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The Essentiality of Money in a Trading Post Economy with Random Matching

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Abstract

This paper studies how the structure of centralized markets may affect the efficient allocation in anonymous decentralized trades. In line with previous studies, we show that efficiency in decentralized markets can be sustained in a moneyless finite-number-of-agents setting if agents are patient enough and the price is observed with noise as long as the noise disappears, but not too fast, as the number of agents grows. We also show that the Levine-Pesendorfer noise can be applied to dynamic games, not only to static games.

Keywords: Essentiality of money, anonymity, noisy prices, trading post

1 Introduction

In any ideal monetary model, money should be *essential* in the sense that it helps to achieve a better allocation compared to a model without it. This guiding principle has led a large segment of the money literature to introduce a variety of frictions in the trading process that make intertemporal trades

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(i.e. trade credit) more difficult.[1] The idea is that money allows agents to trade *quid pro quo* as it is an immediate compensation for the producer of goods, which expands the set of feasible allocations when trade credit is not possible.

A limitation of this literature is that the frictions that impair trades are usually descriptive and insufficiently formalized. For example, agents are merely assumed to meet and trade in pairwise meetings but the physical environment (preferences, technologies, etc.) under which this happens is not made explicit. This prevents a clear understanding of how these frictions reduce the set of feasible allocations.

In an attempt to overcome this limitation, part of the money literature has focused on the underlying economic conditions that affect the essentiality of money. For example, Aliprantis, Camera and Puzzello (2006, 2007c) provide rigorous set-theoretic foundations to bilateral matching theory. In their paper, anonymity is a result rather than an assumption. Araujo (2004) and Aliprantis, Camera, and Puzzello (2007a, 2007b) showed that anonymity in bilateral trades is not sufficient for money to be essential. Lack of communication is also needed.[2] To see this, think of a gift-giving economy without money where each agent agrees to produce a good in a bilateral meeting whenever it is his turn to do so with the expectation that others will do the same in the future when it is their turn to produce. If an agent deviates by not producing in a meeting, then his trading partner –who will never meet him again– can communicate the deviation to other agents who can punish the deviator in future trading. In other words, agents can use social norms and social punishment as a threat to sustain the desired outcome. These studies show that such a scheme is effective as long as agents are patient enough and information spreads quickly enough. They also show that the speed at which the information spreads among agents depends on how markets are structured. For example, it may take several rounds of decentralized trades

[1] These frictions, which are mainly in the form of informational and spatial separations, are made explicit by assuming agents interact in small coalitions. To this end, the literature relied on pairwise random matching models as in Kiyotaki and Wright (1989), Shi (1997), Green and Zhou (1998), and Lagos and Wright (2005), to cite a few. For earlier studies on the essentiality of money, see e.g. Ostroy (1973), Kocherlakota (1998), and Wallace (1998).

[2] Araujo (2004) introduces *social norms* in a setup similar to Kiyotaki and Wright (1989), whereas Aliprantis, Camera and Puzzello (2007a, 2007b) do so in a Lagos and Wright's (2005) framework. See Kandori (1992) for a seminal paper on social norms.

in Kiyotaki and Wright (1989) before the deviator gets punished. In contrast, punishment can take place immediately in centralized economies such as in Lagos and Wright (2005). In this context, Aliprantis, Camera, and Puzzello (2007b) show that money can fail to be essential in large economies if individual's actions are observable and agents are patient enough.

The assumption that individual's actions are observable in large economies is quite strong, though. A more plausible assumption is that aggregate variables, e.g. prices, are perfectly observable. Araujo, Camargo, Minetti, and Puzzello (2012), ACMP hereafter, address this issue by studying a finite-number-of-agents framework similar to Lagos and Wright. They show that there exists a non-monetary equilibrium that implements the first-best if agents observe prices instead of actions. The idea is that individuals are non-zero measure in finite-number-of-agents economies, so there is a one-to-one correspondence between individual's actions and the price. Consequently, a deviation by an agent has a measurable effect on prices and so can be detected regardless of the population size.

A more realistic assumption in large economies is that prices are observed only with noise. ACMP extend their analysis to this case and show that money remains essential if the ratio between the number of agents who participate in trade and the number of goods that are traded in the centralized market is sufficiently large. In other words, they show that the way the centralized market is modeled matters for the essentiality of money.

The focus of the present paper is the same as ACMP's. It differs from theirs in the way we formalize the noise. While ACMP follow the approach in Green (1980) in modeling the noise, we follow Levine and Pesendorfer (1995). In ACMP, the map between the agent's actions and the price is not deterministic, but agents observe the price with certainty. In our model, the map between the actions and the price is deterministic, but the price is observed with noise.

