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Abstract

This study explores the dynamic effects of patent policy on innovation and income inequality in a Schumpeterian growth model with endogenous market structure and heterogeneous households. We find that strengthening patent protection has a positive effect on economic growth and a positive or an inverted-U effect on income inequality when the number of differentiated products is fixed in the short run. However, when the number of products adjusts endogenously, the effects of patent protection on growth and inequality become negative in the long run. We also calibrate the model to US data to perform a quantitative analysis and find that the long-run negative effect of patent policy on inequality is much larger than its short-run positive effect. This result is consistent with our empirical finding from a panel vector autoregression.

JEL classification: D30, O30, O40
Keywords: patent policy, income inequality, innovation, endogenous market structure

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1 Introduction

A recent study by Aghion *et al.* (2019) provides empirical evidence to show that innovation and income inequality have a positive relationship. However, innovation and income inequality are both endogenous variables; therefore, it would be interesting to see how they are both affected by an exogenous factor. Many growth-theoretic studies have explored the effects of patent policy on innovation in the macroeconomy, but these studies often do not consider its microeconomic implications on the income distribution. Therefore, this study analyzes the effects of patent policy on innovation and inequality. Furthermore, the Schumpeterian growth model that we develop allows us to derive how the effect of patent policy on income inequality changes over time. The tractability of this dynamic analysis enables us to compare the transition path of income inequality from the growth model to the impulse response function estimated from a panel vector autoregression (VAR).

We introduce heterogeneous households into a Schumpeterian model with endogenous market structure to explore the effects of patent protection on economic growth and income inequality. The Schumpeterian model with endogenous market structure is based on Peretto (2007, 2011) and features both horizontal innovation (i.e., variety expansion) and vertical innovation (i.e., quality improvement). Although endogenous market structure gives rise to transition dynamics in the aggregate economy, the wealth distribution of households is stationary along the entire transition path due to the stationary consumption-output and consumption-wealth ratios. This useful property makes our analysis tractable. Upon deriving the autonomous dynamics of the average firm size, we are able to also derive the dynamics of economic growth and the evolution of the income distribution.

In this growth-theoretic framework, we find that strengthening patent protection leads to a higher growth rate and causes a positive or an inverted-U effect on income inequality when the number of differentiated products is fixed in the short run. However, when the number of products adjusts endogenously, the effects of patent protection on economic growth and income inequality become negative in the long run. The intuition of the above results can be explained as follows.

Stronger patent protection confers more market power to monopolistic firms, which then charge a higher markup and earn more profits. As a result, strengthening patent protection has a positive effect on innovation and economic growth in the short run. However, the increased profitability also attracts the entry of new products, which in turn reduces the size of the market captured by each product. Given that it is the market size of a product that determines the incentives for quality-improving innovation,[1] the entry of new products caused by stronger patent protection stifles quality-improving innovation, which determines long-run growth.[2] These contrasting effects of patent protection on economic growth at different time horizons have novel implications on the dynamics of income inequality.

In our model, households own different amounts of wealth. This wealth inequality gives rise to income inequality.[3] Given that asset income is determined by the rate of return on assets and the value of assets, an increase in either the real interest rate or asset value

[1] See Laincz and Peretto (2006) for empirical evidence.

[2] See Peretto and Connolly (2007) for a theoretical explanation on why vertical innovation, instead of horizontal innovation, drives growth in the long run.

[3] See Piketty (2014) for the importance of wealth inequality on income inequality.

would raise income inequality. As a result, strengthening patent protection has the following effects on income inequality in the short run. The positive effect of patent protection on the equilibrium growth rate leads to a higher interest rate through the Euler equation of the households; therefore, strengthening patent protection has a positive effect on income inequality. This effect is also present in previous studies, such as Chu (2010) and Chu and Cozzi (2018), who focus on quality improvement without variety expansion. In our model, endogenous entry gives rise to a novel effect. The larger markup as a result of stronger patent protection reduces the demand for intermediate goods, which in turn reduces the value of assets through the entry condition of new products. Therefore, strengthening patent protection also has a negative effect on income inequality.

The above positive and negative effects together generally give rise to an inverted-U relationship between patent protection and income inequality in the short run. However, it is also possible to have only a positive relationship between patent protection and income inequality over the permissible range of the policy instrument. In the long run, the effects of patent protection on economic growth and the real interest rate become negative due to endogenous market structure, and hence, the relationship between patent protection and income inequality also becomes negative. Finally, we calibrate the model to US data to perform a quantitative analysis and find that the long-run negative effect of patent protection on income inequality is much larger than its short-run positive effect. This dynamic pattern of income inequality is consistent with the impulse response function estimated from a panel VAR.

This study relates to the literature on innovation and economic growth. Romer (1990) develops the seminal R&D-based growth model in which economic growth is driven by the invention of new products. Aghion and Howitt (1992), Grossman and Helpman (1991) and Segerstrom *et al.* (1990) consider an alternative growth engine that is the innovation of higher-quality products and develop the Schumpeterian growth model. Subsequent studies, such as Smulders and van de Klundert (1995), Peretto (1998, 1999) and Howitt (1999), develop the second-generation Schumpeterian model with both vertical and horizontal innovation.[4] This study contributes to the literature by developing a second-generation Schumpeterian model with heterogeneous households to explore the effects of patent protection.

Other studies also explore the effects of patent protection on innovation in the R&D-based growth model; see for example, Cozzi (2001), Li (2001), Goh and Olivier (2002), Furukawa (2007), Futagami and Iwaisako (2007), Horii and Iwaisako (2007), Chu (2009, 2011), Acemoglu and Akcigit (2012), Iwaisako (2013), Iwaisako and Futagami (2013), Kiedaisch (2015), Chu *et al.* (2016) and Yang (2018, 2019). These studies focus on models with a representative household; therefore, they do not consider the effects of patent protection on income inequality. This study contributes to the literature by applying an R&D-based growth model with heterogeneous households to explore the effects of patent protection on income inequality in addition to innovation and economic growth.

Some studies in the literature consider heterogeneous workers and explore the effects of innovation on the skill premium or more generally wage inequality; see for example, Acemoglu (1998, 2002), Spinesi (2011), Cozzi and Galli (2014) and Grossman and Helpman

[4]See Laincz and Peretto (2006), Ha and Howitt (2007), Madsen (2008, 2010) and Ang and Madsen (2011) for empirical evidence that supports the second-generation Schumpeterian model.

(2018). This study complements them by considering wealth heterogeneity rather than worker heterogeneity and by exploring income inequality rather than wage inequality. Some studies in the literature consider income inequality and/or wealth inequality in the R&D-based growth model; see for example, Chou and Talmain (1996), Zweimuller (2000), Foellmi and Zweimuller (2006), Jones and Kim (2018) and Aghion *et al.* (2019). These studies focus on the relationship between income inequality and innovation. Our study relates to these interesting studies by exploring how patent policy influences the relationship between innovation and inequality. Chu (2010), Chu and Cozzi (2018) and Kiedaisch (2018) also explore the effects of patent policy on innovation and inequality; however, their model features only one type of innovation and does not feature endogenous market structure. This study contributes to the literature by showing that endogenizing the market structure has novel implications on the dynamic effects of patent protection on income inequality.

The rest of this study is organized as follows. Section 2 presents some stylized facts. Section 3 presents the model. Section 4 analyzes the dynamics of the model. Section 5 explores the effects of patent policy. Section 6 concludes.

2 Stylized facts

This paper examines whether heterogeneity in the strength of patent systems affects income inequality across countries. The Ginarte-Park index of patent rights is a standard measure of patent strength across countries; see Ginarte and Park (1997). However, the index is not available at an annual frequency (available at a quinquennial frequency only), which prevents us from using the index in our panel VAR analysis. Instead, we measure patent protection by using total patent counts, which is an annual time series being useful for a shock analysis. We have plotted the correlation between patent count and the Ginarte-Park index in Figure 1, which is clearly positive on average, indicating that countries with stronger patent rights tend to have higher patent counts.

