influx of newly educated workers - so average pay is higher. Remark that w remains fixed, but the composition of the labour force affects $\mathbb{E}[z]$. The interest rate is higher than when we let markets clear, which benefits a distribution that generates less debtors. The net debtor share and inequality fall. Despite higher income taxes, aggregate welfare is higher when holding prices fixed. The results in TS 50 % follow the same pattern.

Focusing on TS 100% reveals an interesting feature. Almost all distributional effects are isolated as the composition of types remains the same as when we let markets clear. Thus we have something close to a pure price effect since the only differences lie in labour income, interest rates and in the income tax. Nonetheless, while the composition of types is close to identical we can still have differences in asset holdings; the debtor share rises in the partial equilibrium exercise. The interest rate is lower than when markets clear, so lenders receive less asset income. As shown further below (see Figure 17), the distribution of wealth will shift to lower values of b. So despite having more debtors paying less interest, the shifts in the distribution are both bad for lenders and borrowers, dragging down aggregate welfare. Notice how with a lower income tax and higher expected labour earnings, aggregate welfare gains over SF are about four percentage points smaller than when markets clear. As mentioned above, TS 50% goes in the opposite direction, benefiting the most (out of the three TS variants) in the partial equilibrium exercise. It has a relatively higher interest rate, improvements in educational attainment and unambiguously lower inequality. The distribution of wealth places more mass to the right of the general equilibrium result. So in that case there are less debtors, more lenders and higher capital income. These effects dominate a higher income tax rate and slightly lower labour earnings than when markets clear.

In NICL, expected earnings and interest rates are higher than when markets clear. These two results affect welfare in opposite directions. On one hand a higher interest rate increases the risk of educational investments; it is harder to pay back student loans and being hit with unemployment or skill depreciation shocks become more painful. When we combine this with no income contingency protections we lower the attractiveness of earning a college degree. On the other hand graduates receive a higher wage and net savers get more asset income. The net impact delivers an amelioration of the distribution of agents relative to when markets clear, yielding relatively more college grads and less wealth inequality. The welfare gains are more substantial in the ICLs (especially ICL2). The reason behind this is that ICLs shield agents from movements in interest rates, and this is specially the case of ICL2. Besides highlighting how distributions interact with market prices this partial equilibrium exercise gives yet more evidence of the price effects of debt and the importance of evaluating higher education policy changes in general equilibrium.

Finally, I repeat the exercise of section 2.1, on borrowing limits and welfare, but this time in partial equilibrium. Figures (34) and (35) in the appendix show how solving the model in partial equilibrium can over (under) estimate welfare gains at low (high) subsidy rates and student loan debt limits. The over/under estimation follows closely the net debtor share and wealth inequality, illustrating once again how the distributional and price effects of debt are key determinants of aggregate welfare. In the next subsections I evaluate disaggregated comparisons as done earlier but in a partial equilibrium setting.

2.3.1 Disaggregated partial equilibrium results

Disaggregated partial equilibrium in TS: Figure (16) displays two results that stand out. First, all TS variants are superior to SF, everywhere in the state space, in the partial equilibrium exercise. Second, when prices are fixed at SF the welfare gains of TS 100% (which has a higher interest rate in general equilibrium) decrease for wealthy agents. The opposite happens in TS 50% (asset poor agents have smaller welfare gains relative to general equilibrium but asset rich agents gain more). Note that r in SF is not that different of that of TS 75 % when markets clear so there is not much change between the partial and general equilibrium CELs across all of the state space.

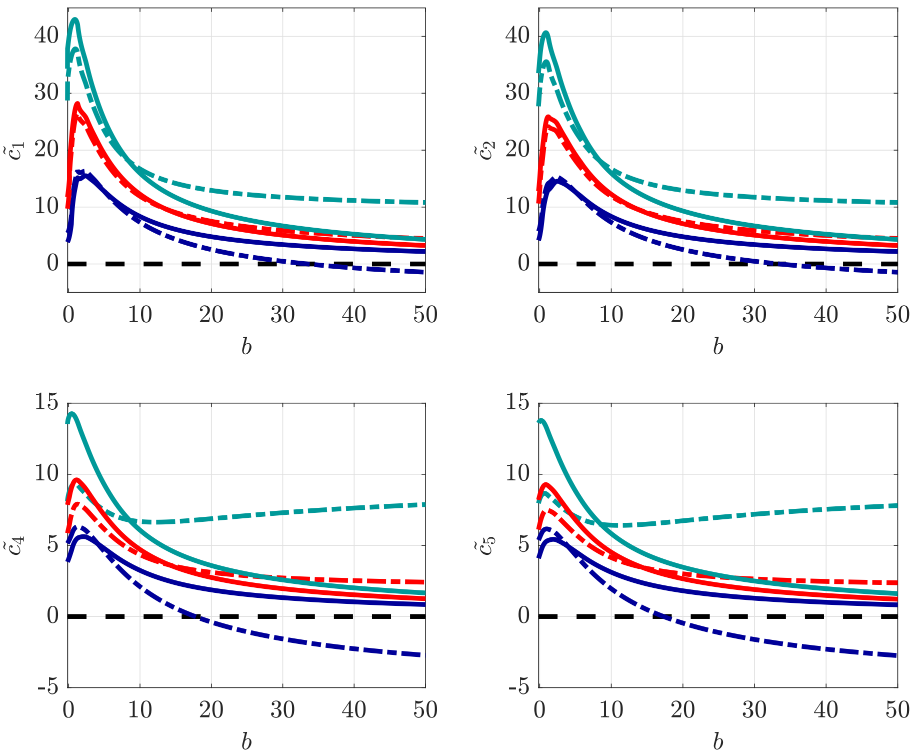

Figure 16: Dash/dotted lines are the same as in Figure (11), solid lines introduce the new educational policy but hold all SF prices and tax rate fixed.

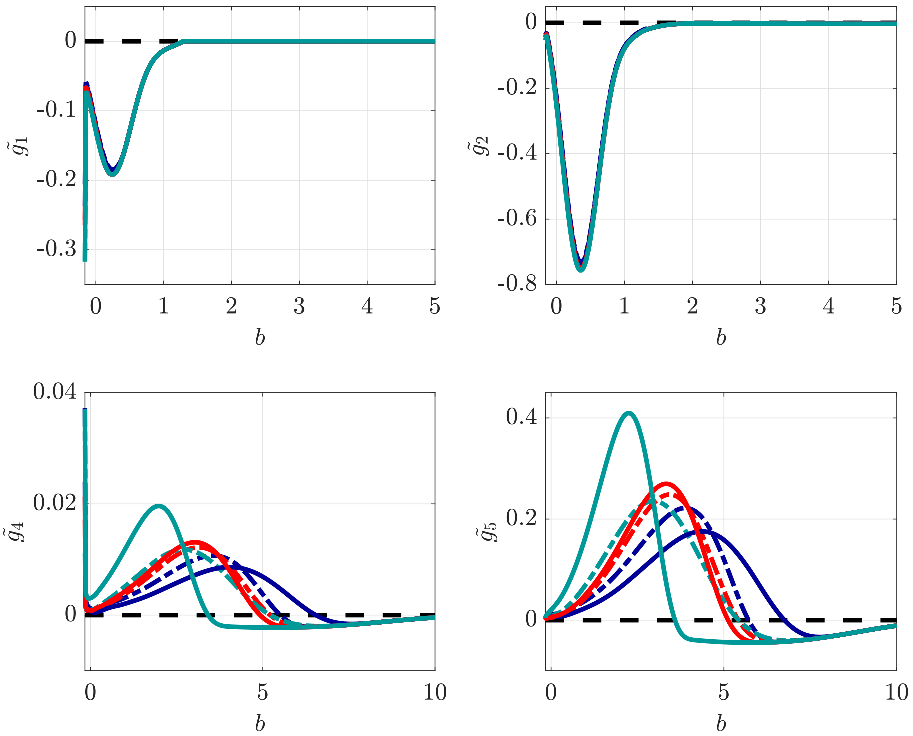

Figure 17: Dash/dotted lines are the same as in Figure (11), solid lines introduce the new educational policy but hold all SF prices and tax rate fixed.