As a result of the way the Levine-Pesendorfer noise is formalized, we obtain an expression for consumption that is different from the usual one which is given by total bids divided total production for the trading post. This is because what an agent gets from the trading post is affected by his observed price which, in turn, is subject to an i.i.d. shock. Hence, the market may not clear. To address this point, we introduce rationing. For simplicity, we assume a rule such that rationing (lump-sum transfer, if there is an excess supply) is equal for all agents.

Using the Levine-Pesendorfer noise, we show that there is a sequential equilibrium that sustains an efficient outcome in the decentralized stage. In this equilibrium, players neither produce nor trade in the centralized stage on the equilibrium path. This effectively turns off the noise in the observed price, and makes it possible to detect deviations. Agents' inactivity in the centralized stage does not result in a loss of efficiency, however, since there are no gains from trade in the trading post. In ACMP, by contrast, there are positive gains from trade, and efficiency is achieved both in the decentralized stage and the centralized stage.

The paper is organized as follows. Next section discusses the Levine and Pesendorfer's (1995) noise and related literature. In Section 3, we present the deterministic setup which is a review of ACMP. Section 4 extends the deterministic setup to noisy observations and establishes the main result.

2 The Levine and Pesendorfer's (1995) noise and related literature

Standard models use the continuum-of-agents assumption. The rationale for this assumption is that it is a useful idealization of large finite-number-of-agents economies. However, in dynamic settings, equilibria can be radically different in economies with finite number of agents and economies with continuum of agents. In a continuum-of-agents model, the play of any zero-measure set of agents (and so of any single agent) is ignored (or negligible). In contrast, in finite-number-of-agents models, each agent is non-zero measure and in principle his action can be observed, directly or indirectly. Consequently, in models with finite number of agents, each agent can be induced to play in a cooperative way by future rewards or punishments depending on his current action. This is the case even if agents are anonymous. Indeed, in finite economies, a slight change in the aggregate statistic (e.g. the price vector, or the total supply or demand) from the equilibrium outcome indicates that *someone must have* deviated. Thus, social norms can be designed that implement cooperative equilibria. These equilibria are ruled out in continuum-of-agents models because player's actions are negligible. This creates a discontinuity problem in finite-number-of-agents models in the limit.

Green (1980) was one of the first to study the discontinuity problem in a Cournot-type setting by showing that firm's actions can be negligible in finite economies if firms are anonymous and aggregate variables are random.

Intuitively, as the significance of a firm becomes small relative to the size of the economy, the noise makes strategic threats and rewards on certain equilibria unforceable in the limit. Green (1980) establishes this result by focusing on some sort of trigger-strategy equilibria. Dubey and Kaneko (1984, 1985) study the relation between information pattern and Nash equilibria in extensive games in a continuum-of-agents economy and finite-number-of-agents economy, respectively. Dubey and Kaneko (1985) propose to solve the discontinuity problem by assuming that agent's deviations cannot be detected unless they exceed some small threshold. This assumption is strong as it implies that agent's actions are not observable irrespective of the number of players. Sabourian (1990) generalizes Green's (1980) results by placing no restriction on strategies.

On the same lines as Green (1980), Levine and Pesendorfer (1995) propose to solve the discontinuity problem by introducing some noise in the model. The two models, however, differ on several aspects. Green (1980) studies infinitely repeated games while Levine and Pesendorfer focus on three–stage games. In Green (1980), the map between the player's actions and the price is random, but players observe the price with certainty. In Levine and Pesendorfer (1995), in contrast, the map between the actions and the price is deterministic, but the price is observed with noise. The way the noise is formalized is also different in the two models. In Levine and Pesendorfer, the noise vanishes as the number of agents tends to infinity, but it does so at a lower rate than the inverse of the number of players. In Green (1980), the noise does not depend on the number of players in the economy. Moreover, Levine and Pesendorfer (1995) focus on a class of games where there is a large player and a (finite or infinite) number of small anonymous players, while Green (1980) studies a more general class of market games.[3]

To the best of our knowledge, ACMP and this paper are the first to study the essentiality of money using a noise a la Green (1980) and Levine and Pesendorfer (1995), respectively. This paper is also the first to use the Levine-Pesendorfer noise in a dynamic setting. Thus, we show that their noise can be applied to dynamic games, not only to static games.

Our paper is also related to the growing literature that uses experiments to study topics that are analyzed here (e.g., the essentiality of money) as well

[3]In particular, Green (1980) compares a *sequence of replica markets* with a *nonatomic market* which is constructed as the limit of a replica sequence of markets.

as other issues in monetary economics and policy.[4] For example, Camera and Casari (2014) and Duffy and Puzzello (2014a, 2014b) discuss the essentiality of money in a model where money is not essential. Similarly, Jiang et al. (2022) study the essentiality of money both theoretically and experimentally using mechanism design and a finite-horizon model. They show that the use of money and welfare are much higher in treatments where money is essential. They also find that money is sometimes used when *it should not*. Duffy and Puzzello (2022) focus on monetary policy instead. They show that the Friedman rule does not improve welfare in the lab and *it is not* superior to a monetary policy with a constant growth rate of money supply. A common denominator of these studies is that they use Lagos and Wright (2005) –or a modified version of it– as a theoretical setting.[5] None of these studies, however, contemplates noisy prices, either in the form of ACMP's or Levine-Pesendorfer's noise.