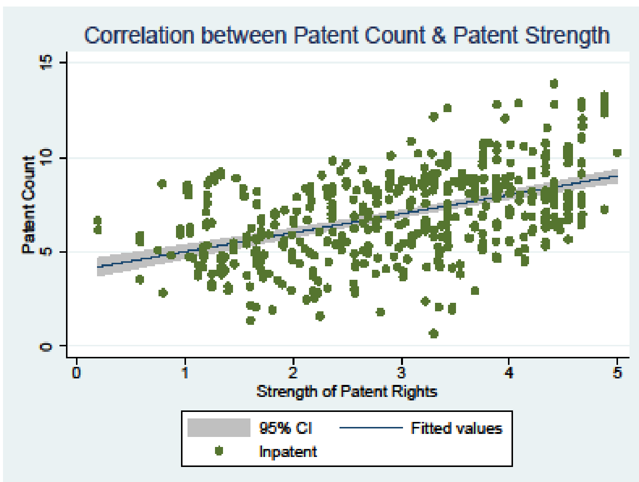

Figure 1

We compile country-level data on income inequality and patent counts. The data series are in annual frequency, giving us an unbalanced panel of 89 countries from 1980 to 2017. The Gini index of household income inequality comes from the Standardized World Income Inequality Database, whereas the number of patents is taken from the World Development Indicators of the World Bank.

We carry out a shock analysis in a panel VAR to examine the dynamic relationship between income inequality and patents. We estimate a recursive panel VAR with a maximum of 3 lags to capture the dynamics in the data. We identify a patent shock by applying the usual Choleski decomposition of variance-covariance matrix of residuals. A panel VAR extends the traditional VAR to panel data and allows for unobserved individual heterogeneity denoted as Λ_n for country n. A first-order VAR model can be specified as follows:

$$Az_{n,t} = \Lambda_n + \Lambda(L)z_{n,t-1} + \varepsilon_{n,t},$$

where $z_{n,t}$ is a $k \times 1$ vector of endogenous variables. As this equation cannot be estimated directly due to contemporaneous correlations between $z_{n,t}$ and $\varepsilon_{n,t}$, the standard reduced form can be derived by pre-multiplying the system by A^{-1} as follows:

$$z_{n,t} = \Gamma_n + \Gamma(L)z_{n,t-1} + e_{n,t},$$

where $\Gamma_n = A^{-1}\Lambda_n$, $\Gamma(L) = A^{-1}\Lambda(L)$ and $e_{n,t} = A^{-1}\varepsilon_{n,t}$. The impulse response functions can now be derived on the basis of the moving average representation of the system as follows:

$$z_{n,t} = \mu_n + \sum_i \Gamma^i(L)e_{n,t-i} = \mu_n + \sum_i \phi_i(L)\varepsilon_{n,t-i},$$

where ϕ_i are the impulse response functions.

We estimate the panel VAR in a generalized method of moments (GMM) framework that can better deal with unobserved country heterogeneity, especially in fixed t and large n settings, providing consistent estimate of the mean effects across countries. We specify the following ordering for $z_{n,t}$ as a 2×1 vector of variables [patents, income inequality] in order to identify the patent shock. The reason behind this specific recursive ordering stems from the theoretical ordering of the variables that should run from the more exogenous variable to the less exogenous one. The variable, patents, is ordered first and followed by income inequality. By undertaking a panel VAR-Granger causality Wald test, we find patent count to be exogenous among the variables.

Our aim here is to track the response of income inequality due to a shock in patents, using a panel VAR in a bivariate setting as a benchmark: the log of patent count and income inequality. As efficiency can be improved by including a longer set of lags in GMM estimation, we estimate the VAR using 3 lags and plot the estimated response coefficients up to a forecast horizon of 10 years. The panel VAR approach helps us assess the common response for the countries to a patent shock.

Figure 2 shows the bootstrapped impulse responses to a patent shock, together with plus/minus one standard-error confidence bands, obtained by bootstrapping (1000 draws). For a one standard deviation positive shock in patents, income inequality initially increases and then the median response converges to a negative level in the long run. The shaded curves represent the confidence interval around the estimated response functions, computed from a typical Monte Carlo integration exercise with 1000 draws, for statistical significance. Following Uhlig (2005) and Alessandri and Mumtaz (2019), we construct 68% confidence bands around the median estimate. The eigenvalue stability condition graph in Figure 3 suggests that as all the eigenvalues lie inside the unit circle, the panel VAR satisfies the stability condition. The short-run positive response and the long-run negative response of income inequality to a patent shock also remain robust even if we extend the panel VAR to a multivariate setting and consider an alternative measure of income inequality.[5]

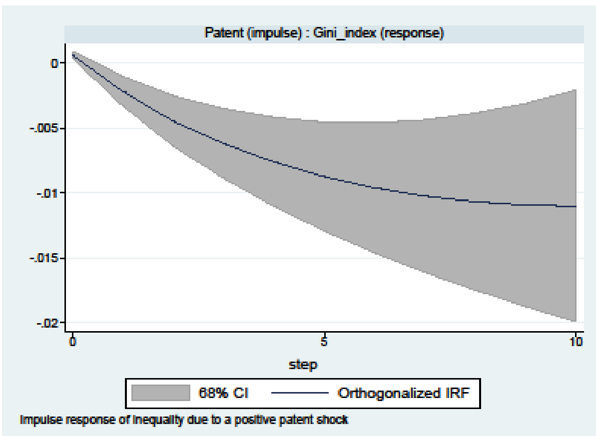

Figure 2

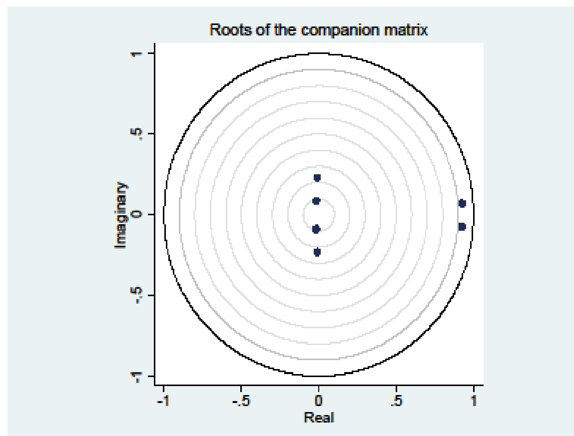

Figure 3

3 A Schumpeterian growth model with heterogeneous households and endogenous market structure

The Schumpeterian model with in-house R&D and endogenous market structure is based on Peretto (2007, 2011). Chu *et al.* (2016) introduce patent protection into the Peretto model to explore its effects on innovation and economic growth. We further introduce heterogeneous households into the Peretto model to analyze the effects of patent protection and endogenous market structure on economic growth and income inequality. Our analysis provides a complete closed-form solution for economic growth and income inequality on the transition path and the balanced growth path.

3.1 Heterogeneous households

The economy features a unit continuum of households, which are indexed by $h \in [0, 1]$. The households have identical homothetic preferences over consumption but own different levels

[5] See the robustness checks in Appendix C.

of wealth. The utility function of household h is given by

$$U(h) = \int_0^\infty e^{-\rho t} \ln c_t(h) dt, \tag{1}$$

where the parameter $\rho > 0$ determines the rate of subjective discounting and $c_t(h)$ is household h's consumption of final good (numeraire). Household h maximizes (1) subject to

$$\dot{a}_t(h) = r_t a_t(h) + w_t L - c_t(h). \tag{2}$$

$a_t(h)$ is the real value of assets owned by household h, and r_t is the real interest rate. Household h supplies L units of labor to earn a real wage rate w_t.[6] From standard dynamic optimization, the familiar Euler equation is

$$\frac{\dot{c}_t(h)}{c_t(h)} = r_t - \rho, \tag{3}$$

which shows that the growth rate of consumption is the same across households such that $\dot{c}_t(h)/c_t(h) = \dot{c}_t/c_t = r_t - \rho$, where $c_t \equiv \int_0^1 c_t(h)dh$ is aggregate consumption.