Figure (17) depicts the density difference of TS with self financing. Changes in the distribution reinforce the variations in welfare that we saw in Figure (16). TS 50 % benefits the most from the relatively higher pre-reform interest rate and moves the distribution to larger values of b. The opposite happens for TS 100%, the distribution shifts towards lower values of b. As a consequence rankings change between these two variants of tuition subsidies. TS 75 % has very similar prices to those of SF, except for the income tax rate, which is slightly higher in general equilibrium, and sees no major changes in aggregate and individual welfare measures; the distribution is also about the same.

Disaggregated partial equilibrium in ICL1: As mentioned previously, given the difference in state space sizes, I will show the partial equilibrium comparison of every grid point in ICL1 against the benchmark calibration of self financing. Figure (18) depicts the difference in CEL, when we hold the SF prices fixed and then switch higher education funding to that of ICL1[30]. The black (green-blue) surface shows consumption equivalent variation in partial (general) equilibrium.

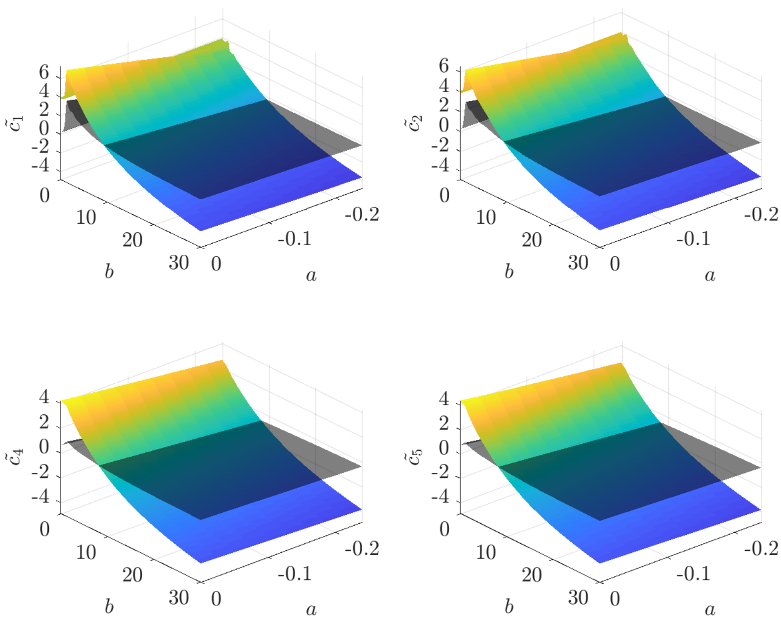

Figure 18: General equilibrium (green-blue) and partial equilibrium (black) disaggregated CEL in ICL1

The equilibrium interest rate in self financing is higher than when we let markets clear at ICL1. Thus, the partial equilibrium CEL surfaces are slightly lower (higher) for low (high) wealth agents relative to those in general equilibrium. The density difference plots are displayed in Figure (33) in the appendix. The figures reveal that the partial equilibrium exercise shifts the distribution towards more educated agents and towards lager values of b. The influx of educated workers does not depress wages, as they are fixed at the higher value in SF. The price effects and the endogenous change in the distribution illustrate how all student loan programs improve under the partial equilibrium experiment (with the largest gains taking place in the ICLs). Those most vulnerable (those with high debt in b and a) benefit the most from the income contingent protections in ICL1. Nonetheless, when repayment of student loan starts, the student loan interest rate is much higher than in other systems since $r_A = r + \lambda_{np}$. As a consequence, it should not surprise us that the group of educated and employed agents benefits the least from having the policy change with prices fixed at SF. Hence, while reductions in income uncertainty,

[30]Disaggregated welfare results and density differences of other student loan programs are left in the appendix. See Figures (30), (31), (32) and (33) in the appendix.

thanks to education financing and the endogenous change in the composition of types, contribute to welfare gains, these are dampened (magnified) by less (more) favourable prices for borrowers (lenders) in partial equilibrium.

It is evident that the price effect described in Obiols-Homs (2011) and the endogenous make up of agent types play an important role in shaping the welfare gains of higher education reforms, especially through the link between the share of the population in net debt and wealth inequality.

3 Transitions

Since there are large steady state welfare differences amongst the five HE systems considered in this paper, is it worth making the transition to the system yielding the largest welfare gain? This section seeks to answer that question. The results shown in Table (2) and Figure (6) indicate that the biggest steady state welfare gain is between SF and TS 100%. Thus the next experiment considers an unexpected, immediate[31] and permanent change of the subsidy rate from %0 to 100%. Figures (19) and (20) show the transitional dynamics of key aggregates.

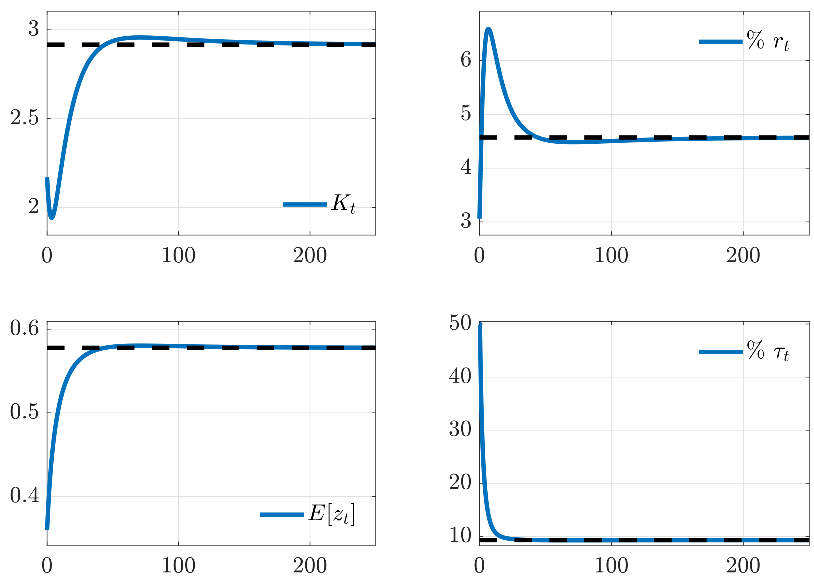

Figure 19: SF to TS %100

After the policy change the share of workers with a university degree increases, roughly doubling in about 12 years. Wealth inequality (wealth Gini and cofficient of variation) and the net debtor population share decrease after an initial spike. During the spike, interest rates remain substantially higher than in the initial and terminal equilibrium. Furthermore, the early years of the transition are accompanied by a large jump in the income tax rate (almost a tenfold increase from the initial SF steady state τ value of 5.83% to 49 %) and a short term drop in the effective labour force (non-college grads workers become students and reduce work hours). As seen earlier, higher net debtor shares and interest rates drag down welfare. Given that agents discount future gains, more weight is placed on the immediate sacrifices incurred during the

[31]Similar experiments where policy changes are announced in advance yield lower gains since agents postpone enrolling in university until the subsidy is in place. This has negative aggregate effects given that it initially lowers θ_5. A perpetual youth or life-cycle formulation would dampen such an effect since the education choice will probably be made once in a single life time and sooner rather than later.

transition. Hence, despite the substantial steady state gains the transition costs make the policy change less appealing than when we just compare steady states.