Our model can be of interest to applied economists for a number of reasons. Firstly, like the studies above, we propose a theoretical setup that is well suited for the laboratory as it features microfoundations, frictions, and tractability. Secondly, it would be worth investigating whether our theoretical findings (Proposition 1) are supported in the lab. Indeed, the existing experimental evidence shows that results in the lab may be at odds with theoretical predictions (Duffy and Puzzello, 2022, and Jiang et al., 2022). Thirdly, introducing noisy observations in the above (and other) laboratory experiments would make the experiment more realistic, especially in treatments with a large number of subjects. Fourthly, models in this literature typically admit multiple equilibria due to their self-fulfilling nature –an agent's willingness to accept money depends on what he thinks other agents will do– and equilibrium results can be quite sensitive to underlying frictions (Gu et al., 2013). Noisy observations are one of these frictions and we conjecture that findings in the lab may be substantially different with and without them, but a deeper investigation is left to future research.

[4] See Duffy (2016, 2021) for surveys on the use of experiments in monetary economics and policy.

[5] Lagos and Wright (2005) is well suited to laboratory implementations. The reason for this is that it is micro-founded and features anonymity, limited commitment, and limited information, all desirable properties. It is also tractable and simple to understand which is crucial when conducting laboratory experiments. Moreover, it provides a precise measure of welfare which allows to assess the impact of different monetary policies.

3 The deterministic case

The deterministic setup is very similar to that in ACMP.[6] Time is indexed by $t = 1, 2, ..., \infty$. There is a finite number N of population in the economy where N is an even number. Agents are indexed by $j \in \{1, ..., N\}$. There are two stages in each period and each stage differs in terms of the matching process, preferences, and technology. In the first stage, agents are randomly matched in pairs. Agents are anonymous in the first stage. In the second stage, trade takes place in a centralized market. Agents discount between periods, but not within periods. The discount factor is denoted by $\delta \in (0, 1)$.

In the decentralized market, agents produce *or* consume a divisible special good. With probability $1/2$ an agent is a producer in a meeting and with probability $1/2$ he is a consumer. Consuming $q \geq 0$ units of the special good yields utility $u(q)$, while producing q units of this good costs $c(q)$. We assume $u'(q) > 0$, $u''(q) < 0$, $c'(q) > 0$, $c''(q) > 0$, $u(0) = c(0) = 0$, $u'(0) = \infty$, and $c'(0) = 0$. We also assume that there exists a $\bar{q} > 0$ such that $u(\bar{q}) = c(\bar{q})$, and there exists a unique efficient quantity produced $q^* > 0$ such that $u'(q^*) = c'(q^*)$.

In the centralized market, agents can consume *and* produce a divisible general good. We assume a trading post protocol in the centralized market. We also assume quasi-linearity. An agent who consumes x units of the general good obtains utility x. So, there are no gains from trade.[7] An agent who produces x units of this good incurs a disutility x. Like ACMP, we impose an upper bound $\bar{x} > 0$ on the amount of goods that an agent can produce in a period. Both the special good and the general good are not storable, so they have to be consumed in the same stage where they are produced.

3.1 The stage game

The stage game is an extensive-form game with one round of decentralized market followed by one round of centralized market. Agents in the first stage can commit themselves to a reaction in the second stage. At the beginning of the first stage, before they are matched pairwise, agents learn their type (i.e. consumers or producers). In a decentralized meeting agents simultaneously (and independently) choose from two actions: say "yes" or "no". If at least

[6] More precisely, we borrow from Araujo, Camargo, Minetti, and Puzzello (2010) which is an earlier version of ACMP where they assume only one trading post.

[7] This means that the trading post acts merely as a coordinating device in our model.

one agent in a match says "no", then no trade takes place and agents walk away. If both agents in a match say "yes", then the producer transfers the efficient quantity q^* of the special good to the consumer.