3.2 Final good

Competitive firms produce final good Y_t using the following production function:

$$Y_t = \int_0^{N_t} X_t^\theta(i)[Z_t^\alpha(i) Z_t^{1-\alpha} L_t/N_t]^{1-\theta} di, \tag{4}$$

where $\{\theta, \alpha\} \in (0, 1)$. $X_t(i)$ denotes the quantity of non-durable intermediate good $i \in [0, N_t]$, and N_t is the mass of available intermediate goods at time t. The productivity of intermediate good $X_t(i)$ depends on its own quality $Z_t(i)$ and also on the average quality $Z_t \equiv \frac{1}{N_t}\int_0^{N_t} Z_t(i)di$ of all intermediate goods capturing technology spillovers. The private return to quality is determined by α, and the degree of technology spillovers is determined by $1-\alpha$. The term L_t/N_t captures a congestion effect of variety and removes the scale effect in the model.[7]

Profit maximization yields the following conditional demand functions for L_t and $X_t(i)$:

$$L_t = (1-\theta)Y_t/w_t, \tag{5}$$

$$X_t(i) = \left(\frac{\theta}{p_t(i)}\right)^{1/(1-\theta)} Z_t^\alpha(i) Z_t^{1-\alpha} L_t/N_t, \tag{6}$$

where $p_t(i)$ is the price of $X_t(i)$. Competitive producers of final good pay $\theta Y_t = \int_0^{N_t} p_t(i)X_t(i)di$ for intermediate goods. The market-clearing condition for labor implies $L_t = L$ for all t.

[6] Our results are robust to allowing for population growth. Derivations are available upon request.

[7] Our results are robust to parameterizing this congestion effect as $L_t/N_t^{1-\xi}$, where $\xi \in (0, 1)$. See the discussion in footnote 12.

3.3 Intermediate goods and in-house R&D

The monopolistic firm in industry i produces the differentiated intermediate good with a linear technology that requires $X_t(i)$ units of final good to produce $X_t(i)$ units of intermediate good $i \in [0, N_t]$. Furthermore, the firm in industry i incurs $\phi Z_t^{\alpha}(i) Z_t^{1-\alpha}$ units of final good as a fixed operating cost. To improve the quality of its product, the firm also devotes $R_t(i)$ units of final good to R&D. The innovation specification is given by

$$\dot{Z}_t(i) = R_t(i). \tag{7}$$

In industry i, the monopolistic firm's (before-R&D) profit flow at time t is

$$\Pi_t(i) = [p_t(i) - 1] X_t(i) - \phi Z_t^{\alpha}(i) Z_t^{1-\alpha}. \tag{8}$$

The value of the monopolistic firm in industry i is

$$V_t(i) = \int_t^{\infty} \exp\left(-\int_t^s r_u du\right) [\Pi_s(i) - R_s(i)]\, ds. \tag{9}$$

The monopolistic firm in industry i maximizes (9) subject to (6), (7) and (8). The current-value Hamiltonian for this optimization problem is

$$H_t(i) = \Pi_t(i) - R_t(i) + \eta_t(i) \dot{Z}_t(i), \tag{10}$$

where $\eta_t(i)$ is the co-state variable on (7).

We solve this optimization problem in the Appendix and derive the unconstrained profit-maximizing markup ratio given by $1/\theta$. To analyze the effects of patent breadth, we introduce a policy parameter $\mu > 1$, which determines the unit cost for imitative firms to produce $X_t(i)$ with the same quality $Z_t(i)$ as the monopolistic firm in industry i.[8] A larger patent breadth μ increases the production cost of imitative firms and allows the monopolistic producer of $X_t(i)$, who owns the patent, to charge a higher markup without losing her market share to potential imitators.[9] Therefore, the equilibrium price becomes

$$p_t(i) = \min\{\mu, 1/\theta\}. \tag{11}$$

We assume $\mu < 1/\theta$. In this case, a larger patent breadth μ leads to a higher markup, and this implication is consistent with Gilbert and Shapiro's (1990) seminal insight on "breadth as the ability of the patentee to raise price".

We follow previous studies to consider a symmetric equilibrium in which $Z_t(i) = Z_t$ for $i \in [0, N_t]$. In this case, the size of intermediate-good firms is also identical across all industries, such that $X_t(i) = X_t$.[10] From (6) and $p_t(i) = \mu$, the quality-adjusted firm size is

$$\frac{X_t}{Z_t} = \left(\frac{\theta}{\mu}\right)^{1/(1-\theta)} \frac{L}{N_t}. \tag{12}$$

[8] Here we assume a diffusion of knowledge from the monopolistic firm to imitators.

[9] Intuitively, the presence of monopolistic profits attracts potential imitators. However, stronger patent protection increases the production cost of imitative products and allows monopolistic firms to charge a higher markup without losing market share to these potential imitators; see also Li (2001), Goh and Olivier (2002), Chu (2011) and Iwaisako and Futagami (2013) for a similar formulation.

[10] Symmetry also implies $\Pi_t(i) = \Pi_t$, $R_t(i) = R_t$ and $V_t(i) = V_t$.

We define the following transformed variable:

$$x_t \equiv \mu^{1/(1-\theta)} \frac{X_t}{Z_t} = \theta^{1/(1-\theta)} \frac{L}{N_t}. \tag{13}$$

x_t is a state variable that is determined by the quality-adjusted firm size, which in turn depends on L/N_t. Lemma 1 derives the rate of return on quality-improving R&D, which is increasing in x_t and μ.

Lemma 1 *The rate of return to in-house R&D is given by*

$$r_t^q = \alpha \frac{\Pi_t}{Z_t} = \alpha \left[\frac{\mu - 1}{\mu^{1/(1-\theta)}} x_t - \phi \right]. \tag{14}$$

Proof. See the Appendix. ■

3.4 Entrants

Following previous studies, we assume that entrants have access to aggregate technology Z_t to ensure symmetric equilibrium at any time t. A new firm pays βX_t units of final good to set up its operation and enter the market with a new variety of products. $\beta > 0$ is a cost parameter, and the cost function βX_t captures the case in which the setup cost is increasing in the initial output volume of the firm. The asset-pricing equation determines the rate of return on assets as

$$r_t = \frac{\Pi_t - R_t}{V_t} + \frac{\dot{V}_t}{V_t}. \tag{15}$$

The free-entry condition is given by[11]

$$V_t = \beta X_t. \tag{16}$$

Substituting (7), (8), (13), (16) and $p_t(i) = \mu$ into (15) yields the return on entry as

$$r_t^e = \frac{\mu^{1/(1-\theta)}}{\beta} \left[\frac{\mu - 1}{\mu^{1/(1-\theta)}} - \frac{\phi + z_t}{x_t} \right] + \frac{\dot{x}_t}{x_t} + z_t, \tag{17}$$

where $z_t \equiv \dot{Z}_t / Z_t$ is the growth rate of aggregate quality.

[11] We treat entry and exit symmetrically (i.e., the scrap value of exiting an industry is also βX_t); therefore, $V_t(i) = \beta X_t$ always holds. If $V_t > \beta X_t$ ($V_t < \beta X_t$), then there would be an infinite number of entries (exits).

3.5 General equilibrium

The equilibrium is a time path of allocations $\{a_t, c_t, Y_t, X_t(i), R_t(i)\}$ and prices $\{r_t, w_t, p_t(i), V_t(i)\}$ such that the following conditions are satisfied:

- households maximize utility taking $\{r_t, w_t\}$ as given;
- competitive firms produce Y_t and maximize profits taking $\{p_t(i), w_t\}$ as given;
- monopolistic firms produce $X_t(i)$ and choose $\{p_t(i), R_t(i)\}$ to maximize $V_t(i)$ taking r_t as given;
- entrants make entry decisions taking V_t as given;
- the value of all existing monopolistic firms adds up to the value of the households' assets such that $N_t V_t = \int_0^1 a_t(h)dh \equiv a_t$;
- the market-clearing condition of labor holds such that $L_t = L$; and
- the following market-clearing condition of final good holds:

$$Y_t = c_t + N_t(X_t + \phi Z_t + R_t) + \dot{N}_t \beta X_t. \tag{18}$$

3.6 Aggregation

Substituting (6) into (4) and imposing symmetry yield the following aggregate production function:

$$Y_t = (\theta/\mu)^{\theta/(1-\theta)} Z_t L, \tag{19}$$

which also uses markup pricing $p_t(i) = \mu$. Therefore, the growth rate of output is

$$\frac{\dot{Y}_t}{Y_t} = z_t, \tag{20}$$

which is determined by the quality growth rate z_t.[12]

4 Dynamics

In this section, we analyze the dynamics of the model. Section 4.1 presents the dynamics of the aggregate economy. Section 4.2 summarizes the dynamics of the wealth distribution, whereas Section 4.3 summarizes the dynamics of the income distribution.