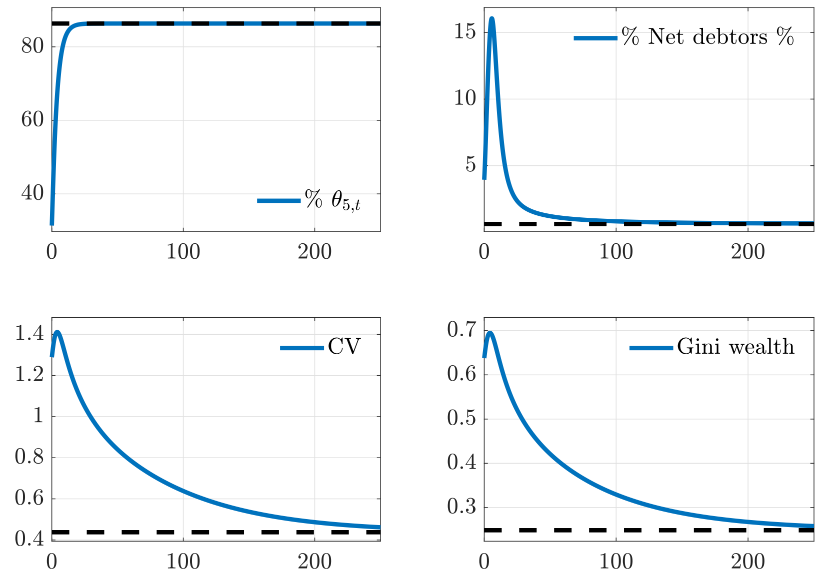

Figure 20: SF to TS %100

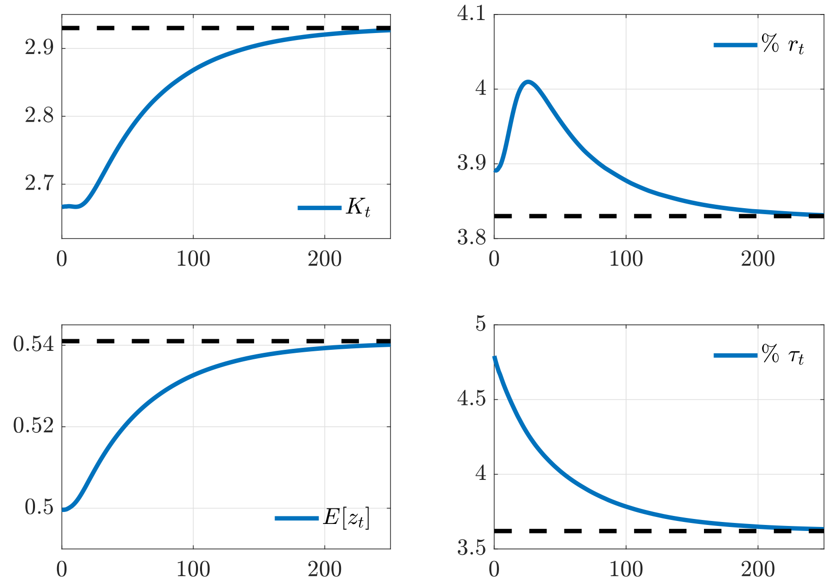

Figure 21: NICL to ICL1

The aggregate CEL of the transition from self financing to full tuition subsidies amounts to a gain of 21.17 %, that is over thirty seven percentage points lower than in the steady state comparisons. This indicates that steady state comparisons alone may be misleading. I repeat the exercise, but this time for a smaller reform, transitioning among two different student loan programs, computing the transition from NICL to ICL1. Could it be that smaller reforms that are less costly make policy changes more viable? The transition paths are shown in Figures (21) and (22).

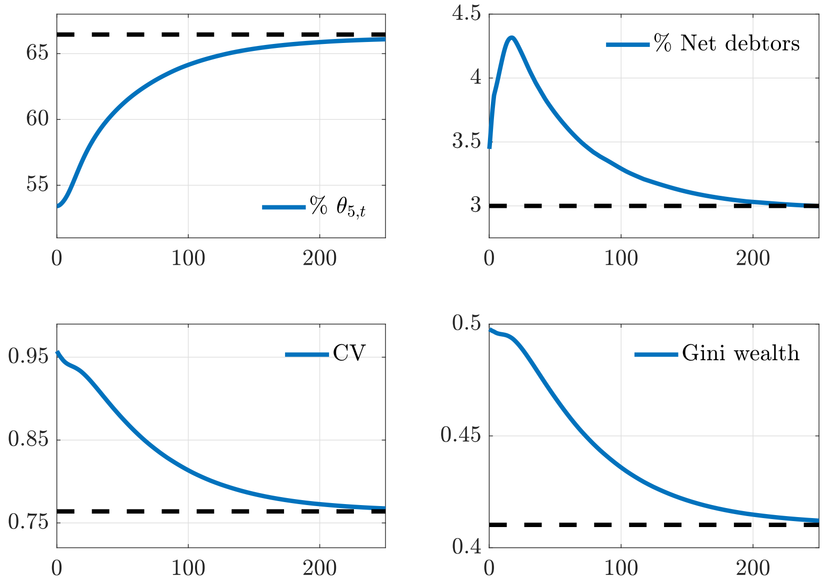

Figure 22: NICL to ICL1

As before, the initial periods are marked by higher taxes, interest rates and indebtedness while gains accrue much later. While the rise in the tax, net debtor population share and interest rate is more muted, the reform yields a positive but small CEL of 0.6223 %. Now that we have an idea of what makes transitions, big or small, costly, the same exercise is repeated for transitions from 1) SF to all possible subsidy rates between 30 and 100 % (say transitioning from SF to TS 30 % and from SF to TS 40 % and so on) and 2) from SF to ICL1 with student debt limits covering 40 to 200% of the costs of education (for instance, from SF to ICL1 with $\underline{a} = -1$ and then from SF to ICL1 with $\underline{a} = -1.05$ and so on), keeping the rest of the parameters in their baseline calibration.

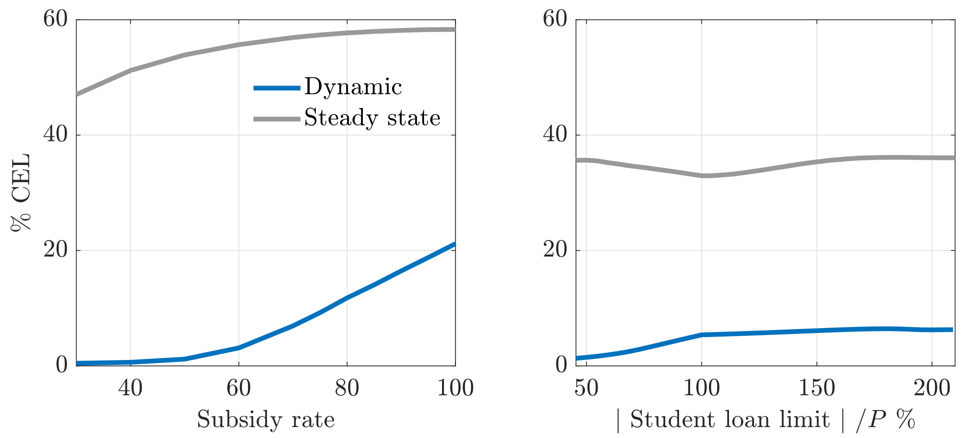

Figure 23: Steady state CEL gain (grey) and dynamic CEL (in blue) - SF to TS - right - SF to ICL1

Figure (23) reveals that when we compute the transition between regimes, the welfare gains can be three to ten times smaller than when just comparing the steady state CELs. Figure (36) in the appendix does not overlay the steady state gains to those factoring the transition costs, so that one can zoom in on the scale of gains. This last experiment reveals that moving towards moderate partial tuition subsidies ($s < 50\%$) is dominated by ICL1. However, generous tuition subsidies (closer to 100 %) outperform ICL1, regardless of the student loan limit. It is clear that policy changes in higher education financing should factor the costs of transitions.

4 Conclusion

In this paper I evaluate the welfare and wealth inequality outcomes of five different higher education financing schemes, with the help of a heterogeneous agent production economy in continuous time, extended to allow for endogenous educational choices. The main contribution of this study is to evaluate the financing of tertiary education under the light of *the price and quantity effects of debt*. When we ignore the pecuniary externalities described in Obiols-Homs (2011), Nuño and Moll (2017) and Angelopoulos et al. (2017) we miss general equilibrium effects that are powerful enough to tilt the balance on which higher education system yields the largest aggregate and individual welfare gains. This article also contributes in identifying when government intervention in tertiary education can increase welfare, reduce inequality and at what cost. This contribution can be broken into four findings.