In the centralized market, an agent can produce a general good. The general good produced by an agent can be consumed directly by him or it can be traded in the trading post. In each period, an agent j simultaneously and independently chooses the quantity z_t^j of the general good to be produced for his own consumption, the quantity y_t^j of the general good to be exchanged at trading post; and the bid $0 \leq b_t^j \leq y_t^j$ to be submitted to the trading post. By definition, the price of the general good in period t is $p_t = \frac{\sum_{j=1}^N b_t^j}{\sum_{j=1}^N y_t^j}$, where $p_t = 0$ if $\sum_{j=1}^N b_t^j = 0$. The quantity of the general good that agent j obtains in the trading post in period t is then $x_t^j = \frac{b_t^j}{p_t}$ where $\sum_{j=1}^N y_t^j = \frac{\sum_{j=1}^N b_t^j}{p_t} = \sum_{j=1}^N x_t^j$. It turns out that the aggregate supply in the trading post is always equal to the aggregate demand. The price p_t is public knowledge.

The amount of goods agent j consumes in the centralized market is given by the sum of two components: what he produces for himself and what he receives from the trading post, $z_t^j + \frac{b_t^j}{p_t}$, while the total amount he produces is given by the sum of two components: the quantity of goods he produces z_t^j for himself and y_t^j for the trading post.

3.2 The repeated game

The environment consists of infinite repetitions of the stage game in the previous subsection. Each agent's history consists of his past actions in both markets, the actions of his past partners in the decentralized market, and the history of prices in the centralized market. A behavior strategy for an agent is a map from the set of all his possible histories into a (mixed) action.

Strategies are described by automata. Let $A_1 = \{\text{yes,no}\}$ be the action set of an agent in the decentralized market and $A_2 = \{a_2 = (z, y, b) : z, y \in \mathbb{R}_+$ and $b \leq y\}$ be the action set of an agent in the centralized market. In our setup, an automaton is a list $(W, w^0, (f_1, f_2), (\tau_1, \tau_2))$ where W is a set of states; $w^0 \in W$ is the initial state; $f_1 : W \to \Delta(A_1)$ and $f_2 : W \to \Delta(A_2)$ are decision rules in the decentralized and centralized markets, respectively; and $\tau_1 : W \times A_1^2 \to W$ and $\tau_2 : W \times A_2 \times R_+ \to W$ are transition rules in the decentralized and centralized markets, respectively. A transition rule is a specification of behavior as a function of states. A transition rule in the decentralized market associates the next state of the automaton with the agent's current state and the profile of actions in his match. More precisely, $\tau_1(w, a_1, a_1')$ is the new state of an agent who enters the decentralized market in state w if the consumer and producer in his match choose a_1 and a_1', respectively. Similarly, a transition rule in the centralized market associates the next state of the automaton with the agent's current state, his action, and the observed price. We restrict attention to symmetric strategy profiles, where all agents behave according to the same automaton. A profile of states for a strategy profile σ is a map $\pi : W \to \{1, ..., N-1\}$ such that $\pi(w)$ is the number of other agents in the population who are in state w, where $\sum_{w \in W} \pi(w) = N - 1$. Denote the set of all state profiles by Π. A belief for an agent is a map $p : \Pi \to [0, 1]$ such that $\sum_{w \in W} \Pr(\pi) = 1$, where $\Pr(\pi)$ is the probability an agent assigns to the event that the profile of states is π, and Δ the set of all possible beliefs. A belief system for an agent is a map $\mu : W \to \Delta$. In an abuse of notation, we use μ to denote the profile of belief systems where all agents have the same belief system μ. We consider sequential equilibria of the repeated game. Note that the repetition of Nash equilibria of the stage game is an equilibrium outcome. The first best is achieved when in every period trade takes place in all meetings in the decentralized market and all agents consume the same amount that they produce in the centralized market.

Let σ^* be the strategy profile where all agents behave according to the following automaton. The set of states is $W = \{C, D, A\}$, where C stands for cooperation, D for deviation, and A for autarky. The initial state is C. The decision rules are

$$
\begin{aligned}
f_1(C) &= f_1(D) = \text{yes},\ f_1(A) = \text{no}, \\
f_2(C) &= (0, \bar{x}, \bar{x}),\ f_2(D) = (0, \bar{x}, 0),\ \text{and}\ f_2(A) = (0, 0, 0).
\end{aligned}
$$

For instance, an agent in state C agrees to trade in the decentralized market (says "yes") and chooses $(0, \bar{x}, \bar{x})$ in the centralized market. The transition rules are

$$
\begin{aligned}
\tau_1(C, a_1, a_1') &= \begin{cases} C \text{ if } (a_1, a_1') = (\text{yes, yes}) \\ D \text{ if } (a_1, a_1') \neq (\text{yes, yes}) \end{cases}, \\
\tau_1(D, a_1, a_1') &= D, \qquad \tau_1(A, a_1, a_1') = A,
\end{aligned}
$$

and

$$\tau_2(w, a_2, p) = \begin{cases} C \text{ if } w \in (C, D) \text{ and } p \in \left\{\frac{N-2}{N}, 1\right\} \\ A \text{ if } w \in (C, D) \text{ and } p \notin \left\{\frac{N-2}{N}, 1\right\} \end{cases},$$
$$\tau_2(A, a_2, p) = A.$$