[12] Parameterizing the congestion effect as $L/N_t^{1-\xi}$ in (4) would yield $Y_t = (\theta/\mu)^{\theta/(1-\theta)} Z_t N_t^{\xi} L$ in which case the growth rate of output is given by $\dot{Y}_t/Y_t = z_t + \xi \dot{N}_t/N_t$, which is nonetheless determined by the rate of return r_t^q in (14) on quality-improving R&D as (22) and (23) show.

4.1 Dynamics of the aggregate economy

We now analyze the dynamics of the economy. In the Appendix, we show that the consumption-output ratio c_t/Y_t jumps to a unique and stable steady-state value. This equilibrium property simplifies the analysis of transition dynamics and ensures the stationarity of the wealth distribution even on the transition path.

Lemma 2 *The consumption-output ratio jumps to a unique and stable steady-state value:*

$$\frac{c_t}{Y_t} = \frac{\beta\theta\rho}{\mu} + 1 - \theta. \tag{21}$$

Proof. See the Appendix. ■

Equation (21) implies that for any given μ, consumption and output grow at the same rate given by

$$g_t \equiv \frac{\dot{Y}_t}{Y_t} = \frac{\dot{c}_t}{c_t} = r_t - \rho, \tag{22}$$

where the last equality uses the Euler equation in (3). Substituting (14) into (22) yields the growth rate of output given by

$$g_t = \alpha \left[\frac{\mu - 1}{\mu^{1/(1-\theta)}} x_t - \phi \right] - \rho, \tag{23}$$

which depends on the state variable x_t. Then, (20) implies that the quality growth rate is also given by

$$z_t = \alpha \left[\frac{\mu - 1}{\mu^{1/(1-\theta)}} x_t - \phi \right] - \rho, \tag{24}$$

which is positive if and only if

$$x_t > \overline{x} \equiv \frac{\mu^{1/(1-\theta)}}{\mu - 1} \left(\frac{\rho}{\alpha} + \phi \right). \tag{25}$$

Intuitively, innovation requires each firm's market size to be large enough so that it is profitable for firms to do in-house R&D. For the rest of the analysis, we assume that $x_t > \overline{x}$. In this case, the dynamics of x_t is derived in Lemma 3.

Lemma 3 *The dynamics of* x_t *is determined by an one-dimensional differential equation:*

$$\dot{x}_t = \mu^{1/(1-\theta)} \left[\frac{(1-\alpha)\phi - \rho}{\beta} \right] - \frac{(1-\alpha)(\mu - 1) - \beta\rho}{\beta} x_t. \tag{26}$$

Proof. See the Appendix. ■

Proposition 1 *Under the parameter restrictions $\rho < \min\{(1-\alpha)\phi, (1-\alpha)(\mu-1)/\beta\}$, the dynamics of x_t is globally stable and x_t gradually converges to a unique steady-state value. The steady-state values $\{x^*, g^*\}$ are given by*

$$x^*(\underset{-}{\mu}) = \mu^{1/(1-\theta)} \frac{(1-\alpha)\phi - \rho}{(1-\alpha)(\mu-1) - \beta\rho} > \overline{x}, \tag{27}$$

$$g^*(\underset{-}{\mu}) = \alpha \left[(\mu-1) \frac{(1-\alpha)\phi - \rho}{(1-\alpha)(\mu-1) - \beta\rho} - \phi \right] - \rho > 0. \tag{28}$$

Proof. See the Appendix. ■

The differential equation in (26) shows that given an initial value x_0, the state variable x_t gradually converges to its steady-state value denoted as x^*, which also determines $N^* = \theta^{1/(1-\theta)} L/x^*$. On the transition path, the market size of each product determines the rate of quality-improving innovation and the equilibrium growth rate g_t according to (23). When x_t evolves toward the steady state, g_t also gradually converges to its steady-state value g^*. The steady-state values of $\{x^*, g^*\}$ are derived in Proposition 1.

4.2 Dynamics of the wealth distribution

In this section, we show that for any given x_t at any time t, the wealth distribution is stationary and determined by its initial distribution that is exogenously given at time 0. It is useful to recall that the aggregate economy features transition dynamics determined by the evolution of x_t. However, the wealth distribution is stationary despite the transition dynamics in the aggregate economy because the consumption-output ratio c_t/Y_t is stationary, which in turn implies that the consumption-wealth ratio c_t/a_t is also stationary as shown in the proof of Lemma 2.

Aggregating (2) across all households yields the following aggregate asset-accumulation equation:

$$\dot{a}_t = r_t a_t + w_t L - c_t. \tag{29}$$

Let $s_{a,t}(h) \equiv a_t(h)/a_t$ denote the share of wealth owned by household h. Then, the growth rate of $s_{a,t}(h)$ is given by

$$\frac{\dot{s}_{a,t}(h)}{s_{a,t}(h)} = \frac{\dot{a}_t(h)}{a_t(h)} - \frac{\dot{a}_t}{a_t} = \frac{c_t - w_t L}{a_t} - \frac{s_{c,t}(h) c_t - w_t L}{a_t(h)}, \tag{30}$$

where $w_t L = (1-\theta) Y_t$ and $s_{c,t}(h) \equiv c_t(h)/c_t$. Given that $\dot{c}_t(h)/c_t(h) = \dot{c}_t/c_t = r_t - \rho$, the consumption share $s_{c,t}(h)$ of any household $h \in [0,1]$ is stationary such that $s_{c,t}(h) = s_{c,0}(h)$, which is endogenous. Proposition 2 derives the dynamics of $s_{a,t}(h)$ and shows that the wealth distribution of households is also stationary (i.e., $s_{a,t}(h) = s_{a,0}(h)$, which is exogenously given at time 0). This stationarity is due to the stationary consumption-output c_t/Y_t and consumption-wealth c_t/a_t ratios along the transition path of the aggregate economy.

Proposition 2 *The dynamics of $s_{a,t}(h)$ is given by an one-dimensional differential equation:*

$$\dot{s}_{a,t}(h) = \rho[s_{a,t}(h) - s_{a,0}(h)]. \tag{31}$$

Also, the wealth distribution is stationary and remains the same as the initial distribution.

Proof. See the Appendix. ■

4.3 Dynamics of the income distribution

In this section, we show that the income distribution is endogenous and nonstationary but still analytically tractable. Although the wealth distribution is stationary, the transition dynamics in the aggregate economy (in particular, the transition dynamics of the real interest rate) gives rise to an endogenous evolution of the income distribution. Therefore, once we trace out the transition dynamics of the real interest rate, we can also trace out the transition dynamics of income inequality.

Income received by household h is given by

$$I_t(h) = r_t a_t(h) + w_t L. \tag{32}$$

Aggregating (32) yields the aggregate level of income as

$$I_t = r_t a_t + w_t L. \tag{33}$$

Let $s_{I,t}(h) \equiv I_t(h)/I_t$ denote the share of income received by household h. Then, we have

$$s_{I,t}(h) = \frac{r_t a_t(h) + w_t L}{r_t a_t + w_t L} = \frac{r_t a_t}{r_t a_t + w_t L} s_{a,0}(h) + \frac{w_t L}{r_t a_t + w_t L}. \tag{34}$$

The coefficient of variation of income is defined as[13]

$$\sigma_{I,t} \equiv \sqrt{\int_0^1 [s_{I,t}(h) - 1]^2 dh} = \frac{r_t a_t}{r_t a_t + w_t L} \sigma_a, \tag{35}$$

where $\sigma_a \equiv \sqrt{\int_0^1 [s_{a,0}(h) - 1]^2 dh}$ is the coefficient of variation of wealth that is exogenously given at time 0. Equation (35) shows that income inequality $\sigma_{I,t}$ is increasing in the asset-wage income ratio $r_t a_t/(w_t L)$ given that wealth inequality drives income inequality in our model.[14] Proposition 3 derives the equilibrium expression for $\sigma_{I,t}$ at any time t. Let's define a composite parameter $\Theta \equiv (1-\theta)/(\theta\beta)$.