First, the ranking between systems depend on the cost of education. If the price of education is low and the time length of study is short, then tuition subsidies or student loans may drag down welfare relative to self financing. That is, when education is relatively easy to achieve, the capital market failures associated with educational investments do not matter enough to warrant government intervention. When the costs of education are calibrated to realistic values, government guaranteed income contingent loans and tuition subsidies are found to be the best choice, out of the five systems considered herein, to finance higher education, with the latter yielding the largest steady state gains.

Second, while I show significant steady state welfare differences between various higher education systems, large transition costs from one regime to another diminish the desirability of policy changes. That is, comparing steady states alone may be misleading for policy, transition costs must be factored in.

Third, balanced budget tax rates can be higher (relative to regimes with tuition subsidies) in systems relying on income contingent student loans. Fourth, public financing of higher education only increases inequality when the cost of education is low; if this cost rises to realistic levels inequality falls as public sector support increases. This is particularly true with tuition subsidies; they yield the lowest inequality outcomes in all the systems considered in this paper.

This paper will be extended along three directions. The first extension addresses the fact that this study has abstracted from important dimensions such as labour skill substitutability, age and ability. A future extension of this work will include these dimensions, emphasise educational investments as a once-in-a-lifetime decision and explore the interactions between higher education funding and intergenerational inequality.

A separate extension will disaggregate the college wage premium, as recent UK based studies - Belfield et al. (2018a) and Belfield et al. (2018b) - have shown that while the average premium has remained stable, if not rising, it is driven by substantial heterogeneity and large outliers, where the returns to higher education vary by subject, institution and agents' gender. Should we fund higher education generously when the distribution of returns is highly skewed? The third avenue that will be pursued seeks to understand how increased longevity and increasing exposure to automation risk may warrant repeated educational investments and additional government support in higher education. Would longer lives lead us to retrain and go back to university at older ages? What role can the public sector play to mitigate the riskiness and capital market imperfections affecting tertiary education?

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5 Appendix

5.1 Market clearing

In this subsection I show a heuristic proof of how to show that $K_S = K_D$ implies $Y = C + I +$ Ed costs. The same steps can be applied to any regime and will lead to the same conclusion. For the sake of brevity, I illustrate this with the TS regime[32]. Following Nuño and Moll (2017) we start by the aggregate law of motion of capital.

$$
\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}\left[b_i g_i\right] &= S_i g + b_i \dot{g}_i \\
\sum_{i=1}^{5}\int_b \frac{\mathrm{d}}{\mathrm{d}t}[b g_i]\mathrm{d}b &= \sum_{i=1}^{5}\int_b S_i g_i \mathrm{d}b + \sum_{i=1}^{5}\int_b b\dot{g}_i \mathrm{d}b \\
\frac{\mathrm{d}}{\mathrm{d}t}\sum_{i=1}^{5}\int_b [b g_i]\mathrm{d}b &= \sum_{i=1}^{5}\int_b S_i g_i \mathrm{d}b + \sum_{i=1}^{5}\int_b b\dot{g}_i \mathrm{d}b \\
\frac{\mathrm{d}}{\mathrm{d}t} K &= \sum_{i=1}^{5}\int_b S_i g_i \mathrm{d}b + \sum_{i=1}^{5}\int_b b\dot{g}_i \mathrm{d}b \\
0 &= \sum_{i=1}^{5}\int_b S_i g_i \mathrm{d}b + \sum_{i=1}^{5}\int_b b\dot{g}_i \mathrm{d}b \quad \text{(in steady state)}
\end{aligned}
\tag{24}
$$

Expanding the first term on the right hand side gives us the following result.

$$
\begin{aligned}
& w\left[(1-\tau)[z_L\theta_2 + z_H\theta_5] + \mu[z_L\theta_1 + z_H\theta_4] + z_L z_s \theta_3\right] + r\sum_{i=1}^{5}\int_{\underline{b}}^{\infty} b g_i \mathrm{d}b - C \\
& \frac{(1-\alpha)Y}{\tilde{L}}\left[z_L\theta_2 + z_H\theta_5 + z_L z_s\theta_3\right] + \left[\frac{\alpha Y}{K_D} - \delta\right] K_S - C - \text{Education subsidies} \\
& Y - I - C - \text{Education subsidies}
\end{aligned}
\tag{25}
$$

The last step requires that $K_S = K_D$, which is what was intended to be shown. Education subsidies are captured by the next expression.

$$
sP\left(\int_{b_1^{\dagger}}^{\infty}[\lambda_{21}g_2 + \lambda_{ex}^{S}g_3 + \lambda_{ex}^{U}g_4]\mathrm{d}b + \int_{b_2^{\dagger}}^{\infty}\left[\lambda_{ex}^{E}g_5 + \lambda_{12}g_1\right]\mathrm{d}b\right) \tag{26}
$$

Expanding the second term on the right hand side of (24) yields aggregate education costs covered by agents (private sector). The KFEs of the single asset economy are given by the following five expressions. Note that dependence on b is made explicit only when wealth drops due to education costs[33].

[32] With two asset models we have to perform multivariate integration parts. I omit the derivation in this paper, but can be reproduced upon request.

[33] Note that inflows to g_1 and g_2 beyond their respective boundaries $b_i^{\dagger}$ are immediately redirected to g_3. In the single asset case these jumps act as wealth shocks and are thus represented as inflows in $g_3(b)$ coming from $g_i(\tilde{b^*}), i = 2,3,4,5$ where $\tilde{b^*} = b + P(1-s)$. While there is some abuse of notation I am conveying that there is no mass in g_1 and g_2 beyond $b_i^{\dagger}$.

$$\partial_t g_1 = -\partial_b[S_1 g_1] + \lambda_{21} g_2 - \lambda_{12} g_1 + \lambda_{ex}^S g_3 + \lambda_{ex}^U g_4 \quad \text{for } b < b_1^\dagger \tag{27}$$

$$\partial_t g_2 = -\partial_b[S_2 g_2] + \lambda_{12} g_1 - \lambda_{21} g_2 + \lambda_{ex}^E g_5 \quad \text{for } b < b_2^\dagger \tag{28}$$

$$\partial_t g_3 = -\partial_b[S_3 g_3] - [\lambda_{ex}^S + \lambda_{34} + \lambda_{35}] g_3 + \lambda_{ex}^S g_3(\tilde{b^*}) + \lambda_{21} g_2(\tilde{b^*}) + \lambda_{ex}^U g_4(\tilde{b^*}) + \lambda_{ex}^E g_5(\tilde{b^*}) + \lambda_{12} g_1(\tilde{b^*}) \tag{29}$$

$$\partial_t g_4 = -\partial_b[S_4 g_4] + \lambda_{54} g_5 - [\lambda_{ex}^U + \lambda_{45}] g_4 + \lambda_{34} g_3 \tag{30}$$

$$\partial_t g_5 = -\partial_b[S_5 g_5] + \lambda_{35} g_3 + \lambda_{45} g_4 - [\lambda_{ex}^E + \lambda_{54}] g_5 \tag{31}$$

Adding the KFEs, multiplying by b and integrating, as in the second term on the right hand side of (24), yields the next result.