An agent in state C in the decentralized market remains in C only if there is trade in his match, otherwise he moves to state D. Likewise, an agent in state C in the centralized market stays in C if the price he observes is either 1 or $\frac{N-2}{N}$, otherwise he moves to state A. Note that no agent is ever in state D after the centralized market (on- and off-the-path-of-play). It is evident that σ^* implements the first best. Indeed, under σ^*, the transaction takes place in every decentralized meeting (everybody says "yes") and agents always choose $(0, \bar{x}, \bar{x})$ in the centralized market.

Consider the belief system μ^* where: an agent in state C believes that all other agents are in state C; an agent in state A believes that all other agents are in state A; an agent in state D believes that there is one other agent in state D, and the remaining $N-2$ agents are in state C.

Now, assume $\bar{x} > c(q^*)$. Then, in equilibrium, the first best is achieved for any $\delta \geq \underline{\delta}$, independent of the number of agents N.

To see this, suppose that $N-2$ agents choose $(0, \bar{x}, \bar{x})$ in the centralized market and the other two agents choose the action $(0, Y, B)$.[8] If $(0, Y, B)$ is such that the price is $N-2/N$, then

$$p = \frac{2B + (N-2)\bar{x}}{2Y + (N-2)\bar{x}} = \frac{N-2}{N},$$

which implies $B/p = BN/(N-2) = Y - \bar{x}$. Hence, the flow payoff of the deviator in the centralized market is $U(B/p) - Y = U(Y - \bar{x}) - Y = -\bar{x} < 0$.

Let us first check incentives in state C. A producer in the decentralized market has no profitable one–shot deviation if

$$-c(q^*) + V^*_{CM} \geq -\bar{x} + \delta V^*_{DM}$$

which is always satisfied since $\bar{x} > c(q^*)$, by assumption. A consumer clearly has no profitable one shot deviation either.

[8] As there are no gains from trade, the choice of Z is irrelevant for an agent's payoff, so we assume $Z = 0$.

Consider now an agent in the centralized market and suppose he chooses $(0, Y, B)$. In this case, his flow payoff is $U(B/p) - Y$, where

$$p = \frac{B + (N-1)\bar{x}}{Y + (N-1)\bar{x}}$$

as he believes all another agents are in state C and produce $\bar{x}$ and bid $\bar{x}$ at the trading post. Since B/p is increasing in B and $B \leq Y$, then the highest flow payoff the agent can obtain is $U(Y) - Y$, which is zero. Therefore, there is no profitable one–shot deviation.

Let us now check incentives in state D. Consider an agent in the centralized market (no agent is ever in state D in the decentralized market). There are two types of one–shot deviations to consider: (1) the agent behaves so that $p \in \{1, (N-2)/N\}$ and so the continuation payoff is V_{DM}^*; (2) the agent behaves so that $p \notin \{1, (N-2)/N\}$ and so the continuation payoff is V_A^*.

Case (1). Clearly, the price can never be 1 as $B \leq Y$. If the agent behaves such that $p = (N-2)/N$, then

$$p = \frac{B + (N-2)\bar{x}}{Y + (N-1)\bar{x}} = \frac{N-2}{N}$$

which implies $BN/(N-2) = Y - \bar{x}$, as one another agent is in state D and produces $\bar{x}$ and bids 0 at the trading post. Thus, the payoff from a one–shot deviation is

$$U(B/p) - Y + V_{DM}^* = U(Y - \bar{x}) - Y + V_{DM}^* = -\bar{x} + V_{DM}^*.$$

Since $\bar{x} > 0$, there is no profitable one-shot deviation.

Case (2). A one-shot deviation is not profitable if

$$-\bar{x} + \delta V_{DM}^* \geq B/p - Y + V_A^*$$

or, equivalently,

$$-\bar{x} + \frac{\delta}{2(1-\delta)} \left[u(q^*) - c(q^*)\right] \geq B/p - Y. \tag{1}$$

Since B/p is increasing in B and $B \leq Y$, it holds that

$$\begin{aligned} B/p - Y &\leq Y \frac{Y + (N-1)\bar{x}}{Y + (N-2)\bar{x}} - Y \\ &= \frac{Y\bar{x}}{Y + (N-2)\bar{x}} \\ &< \bar{x}. \end{aligned}$$

Since the right-hand side of the last inequality is bounded above by $\bar{x}$ and the left-hand side of (1) goes to infinity as δ goes to 1, then there exists a $\underline{\delta}$ such that a one-shot deviation is not profitable if $\delta > \underline{\delta}$. Finally, it is straightforward to see that no one-shot deviation is profitable in state A.