Proposition 3 *The degree of income inequality at any time t is given by*

$$\sigma_{I,t} = \frac{1}{1 + \mu\Theta/r_t} \sigma_a = \frac{1}{1 + \mu\Theta/(\rho + g_t)} \sigma_a. \tag{36}$$

Proof. See the Appendix. ■

[13] In Appendix B, we show that the Gini coefficient of income is also given by $\sigma_{I,t} = \frac{r_t a_t}{r_t a_t + w_t L} \sigma_a$, where σ_a is the Gini coefficient of wealth.

[14] See Madsen (2017) for evidence that asset returns are an important determinant of income inequality.

5 Effects of patent breadth on growth and inequality

This section analyzes the effects of patent breadth μ on economic growth g_t and income inequality $\sigma_{I,t}$. Equation (23) shows that the initial impact of a larger μ on the growth rate g_t is positive because x_t is fixed in the short run. This is the standard positive *profit-margin* effect, captured by $(\mu - 1)/\mu^{1/(1-\theta)}$ in (23), of patent breadth on monopolistic profits and innovation as in previous studies, such as Li (2001) and Chu (2011), which feature an exogenous market structure. However, in our model, the market structure is endogenous and the number of firms gradually adjusts. The higher profit margin attracting entry of new products reduces the market size x_t of each product and the rate of return r_t^q on quality-improving innovation as (14) shows. In the long run, this negative *entry* effect dominates the positive profit-margin effect such that the new steady-state growth rate g^* in (28) is lower than the initial steady-state growth rate; see Figure 4 for an illustration in which patent breadth increases at time t. In summary, endogenous market structure gives rise to opposite short-run and long-run effects of patent protection on growth as in Chu *et al.* (2016).

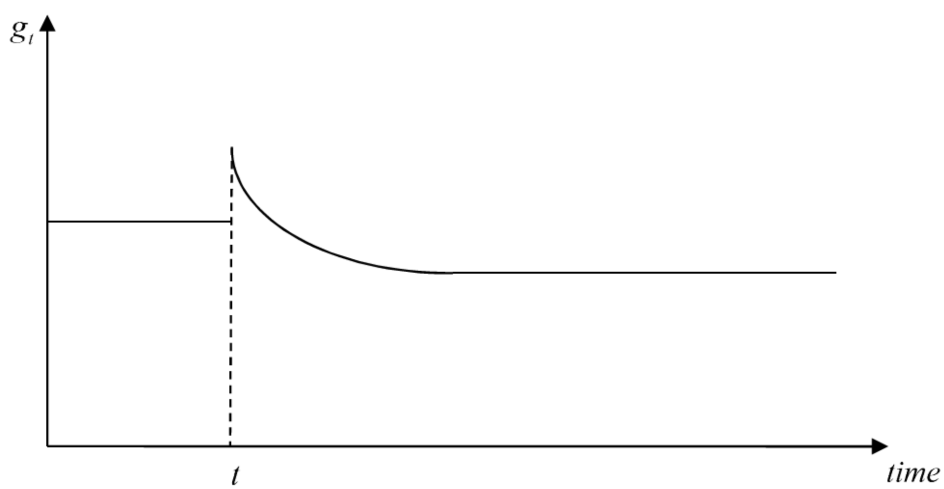

Figure 4: Transitional effects of patent breadth on economic growth

The above contrasting effects of patent protection on economic growth at different time horizons have novel implications on income inequality, which is determined by the rate of return on assets and the value of assets as (35) shows. The initial impact of a larger patent breadth μ has both a positive effect and a negative effect on income inequality $\sigma_{I,t}$. The positive effect arises because a larger patent breadth initially increases the growth rate g_t and the interest rate r_t as in Chu (2010) and Chu and Cozzi (2018), who focus on quality improvement without endogenous entry. In our model, endogenous entry gives rise to a negative effect on income inequality because a larger patent breadth reduces the demand for intermediate goods X_t, which in turn reduces asset value via the entry condition in (16). These positive and negative effects together generally give rise to an inverted-U relationship between patent protection and income inequality in the short run. However, it is also possible to yield only a positive relationship between patent protection and income inequality over the permissible range of patent breadth μ. In the long run, the effect of a larger patent breadth on the growth rate g_t and the interest rate r_t becomes negative due to endogenous market structure. Therefore, increasing patent breadth causes a negative effect on income inequality in the long run; see Figure 5 for an illustration in which case 1 (case 2) refers to a small (large) increase in patent breadth at time t. Proposition 4 summarizes these results.

Proposition 4 *Strengthening patent protection has the following effects on economic growth and income inequality at different time horizons: (a) it causes a positive effect on economic growth and a positive or an inverted-U effect on income inequality in the short run; and (b) it causes a negative effect on both economic growth and income inequality in the long run.*

Proof. See the Appendix. ■

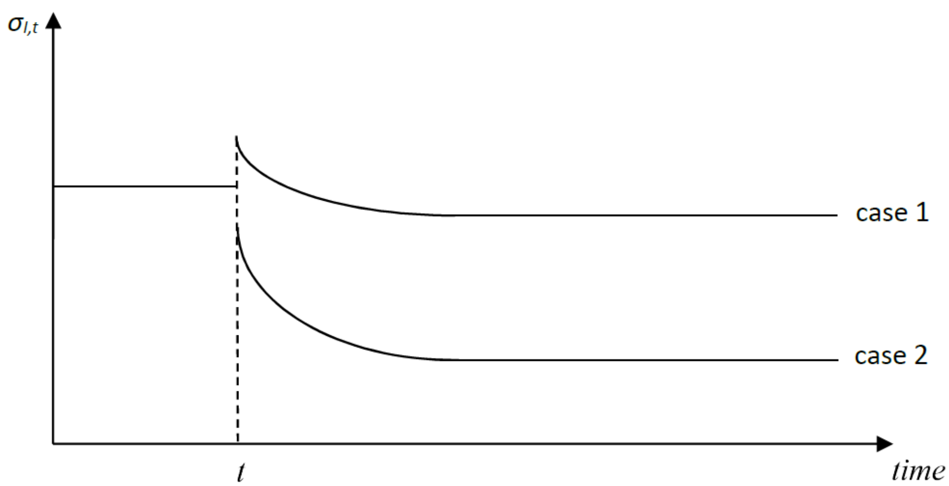

Figure 5: Transitional effects of patent breadth on income inequality

5.1 Quantitative analysis

In this section, we calibrate the model to aggregate US data in order to perform a quantitative analysis. The model features the following parameters: $\{\alpha, \rho, \theta, \beta, \phi, \mu\}$. We follow Iacopetta *et al.* (2019) to set the degree of technology spillovers $1 - \alpha$ to 0.833. We set the discount rate ρ to 0.03 and the markup μ to 1.40, which is at the upper bound of the range of values reported in Jones and Williams (2000). Then, we calibrate $\{\theta, \beta, \phi\}$ by matching the following moments in the US economy. First, labor income as a share of output is 60%. Second, the consumption share of output is 64%. Third, the growth rate of output per capita is 2%. Table 1 summarizes the calibrated parameter values.

Table 1: Calibrated parameter values

α	ρ	θ	β	ϕ	μ
0.167	0.040	0.400	4.667	0.499	1.400

We simulate the effects of patent breadth μ on the quality-adjusted firm size x_t, the growth rate g_t and income inequality $\sigma_{I,t}$. The baseline value of markup μ is 1.40, and we raise μ by 0.01 to 1.41. Figure 6 presents the transitional path of the quality-adjusted firm size x_t. Figure 7 presents the transitional path of the growth rate g_t. Figure 8 presents the transitional path of income inequality $\sigma_{I,t}$ in terms of percent changes from its initial value. When patent protection strengthens, the growth rate increases from 2.00% to 2.17%, which in turn raises income inequality by 2.43% on impact. Gradually, more products enter the market, resulting into a gradual decrease in the quality-adjusted firm size x_t from 3.50 to 3.39. This smaller firm size leads to a decrease in the steady-state growth rate to 1.77%, which in turn decreases income inequality by 4.80% in the long run. Therefore, the negative effect of patent breadth on income inequality in the long run is much larger in magnitude than its positive effect in the short run. This result is consistent with the stylized facts documented in Section 2.