$$\begin{aligned}
\sum_{i=1}^{5} \int_{\underline{b}}^{\infty} b\dot{g}_i \mathrm{d}b = &- \sum_{i=1}^{5} \int_{\underline{b}}^{\infty} b\partial_b[S_i g_i]\mathrm{d}b - \int_{b_1^\dagger}^{\infty} b[\lambda_{21} g_2 + \lambda_{ex}^S g_3 + \lambda_{ex}^U g_4]\mathrm{d}b - \int_{b_2^\dagger}^{\infty} b\left[\lambda_{ex}^E g_5 + \lambda_{12} g_1\right]\mathrm{d}b \\
&+ \int_{b_1^\dagger - P(1-s)}^{\infty} b[\lambda_{21} g_2(\tilde{b^*}) + \lambda_{ex}^S g_3(\tilde{b^*}) + \lambda_{ex}^U g_4(\tilde{b^*})]\mathrm{d}b \\
&+ \int_{b_2^\dagger - P(1-s)}^{\infty} b\left[\lambda_{ex}^E g_5(\tilde{b^*}) + \lambda_{12} g_1(\tilde{b^*})\right]\mathrm{d}b \\
\sum_{i=1}^{5} \int_{\underline{b}}^{\infty} b\dot{g}_i \mathrm{d}b = &- \sum_{i=1}^{5} \int_{\underline{b}}^{\infty} b\partial_b[S_i g_i]\mathrm{d}b \\
&- P(1-s)\left(\int_{b_1^\dagger}^{\infty} [\lambda_{21} g_2 + \lambda_{ex}^S g_3 + \lambda_{ex}^U g_4]\mathrm{d}b + \int_{b_2^\dagger}^{\infty} \left[\lambda_{ex}^E g_5 + \lambda_{12} g_1\right]\mathrm{d}b\right)
\end{aligned} \tag{32}$$

Using integration by parts one can show that the first term on the right hand side of (32) is equal to zero. The second term captures education costs covered by agents, as the difference between b^* and $b^\dagger$ and $\tilde{b^*}$ and b is $P(1-s)$. Putting everything together gives us the national accounting identity, augmented with aggregate flow educational expenditure. Since we are solving for the steady state, the inflow of new graduates will be equal to the depreciation of the stock of those with higher education[34].

$$\begin{aligned}
0 &= Y - I - C - P\left(\int_{b_1^\dagger}^{\infty} [\lambda_{21} g_2 + \lambda_{ex}^S g_3 + \lambda_{ex}^U g_4]\mathrm{d}b + \int_{b_2^\dagger}^{\infty} \left[\lambda_{ex}^E g_5 + \lambda_{12} g_1\right]\mathrm{d}b\right) \\
Y &= I + C + \text{Education costs}
\end{aligned} \tag{33}$$

The derivations for models with student loans, although more tedious due to two types of assets, lead to similar results and can be shown following the same steps. χ_1 and χ_2 are analogous to the flows $[\lambda_{21} g_2 + \lambda_{ex}^S g_3 + \lambda_{ex}^U g_4]$ and $\lambda_{ex}^E g_5 + \lambda_{12} g_1$, respectively, in the single asset case.

[34] An easier way to compute aggregate education costs is $P\left[\lambda_{ex}^S \theta_3 + \lambda_{ex}^U \theta_4 + \lambda_{ex}^E \theta_5\right]$. The three terms inside the brackets are the outflows from college graduates to non-college-graduates. In steady state outflows from one group have to equal its inflows. Hence, inflows into non-college graduates are equal to its outflow (flows from non college graduates to students). Yet another alternative is to compute $P\theta_3\left[\lambda_{ex}^S + \Delta_{ed}\right]$ (outflows of student type).

5.2 Portfolio problem and pecking order

An earlier version of this model allowed agents to maximise V_3 by choosing how to pay P with the best feasible combination of b and a in the NICL, ICL1 and ICL2 regimes. The results are identical to the ones presented here. Except for unrealistic calibrations, agents rarely pay for university using exclusively b. This motivated the use of a so-called 'pecking order' mechanism to model the decision of how to cover P. This is computationally less expensive. An example of how this works is show in Figure (24).

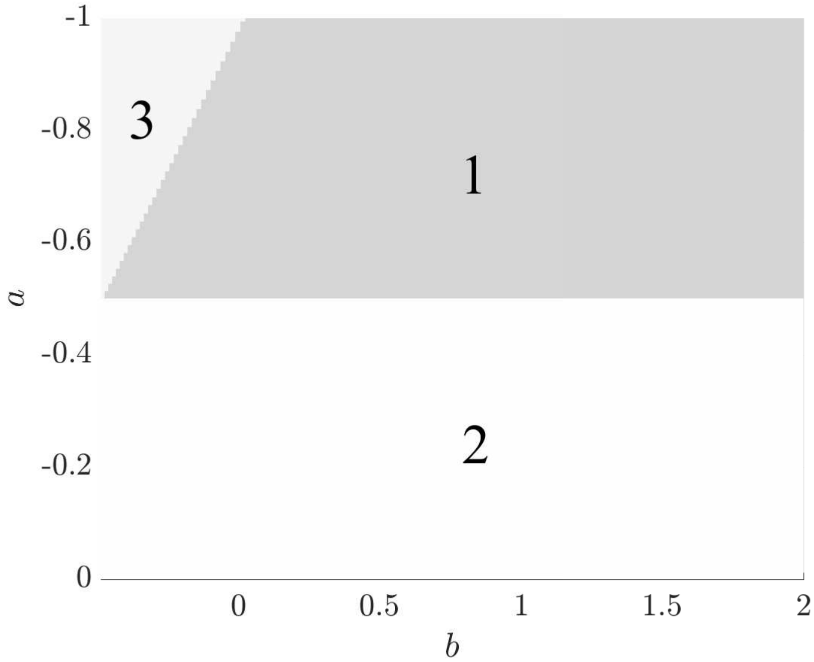

Figure 24: 1 - Cover tuition with mix of b and a. 2 - Cover tuition with a only. 3 - Cannot afford to go to university.

Suppose that $P = 0.5$, $\underline{a} = -1$ and $\underline{b} = -0.5$. The agent will first try to cover P exclusively with student loans, a situation depicted in region 2. If the agent has more than 0.5 in student loans, it will only be able to cover the difference between $\underline{a}$ and -0.5 in new student loans, and cover the rest of P with b. This case is that of region 1. Finally, if the agent has little b and a large stock of a, then it will not be able to go to university, the case of region 3.

5.3 Benchmark calibration of P

In 2018 U.S. GDP per capita stood at \$54541 according to the World Bank. The U.S. Bureau of Economic Analysis reports that U.S. population reached 327.436 million while the OECD estimates that the working age population in the U.S. stood at 206.513 million in 2018. Given that in the 'NICL' regime I set $s = 0$, I calibrate P to the cost of attending a private non-profit university. According to the College Board, the average annual cost of an American private non-profit university in 2018 was \$37430 (tuition) and \$48510 (tuition and board). The benchmark calibration of P can be thus set to:

$$\underline{P}_B = \frac{37430 * 4}{\frac{327.436}{206.513} * 54541} * w\mathbb{E}[z] \qquad \text{Lower bound}$$
$$\overline{P}_B = \frac{48510 * 4}{\frac{327.436}{206.513} * 54541} * w\mathbb{E}[z] \qquad \text{Upper bound}$$

When equilibria deliver a positive amount of college graduates, $w\mathbb{E}[z]$ is fairly stable between 0.5 and 0.59, with an average of 0.56. So for the benchmark calibration I choose 0.56. Given that $w\mathbb{E}[z]$ is an endogenous result and the uncertainty around to what extent do we include student accommodation expenses in the costs of education, the reader is asked to consider P as an arbitrary yet reasonable pick for a benchmark. The sensitivity analysis in P gives further indication of how each regime fares with a large range of educational prices. As mentioned earlier, a perpetual youth extension of this paper endogenises P and matches it's relationship to average income in the United States. The results are fairly consistent with what is shown here.

5.4 Results when P is low and Δ_{ed} is high

When the price of education is low and it takes less time to become educated, the capital market imperfection in educational investment is no longer large enough to justify government support, either with student loans or tuition subsidies. As shown in Table (4) the CEL over self financing is now negative for all regimes. All HE systems give more or less the same share of educated workers. Nonetheless, income taxes are lower in self financing than in any other system. As mentioned earlier, the relationship between inequality and government support is now reversed; more government support fosters inequality. Additionally, the rankings have mostly reversed. Tuition subsidies yield the lowest CEL. The relationship between CEL and the population net debtor share seems to have weakened.