The main idea is that, if a defection occurs in the decentralized trading stage, players can use prices in the centralized trading stage to "broadcast" their (anonymous) partner's cheating behavior. This deters defections in the decentralized stage through the threat of future punishments if agents are patient enough. In the next section, we show that under some conditions this "broadcasting" ability is not diminished by adding the Levine-Pesendorfer noise.

4 The noisy case

In what follows, we depart from ACMP by assuming a different noise, a noise a la Levine and Pesendorfer (1995).

Suppose that each agent can observe the price in the trading post only with some degree of imperfection. Let p_t be the actual price of the general good in period t, and suppose that agent j observes a random variable v_t^j, defined as

$$v_t^j = \frac{p_t}{1+\varepsilon_t^j}, \text{ where} \tag{2}$$

$$p_t = \frac{\sum_{j=1}^N b_t^j}{\sum_{j=1}^N y_t^j}, \text{ and} \tag{3}$$

$$\varepsilon_t^j = \alpha \gamma_t^N \eta_t^{N,j}. \tag{4}$$

The term γ_t^N is a positive number, $\gamma_t^N \eta_t^{N,j}$ is an observational error, and $0 < \alpha < \sqrt{2}$ a constant parameter. For simplicity, we assume that $\eta_t^{N,j}$ is a continuous i.i.d. random variable with support $[-1, 1]$ and zero mean.

We assume $\gamma_t^N = \frac{1}{\sqrt{N}}$ as a functional form. It is easy to see that this choice of γ_t^N satisfies the Levine-Pesendorfer (1995) condition (p.1164), i.e. $\gamma_t^N \to 0$, and $N\gamma_t^N \to \infty$ as $N \to \infty$. This condition is key for our results. The first assumption, $\gamma_t^N \to 0$ as $N \to \infty$, ensures that the error term vanishes as the number of agents goes to infinity. The second assumption, $N\gamma_t^N \to \infty$ as $N \to \infty$, ensures that the error term vanishes at a rate less than $\frac{1}{N}$ as the number of agents increases.

Note that $\gamma^N \eta^{N,j}$ has support $\left[-\frac{1}{\sqrt{N}}, \frac{1}{\sqrt{N}}\right]$ and the observational error $\varepsilon_t^j = \alpha\gamma^N \eta^{N,j}$ has support $\left[-\frac{\alpha}{\sqrt{N}}, \frac{\alpha}{\sqrt{N}}\right]$. Also note that $\alpha\gamma_t^N$ satisfies the Levine-Pesendorfer condition as well. From (2), the noisy variable, v_t, has support $[\underline{v}_t, \overline{v}_t]$ with $\underline{v}_t = \frac{p_t}{1+\frac{\alpha}{\sqrt{N}}}$ and $\overline{v}_t = \frac{p_t}{1-\frac{\alpha}{\sqrt{N}}}$.

4.1 Rationing

The usual expression for own consumption defined as own bid divided by the price, $x^j = \frac{b^j}{v^j}$, does not guarantee market clearing in the noisy case. To see this, note that $\sum_{j=1}^N \frac{b^j}{v^j}$ differs from $\sum_{j=1}^N \frac{b_j}{p}$ because of the i.i.d. error term. So, we need to rely on some form of rationing/transfer for the market to clear. For simplicity, we assume that the per-capita rationing (transfer, if there is excess supply) is the same for all agents; i.e. individual consumption for all agents is reduced (increased) by the same quantity accordingly. Then, what an agent j obtains from the trading post is

$$x_j = \frac{b_j\left(1+\varepsilon^j\right)}{p} - \frac{1}{N}\sum_{i=1}^N \frac{b_i \varepsilon^i}{p}.$$

The first term on the right-hand-side is the usual expression for own consumption in the absence of rationing, i.e. own bid divided by the (observed) price. The second term is new and can be interpreted as the per-capita rationing/transfer component, whereas $\sum_{i=1}^N \frac{b_i\varepsilon^i}{p}$ is the aggregate excess demand (excess supply, if negative). Total consumption with rationing is

$$\sum_{j=1}^N x_j = \sum_{j=1}^N \left[\frac{b_j\left(1+\varepsilon^j\right)}{p} - \frac{1}{N}\sum_{i=1}^N \frac{b_i\varepsilon^i}{p}\right] = \sum_{j=1}^N \frac{b_j}{p} = \sum_{j=1}^N y_j.$$

Hence, the trading post always clears.

4.2 The repeated game

The infinitely repeated game when prices are noisy is the same as the one described in the deterministic case, except that agents now observe v_t, not the actual prices p_t. Hence, the transitions rules depend on the observed prices v_t, rather than the actual prices p_t.