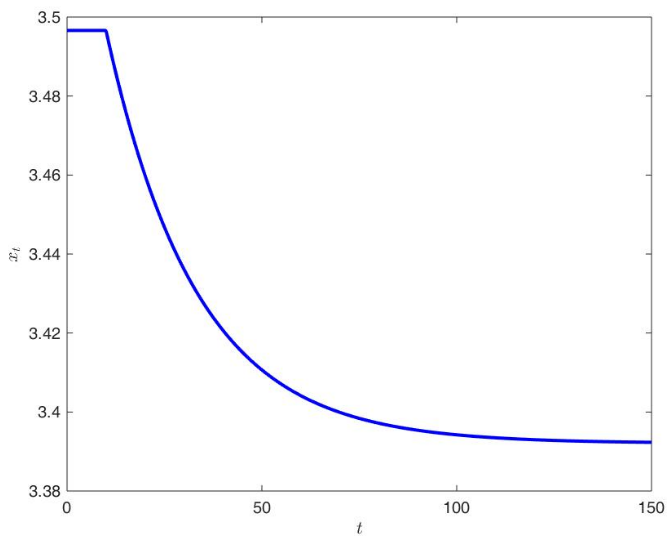

Figure 6: Transitional path of the firm size

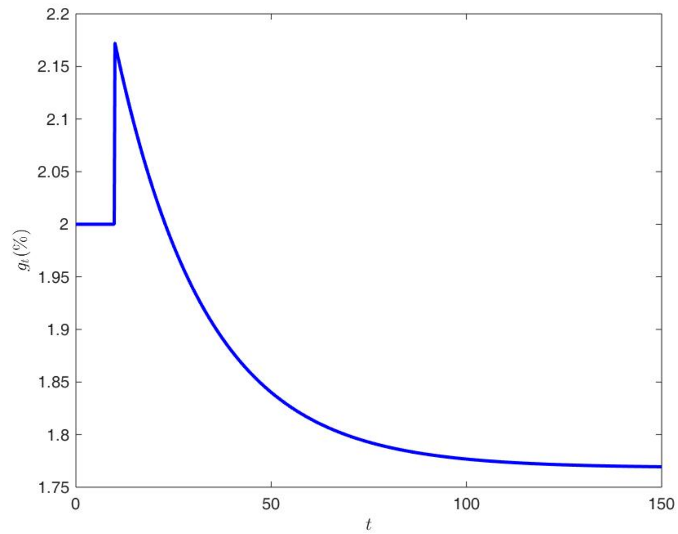

Figure 7: Transitional path of the growth rate

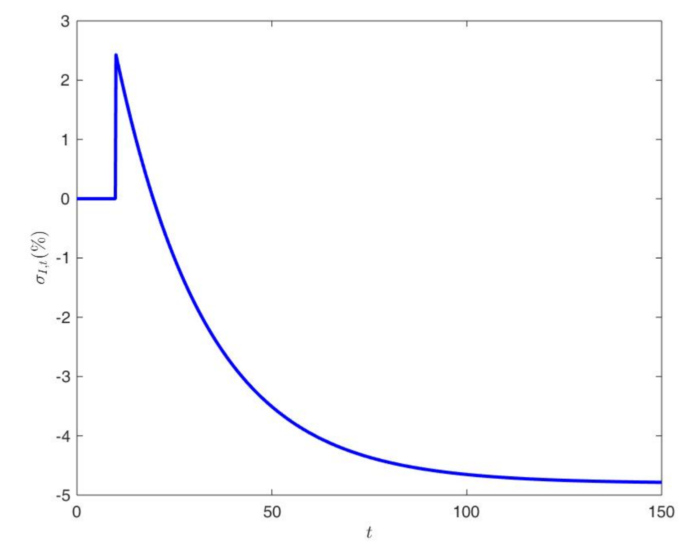

Figure 8: Transitional path of income inequality

In this numerical exercise, we consider a conservatively low discount rate ρ and a relatively large markup μ. Considering a larger ρ or a smaller μ would lead to an even more significant decrease in economic growth g and income inequality σ_I in the long run. In the following tables that report results for $\rho \in \{0.03, 0.04, 0.05\}$ and $\mu \in \{1.20, 1.30, 1.40\}$,[15] we present the equilibrium growth rates and the percent changes in income inequality on impact when

[15] Here we recalibrate the other parameters $\{\theta, \beta, \phi\}$ to match the same moments as before.

μ increases by 0.01 and also when the economy reaches the new balanced growth path. The tables show that strengthening patent protection can lead to a decrease in the steady-state growth rate to as low as 0.79% and a decrease in income inequality by as much as 16.74% in the long run. Therefore, we present the relatively conservative results under $\rho = 0.03$ and $\mu = 1.40$ as our benchmark.

Table 2: Effects of patent protection on economic growth

Short-run effects				Long-run effects			
$\rho =$	0.03	0.04	0.05		0.03	0.04	0.05
$\mu = 1.20$	2.28%	2.34%	2.40%	1.20	1.15%	0.97%	0.79%
1.30	2.22%	2.26%	2.31%	1.30	1.64%	1.56%	1.48%
1.40	2.17%	2.21%	2.25%	1.40	1.77%	1.72%	1.67%

Table 3: Effects of patent protection on income inequality

Short-run effects				Long-run effects			
$\rho =$	0.03	0.04	0.05		0.03	0.04	0.05
$\mu = 1.20$	4.18%	4.28%	4.35%	1.20	−16.19%	−16.52%	−16.74%
1.30	3.19%	3.27%	3.32%	1.30	−7.24%	−7.39%	−7.49%
1.40	2.43%	2.49%	2.54%	1.40	−4.80%	−4.90%	−4.96%

6 Conclusion

This study introduces heterogeneous households into a Schumpeterian growth model with endogenous market structure. Although endogenous market structure causes the aggregate economy to feature transition dynamics, the wealth distribution of households is stationary, which in turn allows us to derive the dynamics of the income distribution. In summary, we find that strengthening patent protection increases economic growth and causes a positive or an inverted-U effect on income inequality in the short run when the number of differentiated products is fixed. However, when the number of products adjusts endogenously, the effects of patent protection on economic growth and income inequality eventually become negative. This finding highlights the importance of endogenous market structure, which gives rise to different effects of patent policy on innovation and inequality at different time horizons. Therefore, previous studies that neglect the endogenous adjustment of the market structure may have identified only the short-run effects of patent policy on innovation and inequality. Finally, to maintain the tractability of the dynamics of income inequality, we have focused on the effects of the aggregate economy on the evolution of the income distribution, without allowing for a potential feedback effect from the income distribution to the aggregate economy. We leave this interesting extension to future research.