	SF	NICL	ICL1	ICL2	TS50 %	TS75 %	TS100 %
CEL %	0.0000	-0.1787	-0.2314	-0.2276	-0.2464	-0.4203	-0.6041
K	2.9882	2.9709	2.9623	2.9630	2.9244	2.9052	2.8923
$\mathbb{E}[z]$	0.6014	0.6003	0.5997	0.5998	0.5973	0.5960	0.5952
r	0.0472	0.0476	0.0478	0.0478	0.0486	0.0491	0.0494
Y	0.8802	0.8786	0.8777	0.8778	0.8742	0.8723	0.8711
θ_1	0.0003	0.0002	0.0002	0.0002	0.0001	0.0000	0.0000
θ_2	0.0007	0.0005	0.0006	0.0006	0.0001	0.0000	0.0000
θ_3	0.0245	0.0246	0.0246	0.0246	0.0246	0.0246	0.0246
θ_4	0.0484	0.0484	0.0484	0.0484	0.0484	0.0484	0.0484
θ_5	0.9262	0.9263	0.9263	0.9263	0.9269	0.9270	0.9270
τ	0.0200	0.0205	0.0218	0.0216	0.0321	0.0382	0.0443
τ_{UI}	0.0200	0.0200	0.0200	0.0200	0.0200	0.0200	0.0199
Gini$_w$	0.2080	0.2146	0.2257	0.2293	0.2445	0.2690	0.2939
Net debtors	0.0014	0.0011	0.0013	0.0013	0.0008	0.0007	0.0010

Table 4: $\Delta_{ed} = 1$ - $P = 0.5$

Figure (25) illustrates that steady state CEL gains are robust to different debt limits in b and a. For sake of brevity I only show the cases for NICL (in red) and ICL1 (in yellow/blue)[35].

[35]Results for TS and ICL2 can be produced upon request.

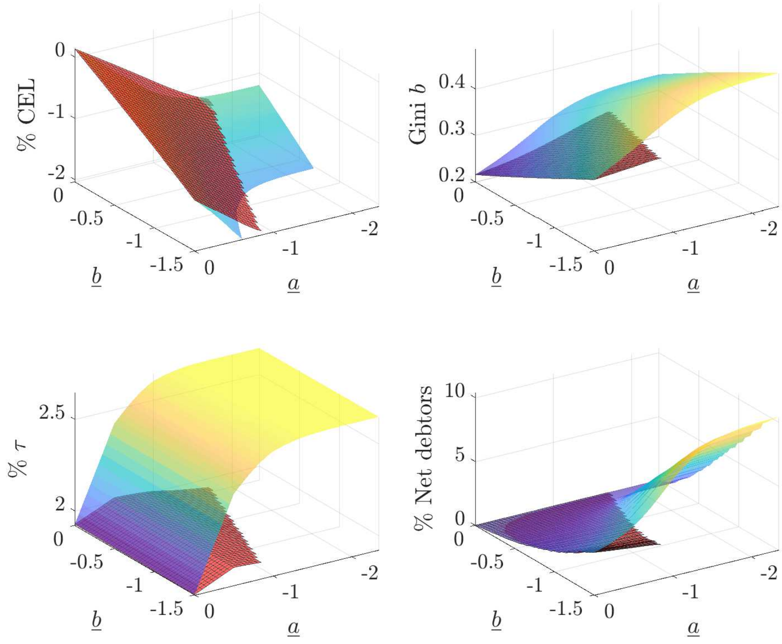

Figure 25: CEL, wealth Gini, tax rate and net debtor share in NICL and ICL1 with $P = 0.5$ and $\Delta_{ed} = 1$. The red (blue/yellow) surface depicts NICL (ICL1).

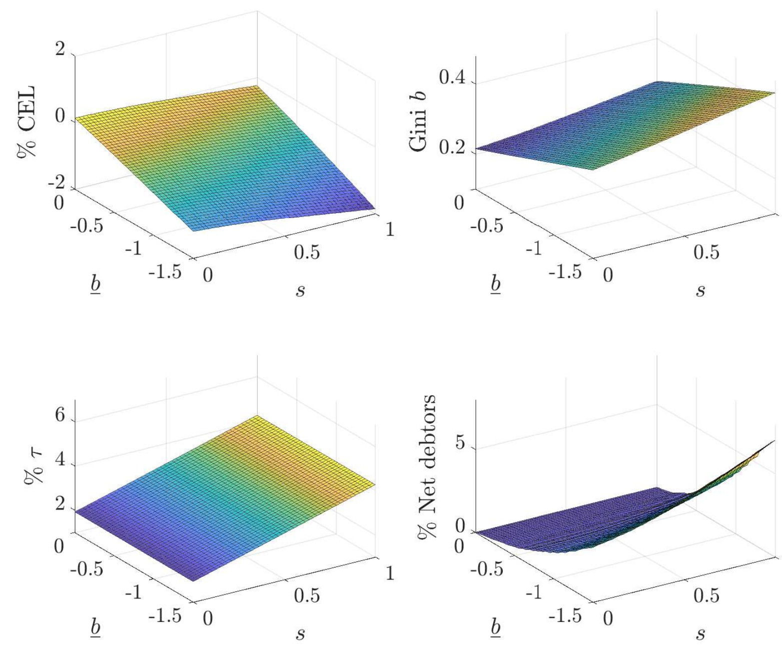

Figure 26: CEL, wealth Gini, tax rate and net debtor share in TS with $P = 0.5$ and $\Delta_{ed} = 1$

5.5 Additional tables and figures

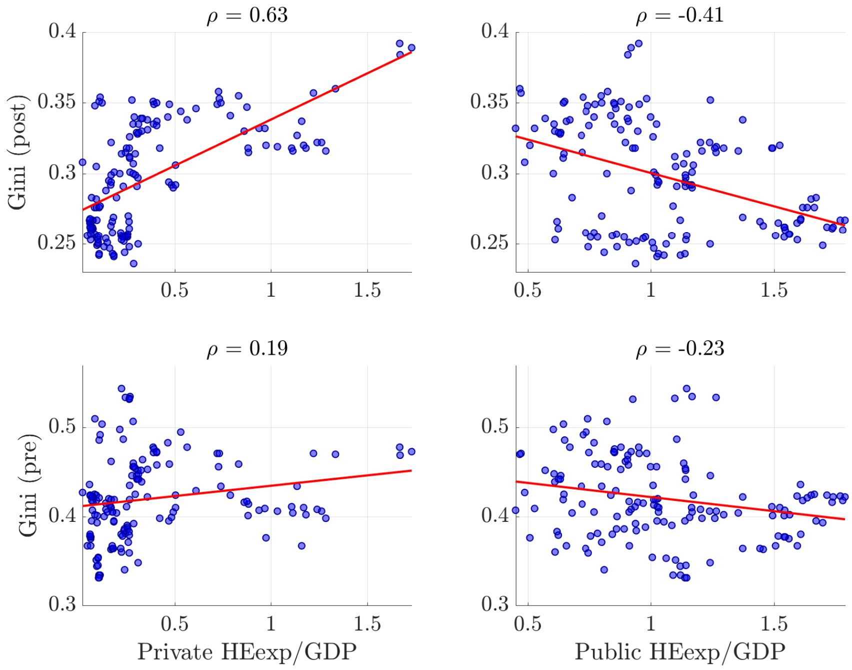

Figure 27: Working age income Gini (post and pre redistribution) vs. HE expenditure relative to GDP and its correlation during 2000-2016 in the following countries: AUS, AUT, CAN, CZE, DEU, DNK, ESP, EST, FIN, FRA, GBR, GRC, IRL, ISL, ISR, ITA, JPN, KOR, LUX, NLD, NOR, NZL, PRT, SVK, SVN, SWE, USA. Source: OECD.