The decision rule f_1 is also the same but f_2 now differs, as described below. Let σ^{**} be the strategy profile where all agents behave according to the following automaton. The decision rules f_1 and f_2 are now given by

$$\begin{aligned} f_1(C) &= f_1(D) = \text{yes},\ f_1(A) = \text{no}, \\ f_2(C) &= f_2(A) = (0,0,0),\ \text{and } f_2(D) = (0,\bar{x},\bar{x}). \end{aligned}$$

The initial state is state C. Then, the transition rules τ_1 and τ_2 are

$$\begin{aligned} \tau_1(C, a_1, a_1') &= \begin{cases} C \text{ if } (a_1, a_1') = (\text{yes, yes}) \\ D \text{ if } (a_1, a_1') \neq (\text{yes, yes}) \end{cases}, \\ \tau_1(D, a_1, a_1') &= D, \qquad \tau_1(A, a_1, a_1') = A, \end{aligned}$$

and

$$\begin{aligned} \tau_2(w, a_2, v_t) &= \begin{cases} C \text{ if } w \in (C, D) \text{ and } v_t = 0 \\ A \text{ if } w \in (C, D) \text{ and } v_t \neq 0 \end{cases}, \\ \tau_2(A, a_2, v_t) &= A. \end{aligned}$$

Consider the belief system μ^{**} where: (i) an agent in state C believes that all other agents are in state C; (ii) an agent in state A believes that all other agents are in state A; (iii) an agent in state D believes that there is one other agent in state D, and the remaining $N-2$ agents are in state C. Under σ^{**}, the transaction takes place in every decentralized meeting (everybody says "yes") and all agents are in state C in the centralized market. Hence, σ^{**} implements the first best.

Proposition 1 *For any* $\widetilde{\delta} \leq \delta < 1$, (σ^{**}, μ^{**}) *is a sequential equilibrium.*

Proof. Let V_{DM}^{**} and V_{CM}^{**} be the discounted lifetime utility of an agent in state C before he enters the decentralized and centralized market, respectively. Then,

$$V_{DM}^{**} = \frac{1}{2(1-\delta)} \left[u(q^*) - c(q^*) \right] \text{ and } V_{CM}^{**} = \delta V_{DM}^{**}.$$

Let V_D^{**} be the lifetime utility of an agent in state D before he enters the centralized market. Then,

$$V_D^{**} = \delta \left[\lambda V_{DM}^{**} + (1-\lambda) V_A^{**} \right]$$

where λ is the probability that $v = 0$ and $1 - \lambda$ is the probability that $v \neq 0$. Now, let's assume an agent is in state D in the centralized market. There can be two cases: (1) he can follow the rule or (2) he can deviate.

Case (1). Assume the agent follows the rule. Then, he bids $\bar{x}$ and produces $\bar{x}$ to the trading post. Because there is one other agent in state D, who does exactly the same, while everyone else do not participate in the trading post, then the actual price is $p_D = 1$. In this case, the price an agent j observes is $v^j = p_D/(1+\varepsilon^j)$ which is different from zero with probability 1 ($\lambda = 0$) as the distribution of v^j is continuous. Therefore, the observed price signals that a deviation occurred. Hence, the economy moves to state A with certainty next period and $V_D^{**} = \delta V_A^{**} = 0$. There is no one-shot deviation that restores state C once an agent is in state D; permanent autarky is the only equilibrium.

Case (2). Assume the agent deviates and chooses $(0, Y, B)$. Since one other agent is in state D who bids $\bar{x}$ and produces $\bar{x}$ to the trading post, the actual price is $p'_D = \frac{B+\bar{x}}{Y+\bar{x}} > 0$. In this case, the price an agent j observes is $v^j = p'_D/(1 + \varepsilon^j)$ which again is different from zero. This signals that a deviation occurred that moves the economy to state A next period and $V_D^{**} = \delta V_A^{**} = 0$. No one-shot deviation restores state C once an agent is in state D and permanent autarky is the only equilibrium.

We now check agents' incentives. Let us first check incentives in state C. If the agent is a producer in the decentralized market, then he has no profitable deviation if

$$-c\left(q^*\right) + V_{CM}^{**} > V_D^{**} = 0 \Leftrightarrow \frac{\delta}{2\left(1-\delta\right)}\left[u\left(q^*\right) - c\left(q^*\right)\right] > c\left(q^*\right)$$

which is always satisfied as long as δ is close enough to 1. A consumer has no profitable deviation either. Consider now an agent in the centralized market. If he cooperates, he has zero flow payoff as nobody participates to the trading post. If he chooses $(0, Y, B)$ with $Y > 0$, his expected flow payoff is

$$\mathbb{E}\left[\frac{B\left(1+\varepsilon^j\right)}{p'_C} - \frac{1}{N}\frac{B\varepsilon^j}{p'_C} - Y\right] = \frac{B}{p'_C} - Y,$$

as $E(\varepsilon^j) = 0$, by definition, where ε^j is his noise and the first two terms within brackets denote his consumption x^j. The actual price resulting from the deviation is $p'_C = \frac{B}{Y}$ since he believes he is the only one to use the trading post. Hence, his expected flow payoff from a deviation is zero. Since V_{DM}^{**} is already the highest continuation payoff possible, the agent has no profitable one-shot deviation.