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Appendix A: Proofs

Proof of Lemma 1. The current-value Hamiltonian for monopolistic firm i is given by (10). To introduce the upper bound μ on price $p_t(i)$, we modify (10) as follows:

$$H_t(i) = \Pi_t(i) - R_t(i) + \eta_t(i)\dot{Z}_t(i) + \omega_t(i)[\mu - p_t(i)], \tag{10'}$$

where $\omega_t(i)$ is the multiplier on $p_t(i) \le \mu$. Substituting (6)-(8) into (10'), we can derive

$$\frac{\partial H_t(i)}{\partial p_t(i)} = 0 \Rightarrow \frac{\partial \Pi_t(i)}{\partial p_t(i)} = \omega_t(i), \tag{A1}$$

$$\frac{\partial H_t(i)}{\partial R_t(i)} = 0 \Rightarrow \eta_t(i) = 1, \tag{A2}$$

$$\frac{\partial H_t(i)}{\partial Z_t(i)} = \alpha\left\{[p_t(i) - 1]\left[\frac{\theta}{p_t(i)}\right]^{1/(1-\theta)}\frac{L_t}{N_t} - \phi\right\} Z_t^{\alpha-1}(i)\, Z_t^{1-\alpha} = r_t\eta_t(i) - \dot{\eta}_t(i). \tag{A3}$$

If $p_t(i) < \mu$, then $\omega_t(i) = 0$. In this case, $\partial\Pi_t(i)/\partial p_t(i) = 0$ yields $p_t(i) = 1/\theta$. If the constraint on $p_t(i)$ is binding, then $\omega_t(i) > 0$. In this case, we have $p_t(i) = \mu$, proving (11). Given that we assume $\mu < 1/\theta$, $p_t(i) = \mu$ always holds. Substituting (A2), (13) and $p_t(i) = \mu$ into (A3) and imposing symmetry yield (14). ■

Proof of Lemma 2. Substituting (16) into the total asset value $a_t = N_t V_t$ yields

$$a_t = N_t \beta X_t = (\theta/\mu)\beta Y_t, \tag{A4}$$

where the second equality uses $\theta Y_t = N_t(\mu X_t)$.[16] Differentiating (A4) with respect to t yields

$$\frac{\dot{Y}_t}{Y_t} = \frac{\dot{a}_t}{a_t} = r_t + \frac{w_t L}{a_t} - \frac{c_t}{a_t}, \tag{A5}$$

where the second equality uses (2) with $a_t \equiv \int_0^1 a_t(h)h$ and $c_t \equiv \int_0^1 c_t(h)dh$. Using (3) for r_t, (5) for w_t, and (A4) for a_t, we can rearrange (A5) to obtain

$$\frac{\dot{c}_t}{c_t} - \frac{\dot{a}_t}{a_t} = \frac{c_t}{a_t} - \left[\rho + \frac{\mu(1-\theta)}{\beta\theta}\right], \tag{A6}$$

the right-hand side of which is increasing in c_t/a_t with a strictly negative y-intercept. Therefore, c_t/a_t must jump to the steady state. Then, we have (21), noting (A4). ■

Proof of Lemma 3. Substituting $z_t = r_t - \rho = r_t^e - \rho$ into (17) yields

$$\frac{\dot{x}_t}{x_t} = \rho - \frac{\mu^{1/(1-\theta)}}{\beta}\left[\frac{\mu - 1}{\mu^{1/(1-\theta)}} - \frac{\phi + z_t}{x_t}\right], \tag{A7}$$

where we have also used the expression of z_t in (24) to obtain (26). ■

[16] We derive this by using $p_t(i) = \mu$ and $X_t(i) = X_t$ for $\theta Y_t = \int_0^{N_t} p_t(i)X_t(i)di$.

Proof of Proposition 1. One can rewrite (26) simply as $\dot{x}_t = d_1 - d_2 x_t$. This linear system for x_t has a unique (non-zero) steady state that is globally (and locally) stable if

$$d_1 \equiv \mu^{1/(1-\theta)} \left[\frac{(1-\alpha)\,\phi - \rho}{\beta} \right] > 0, \tag{A8a}$$

$$d_2 \equiv \frac{(1-\alpha)\,(\mu - 1) - \beta\rho}{\beta} > 0, \tag{A8b}$$

from which we obtain $\rho < \min\{(1-\alpha)\phi, (1-\alpha)(\mu-1)/\beta\}$. Then, $\dot{x}_t = 0$ yields the steady-state value $x^* = d_1/d_2$, which gives (27). Substituting (27) into (23) yields (28). ■

Proof of Proposition 2. Manipulating (2) yields

$$\frac{\dot{a}_t(h)}{a_t(h)} = r_t + \frac{w_t L}{a_t(h)} - \frac{c_t(h)}{a_t(h)}. \tag{A9}$$

Then, the growth rate of $s_{a,t}(h) \equiv a_t(h)/a_t$ is

$$\frac{\dot{s}_{a,t}(h)}{s_{a,t}(h)} = \frac{\dot{a}_t(h)}{a_t(h)} - \frac{\dot{a}_t}{a_t} = \frac{w_t L - c_t(h)}{a_t(h)} - \frac{w_t L - c_t}{a_t}, \tag{A10}$$

which becomes

$$\dot{s}_{a,t}(h) = \frac{c_t - w_t L}{a_t} s_{a,t}(h) - \frac{s_{c,t}(h) c_t - w_t L}{a_t}. \tag{A11}$$

We use (5) for w_t, (21) for c_t/Y_t and (A4) for a_t/Y_t in (A11) to derive

$$\dot{s}_{a,t}(h) = \rho s_{a,t}(h) - s_{c,t}(h) \frac{\beta\theta\rho + \mu\,(1-\theta)}{\beta\theta} + \frac{\mu\,(1-\theta)}{\beta\theta}. \tag{A12}$$

To achieve stability of $s_{a,t}(h)$, $\dot{s}_{a,t}(h) = 0$ must hold for any $t \geq 0$ because $s_{a,t}(h)$ is a predetermined variable and its coefficient is positive. We can achieve this if and only if $s_{c,t}(h)$ jumps into a stationary level at $t = 0$ that ensures $s_{a,t}(h)$ to be stationary. Then, we have

$$s_{c,0}(h) = \frac{\beta\theta\rho s_{a,0}(h) + \mu\,(1-\theta)}{\beta\theta\rho + \mu\,(1-\theta)}, \tag{A13}$$

and $s_{c,t}(h) = s_{c,0}(h)$ for any $t \geq 0$. Substituting (A13) into (A12) yields (31). ■

Proof of Proposition 3. By (35), we have

$$\sigma_{I,t} = \frac{1}{1 + [w_t L/(r_t a_t)]} \sigma_a. \tag{A14}$$

Using (5) for w_t and (A4) for a_t/Y_t, we obtain

$$\frac{w_t L}{r_t a_t} = \mu \left(\frac{1-\theta}{\beta\theta} \right) \frac{1}{r_t}, \tag{A15}$$

where $r_t = \rho + g_t$. Combining (A14) and (A15) yields (36). ■

Proof of Proposition 4. With $r_t^q = r_t$, it is straightforward to show from (14) that for a given x_t, r_t is increasing in $\mu \in (1, 1/\theta)$. Thus, the short-run effect of μ on $r_t = g_t + \rho$ is positive. To see the short-run effect of μ on inequality, we use (A14) and (A15) to write

$$\sigma_{I,t} = \frac{(r_t/\mu)}{(r_t/\mu) + \Theta}\sigma_a, \tag{A16}$$

noting $r_t = g_t + \rho$. It shows that $\sigma_{I,t}$ is increasing in r_t/μ, in which[17]

$$\frac{r_t}{\mu} = \frac{\alpha}{\mu}\left[\frac{\mu - 1}{\mu^{1/(1-\theta)}}x_t - \phi\right], \tag{A17}$$

which uses (14) and $r_t^q = r_t$. For a given x_t, we can show that

$$\frac{d}{d\mu}\left(\frac{r_t}{\mu}\right) > 0 \Leftrightarrow (\mu - 1) - \frac{\phi\mu^{1/(1-\theta)}}{x_t} - \frac{1 - \mu\theta}{1 - \theta} \equiv \varkappa(x_t, \mu) < 0. \tag{A18}$$

It is useful to note that for a given x_t, $\varkappa(x_t, \mu)$ is a monotonically increasing function in both x_t and μ.[18] At both ends of the original domain of $\mu \in (1, 1/\theta)$, the signs of $\varkappa(x_t, \mu)$ are opposite such that

$$\lim_{\mu \to 1} \varkappa(x_t, \mu) = -\left(\frac{\phi}{x_t} + 1\right) < 0 \tag{A19a}$$

and

$$\lim_{\mu \to 1/\theta} \varkappa(x_t, \mu) = \left(\frac{1-\theta}{\theta}\right)\left[1 - \frac{\alpha\phi}{\alpha\phi + \rho}\frac{\overline{x}}{x_t}\right] > 0, \tag{A19b}$$

noting $\overline{x}/x_t < 1$. As shown in Figure 9, there uniquely exists a threshold value of μ, denoted as $\hat{\mu}(x_t) \in (1, 1/\theta)$, such that the effect of μ on $\sigma_{I,t}$ is positive for a sufficiently small $\mu \in (1, \hat{\mu}(x_t))$ and negative for a sufficiently large $\mu \in (\hat{\mu}(x_t), 1/\theta)$. This implies that the unconstrained short-run effect of μ on $\sigma_{I,t}$ follows an inverted-U shaped. However, to ensure $x^* > \overline{x}$, there is an upper bound of μ, that is,

$$\mu < 1 + \beta\left(\alpha\phi + \rho\right) \equiv \overline{\mu}. \tag{A20}$$

Thus, if $\overline{\mu} < \hat{\mu}(x_t)$, then only the positive part of an inverted-U effect appears in the feasible range of $\mu \in (1, \overline{\mu})$.