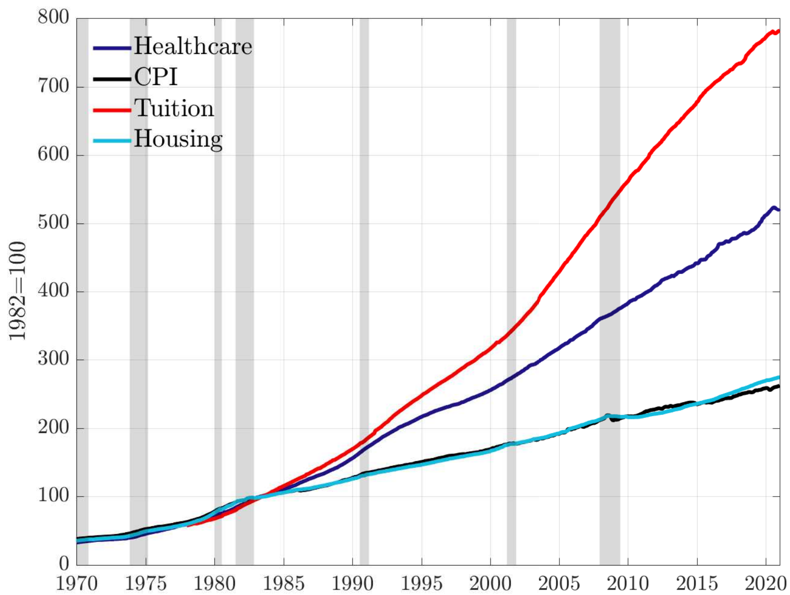

Figure 28: Tuition inflation. Source: US Bureau of Labor Statistics (2020)

5.5.1 Borrowing limits and welfare in ICL2

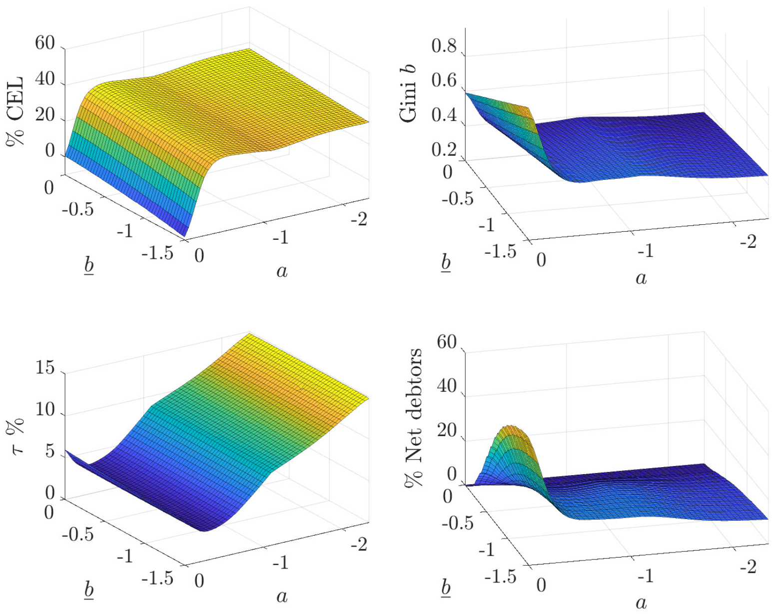

Figure 29: CEL, wealth Gini, tax rate and net debtor share in ICL2

5.5.2 NICL vs. ICL1 at benchmark calibration

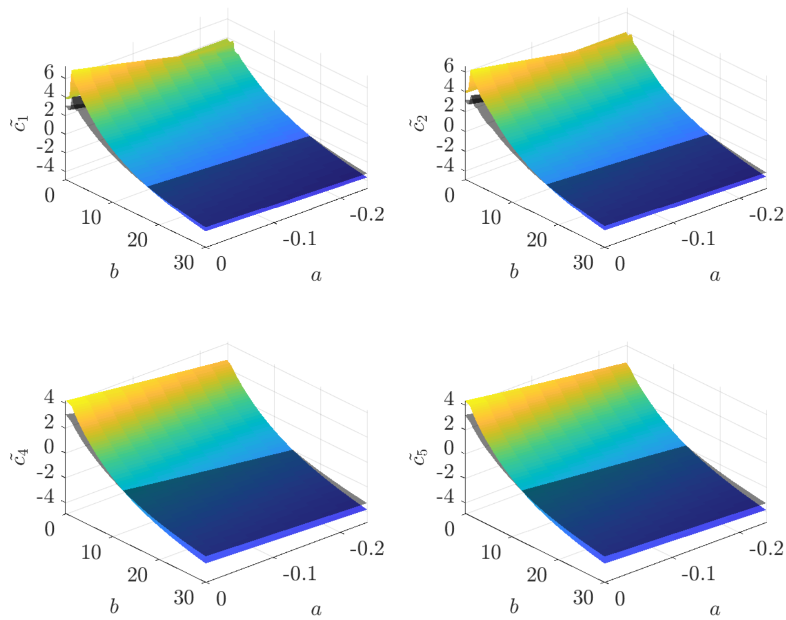

Figure 30: Disaggregated CEL over SF of NICL (in black) and ICL1 (in colour) with baseline calibration

5.5.3 Density difference amongst student loan programs

The most salient fact is that ICL1 places more mass in zero student debt workers with a college degree. This is probably due the interest and amortisation subsidies and especially due to debt cancellation. Additionally, NICL places more mass, relative to ICL1, on groups 1 and 2; this is reflected on a smaller share of workers with a college degree in Table (2). Comparisons between NICL and ICL2 are broadly similar, except that the latter places less mass in groups 4 and 5 (and thus has a larger share of non college-educated workers).