Consider now an agent in state D in the centralized market. If he chooses $(0, Y, B)$ with $Y > 0$, then $p'_D = \frac{B+\bar{x}}{Y+\bar{x}}$ is positive as one other agent is in state D who bids $\bar{x}$ and produces $\bar{x}$ to the trading post. Then his expected flow payoff is

$$\mathbb{E}\left[\frac{B\left(1+\varepsilon^j\right)}{p'_D} - \frac{1}{N}\frac{B\varepsilon^j + \bar{x}\varepsilon^i}{p'_D} - Y\right] = B\frac{Y+\bar{x}}{B+\bar{x}} - Y,$$

as $E(\varepsilon^j) = E(\varepsilon^i) = 0$, by definition, where ε^j is his noise and ε^i is the other agent in state D's noise. It is evident that his expected flow payoff is maximized at $Y = B$ and always non-positive. As we have shown above, a positive price triggers permanent autarky, so his continuation payoff from choosing $(0, Y, B)$ is zero. Therefore, the agent has no profitable one-shot deviation.

Now, suppose he chooses $(0, Y, B)$ with $Y = 0$. Since $B \leq Y$, then $B = 0$ and so his expected flow payoff is

$$\mathbb{E}\left[-\frac{1}{N}\frac{\bar{x}\varepsilon^i}{p'_D}\right] = 0,$$

since the other agent in state D's noise is such that $E(\varepsilon^i) = 0$. The economy moves to autarky next period, so his continuation payoff is zero. Consequently, there are no profitable one-shot deviations for an agent in state D.

It is evident that there are no profitable one-shot deviations in state A.

■

The idea of the proof of Proposition 1 is as follows. If agents are patient enough, the threat of punishment from a deviation is enough to sustain cooperation in the decentralized stage. In contrast, if agents are not sufficiently patient, the punishment is not severe enough to deter them from deviating; permanent autarky is the only outcome. Note that agents cannot use the centralized market to punish a deviation directly, as there are no gains from trade. They can only use it as a coordination device to punish past deviations in future decentralized market meetings. If they observe no deviation in the decentralized market, agents *do not participate* in the next round of trading post. This results in $p = 0$ which effectively turns off the noise. If a deviation is observed in the decentralized market, *at least one agent* participates in the next round of trading post. This results in $p > 0$ which turns on the noise. Since the distribution of v is continuous, $v = 0$ is a zero probability event when $p > 0$. Hence, a deviation in the decentralized market is detected with certainty in the next round of trading post. Permanent autarky as the only punishment for a deviation in the decentralized market is made here for simplicity. A less severe punishment, such as a finite number of periods in autarky before cooperation is restored, would still be a deterrent if agents were patient enough.

It is important to emphasize that the observed price v and the rationing amount convey different information about the noise. The price v_j observed by an agent j depends on his *own* noise, ε_j, but not on the other agents' noise. In contrast, the rationing amount depends on the noise of *all* agents, $\varepsilon_1, \varepsilon_{2,...}, \varepsilon_N$. Thus, rationing provides some information about the noise distribution that is not in v. In principle, this information can be used strategically to achieve a better outcome if $\delta < \widetilde{\delta}$. Indeed, by just observing v, the current strategy already achieves the highest payoffs if $\delta > \widetilde{\delta}$. A further investigation of this aspect would be very interesting but we leave it to future research.

5 Conclusion

Araujo, Camargo, Minetti, and Puzzello (2012) show that, in economies with alternating centralized and decentralized markets, the way one models the former matters for the allocation in the latter. We show that this result is robust to a change in the noise specification. Using the noise specification in Levine and Pesendorfer (1995), we show that the efficient allocation in anonymous decentralized trades can be sustained when the noise goes to zero –but not too fast– as the population grows. We also show that the Levine and Pesendorfer's (1995) noise can be applied to dynamic games, not only to static games. We propose two directions for future research. Theoretically, it would be worthwhile to analyze the effects of noisy observations when agents are not anonymous, but the number of actions is bounded as in Fudenberg, Levine, and Pesendorfer (1998). Experimentally, it would be interesting to study noisy prices in the lab and compare the results with those in previous experiments. This may also help to verify whether our theoretical predictions (Proposition 1) are supported in the laboratory.

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