[17] The lower bound of the right-hand side of (A17) at $x_t = \overline{x}$, defined in (25), is strictly positive, which implies $r_t/\mu > 0$.

[18] $\varkappa(x_t, \mu)$ being increasing in x_t is obvious. As for μ, note

$$\frac{d}{d\mu}\varkappa(x_t, \mu) = \frac{1}{1-\theta}\frac{1}{x_t}\left[x_t - \overline{x}\left(\frac{\alpha\phi}{\alpha\phi + \rho}\right)\left(1 - \frac{1}{\mu}\right)\right] > 0,$$

in which the inequality always holds due to $x_t > \overline{x}$ in (25).

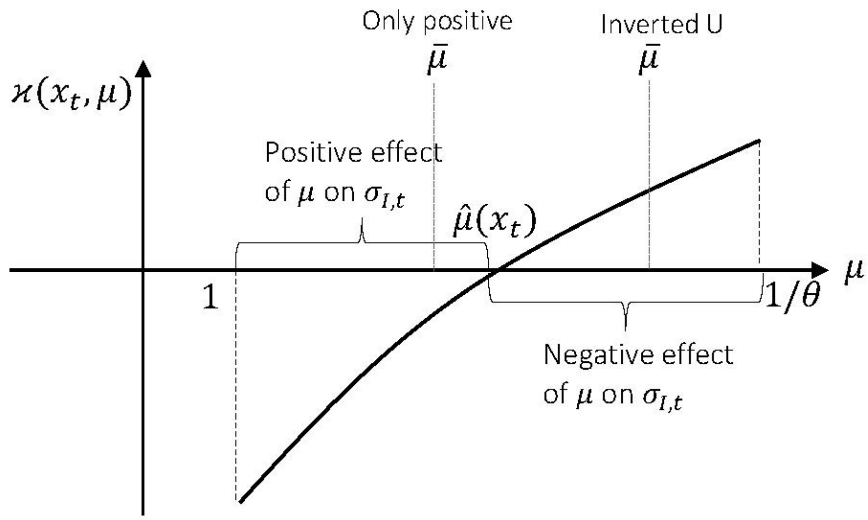

Figure 9: Proof of Proposition 4

Finally, concerning the long-run effects of μ, we differentiate (28) with respect to μ to derive

$$\frac{d}{d\mu}g^* = -\frac{\alpha\beta\rho\left[(1-\alpha)\phi - \rho\right]}{\left[(1-\alpha)(\mu-1) - \beta\rho\right]^2} < 0, \tag{A23}$$

showing the negative effect of μ on the long-run growth rate g^*. Given that $r^* = g^* + \rho$, an increase in μ leads to a decrease in the long-run interest rate r^* and also a decrease in the steady-state ratio r^*/μ. Therefore, the long-run effect of μ on income inequality $\sigma_{I,t}$ is also negative. ■

Appendix B: Gini coefficient

Income received by household h is given by

$$I(h) = ra(h) + wL = s_a(h)ra + wL, \tag{B1}$$

where the identity index h is uniformly distributed between 0 and 1. We now order the households in an ascending order of income. The Gini coefficient of income is given by $\sigma_I = 1 - 2b_I$, where

$$b_I \equiv \int_0^1 \mathcal{L}_I(h)dh. \tag{B2}$$

The Lorenz curve $\mathcal{L}_I(h)$ of income is given by

$$\mathcal{L}_I(h) \equiv \frac{\int_0^h I(\chi)d\chi}{\int_0^1 I(\chi)d\chi} = \frac{ra\int_0^h s_a(\chi)d\chi + wL\int_0^h 1d\chi}{ra + wL}, \tag{B3}$$

where $\int_0^h 1d\chi = h$ and $\int_0^h s_a(\chi)d\chi$ is the Lorenz curve $\mathcal{L}_a(h)$ of wealth. To see this,

$$\mathcal{L}_a(h) \equiv \frac{\int_0^h a(\chi)d\chi}{\int_0^1 a(\chi)d\chi} = \frac{\int_0^h a(\chi)d\chi}{a} = \int_0^h s_a(\chi)d\chi. \tag{B4}$$

Substituting (B3) and (B4) into (B2) yields

$$b_I = \frac{ra}{ra + wL}\int_0^1 \mathcal{L}_a(h)dh + \frac{wL}{ra + wL}\int_0^1 hdh, \tag{B5}$$

where $\int_0^1 hdh = 0.5$ and $\int_0^1 \mathcal{L}_a(h)dh \equiv b_a$. Recall that the Gini coefficient of wealth is given by $\sigma_a = 1 - 2b_a$. Therefore, substituting (B5) into $\sigma_I = 1 - 2b_I$ yields the Gini coefficient of income given by

$$\sigma_I = \frac{ra}{ra + wL}\sigma_a, \tag{B6}$$

which is the same as (35) except that σ_a is now the Gini coefficient of wealth.

Appendix C: Robustness checks of the panel VAR

In this appendix, we present some robustness checks to our panel VAR in section 2. First, we extend the bivariate setting to a multivariate setting by including per capita GDP growth in the analysis. Figure 10 presents the impulse response function. The initial impact of income inequality in response to a patent shock continues to be positive and significant. Furthermore, we continue to see a significant negative response for a 10 year forecast horizon. The result also holds even if we exclude non-resident patents from the patent counts data.

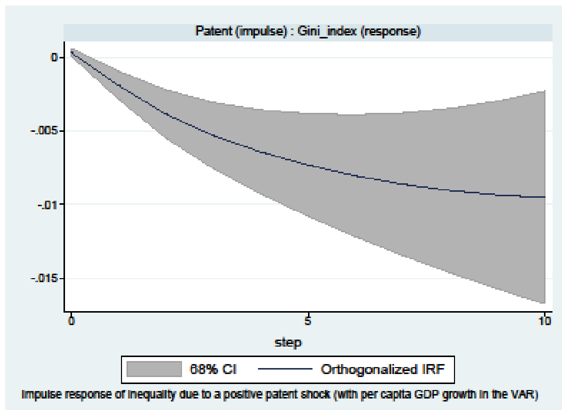

Figure 10

We further test the relation by changing the inequality measure. Instead of using the Gini index of inequality in household market (pre-tax, pre-transfer) income, we now consider the Gini index of inequality in household disposable (post-tax, post-transfer) income. The impulse response function using this inequality measure is shown in Figure 11. With the disposable-income-based Gini index, we find a similar response as the benchmark in Figure 2. Furthermore, regardless of the measure of inequality, the initial positive response disappears in the subsequent periods, converging to a negative response in the long run.

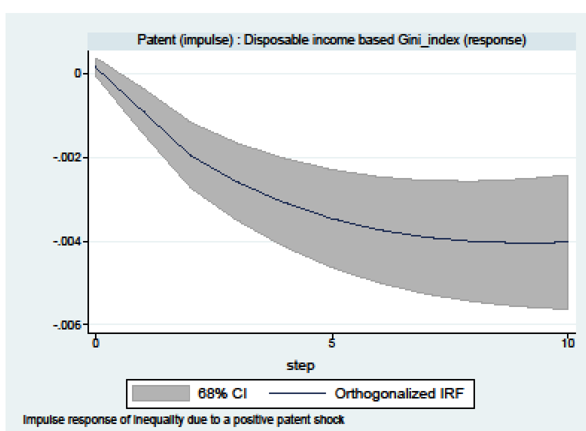

Figure 11