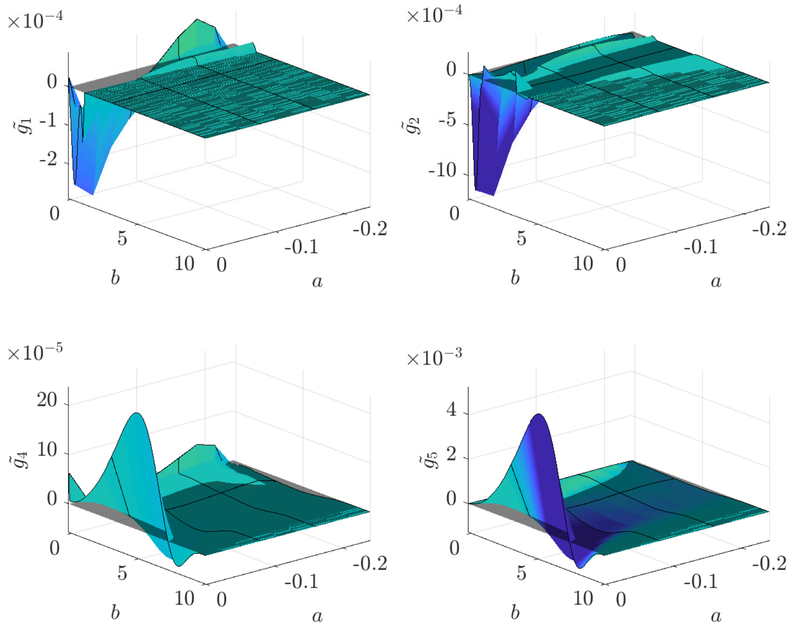

Figure 31: Density difference - $g_{ICL1} - g_{NICL}$ in baseline calibration

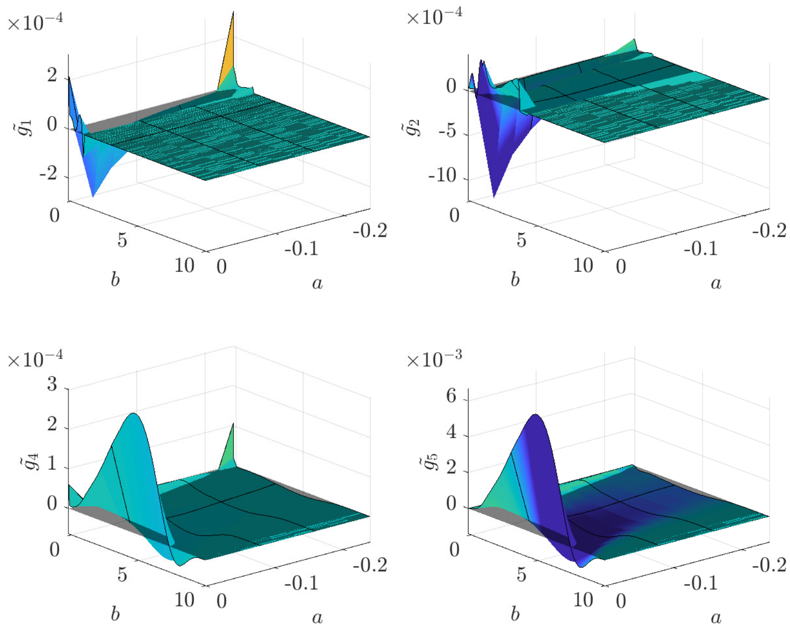

Figure 32: Density difference $g_{ICL2} - g_{NICL}$ in baseline calibration

5.5.4 Density difference of student loan programs in partial and general equilibrium

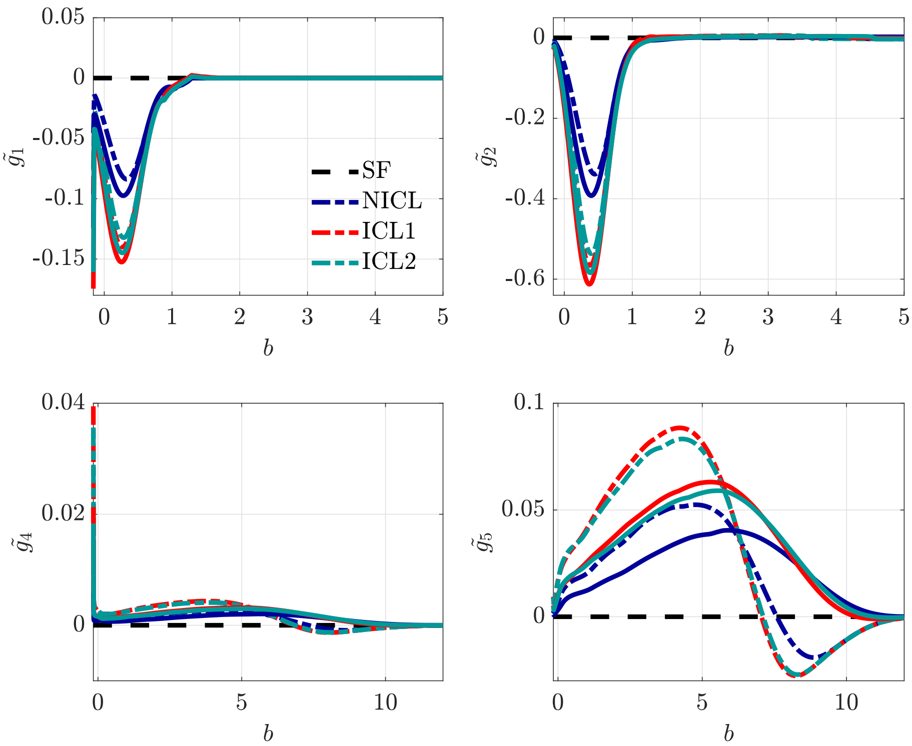

Figure 33: Density difference $g_{ICL1} - g_{SF}$ in partial and general equilibrium. As in Figure(14), dash dotted (solid) lines represent general (partial) equilibrium results and NICL, ICL1 and ICL2 are represented in blue, red and turquoise, respectively.

5.5.5 Borrowing limits and welfare - GE vs PE

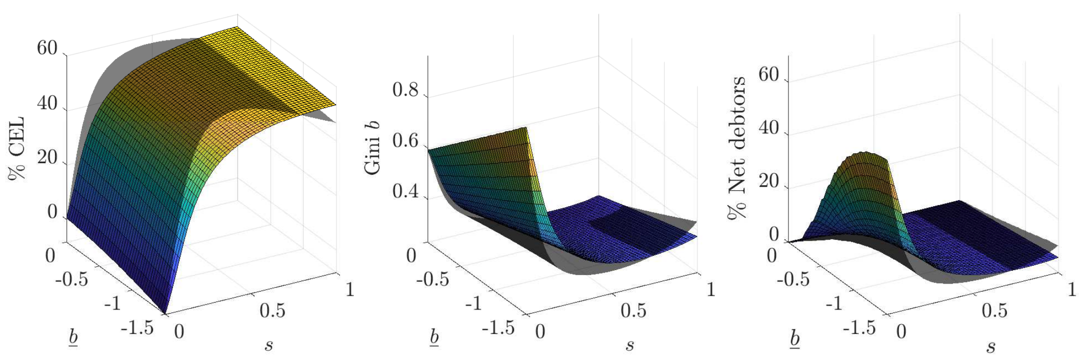

Figure 34: CEL, wealth Gini, tax rate and net debtor share in SF and TS - PE (black). $P = 1.1$ and $\Delta_{ed} = 0.25$

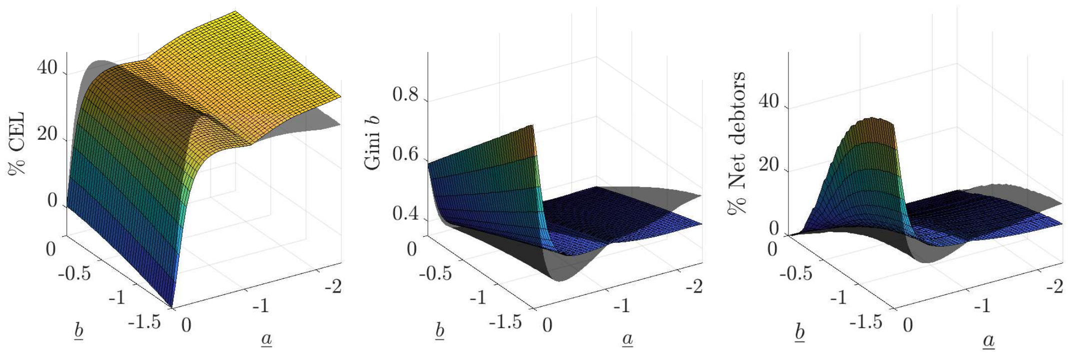

Figure 35: CEL, wealth Gini, tax rate and net debtor share in ICL1 - PE (black). $P = 1.1$ and $\Delta_{ed} = 0.25$

5.5.6 Zooming in transition CEL

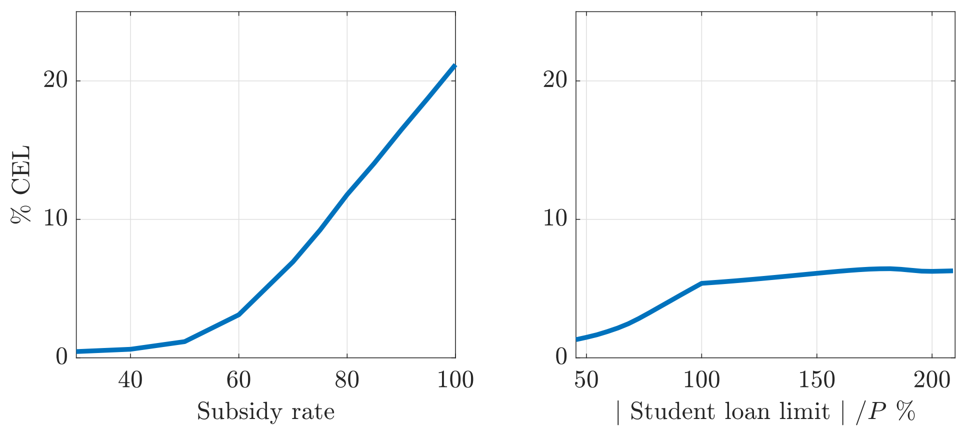

Figure 36: Left - SF to TS for $s \in [0.3, 1]$ - right - SF to ICL1 $\underline{a} \in [-2.35, 0.5]$