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A Model of Rent Seeking and Inequality

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Abstract

Social scientists have argued that inequality fosters rent seeking and that rent seeking is likely to reinforce existing inequalities. In this paper, I formalize these interactions by modelling rent seeking in an unequal endowment economy where agents can choose to be rentiers or not. I find that when the cost of rent seeking is exogenous, more inequality fosters a greater proportion of rentiers, which in turn further skews the distribution of resources. I endogenize the cost of rent seeking by assuming that the rentiers pay the cost to a central institution, which chooses the cost per rentier to maximize its revenue. In this setting, the revenue-optimizing cost of rent seeking per rentier increases with more inequality, which results in a lower proportion of rentiers. However, ex-post inequality still increases. The results show how economies can end up with persistent inequality in the presence of rent seeking.

Keywords: inequality, rent-seeking, property rights, institutions, corruption

JEL code: D23, D31, D63, D72

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1 Introduction

Does rising inequality affect rent seeking? Does rent seeking, in turn, affect the distribution of wealth? These questions have been at the heart of the development discourse and political debates for over a decade and their answers will shape development policy in the coming decades. The World Development Report (2006) brought the issue of equity to the forefront of the development world. In recent years, researchers such as Piketty (2014) and Milanovic (2011, 2016) have revolutionized our understanding and added further interest to the political economy of inequality.

However, much remains to be understood. Specifically, there is a need to examine the causes of extreme inequality beyond theories of marginal productivity. In her seminal work where she coined the phrase 'rent seeking', Krueger (1974) said that societies typically perceive high incomes as a product of high social output in a relatively free market. Inequality is considered incidental as a part of the growth process. If, however, it is believed that economic success or even survival cannot be ensured without exerting influence, the association between economic gains and social product weakens. Keeping this in mind, recently a prominent research agenda in economics has been to understand and explain persistent inequality induced by factors other than productivity differences. All of them point to rent-seeking behavior as the major driver of inequality in recent decades (Alkire et al., 2016; Bourguignon et al., 2007; Deaton, 2017; Jacobs, 2015; Kanbur & Stiglitz, 2016; Stiglitz, 2012, 2015).

In this paper, I address the above questions by modelling inequality in an economy characterized by rent seeking. I do so by formalizing the idea that a more unequal society is more vulnerable to rent seeking. In turn, rent seeking itself transforms the distribution of wealth, skewing it further. The two feed back into each other, creating a cycle of endemic inequality. The equilibrium rent seeking and inequality are history-dependent in that they are a function of the initial level of inequality. Thus, the paper presents rent seeking as both a cause as well as a consequence of inequality.

The idea that unequal societies are more susceptible to rent seeking has been present in the social sciences. Economists have argued that high inequality provides the rich with the resources and reasons to maintain their economic status (Acemoglu & Robinson, 2006; Banerjee et al., 2001). Large distributional inequities place a disproportionate amount of power with a few, be it bargaining power, influence, personal connections or resources to bend the rules in their favor. As Glaeser et al. (2003) point out, if courts are corruptible, then the legal system will favor the rich over the just. Similarly, if regulatory institutions can be 'captured' [1] by wealth or influence, they will gratify the influential, not the efficient. Moreover, if political actors value campaign contributions, they

[1] The pioneering work on regulatory capture was done by Stigler (1971). For a more recent review of developments in this literature, see Dal Bó (2006).

will accommodate and even facilitate special interests[2]. In political science, prominent work has been done by Uslaner (2004) who argues that inequality breeds lower trust and more rent seeking. Jong-Sung & Khagram (2005) provide cross-country evidence that high inequality leads to an increase in perceptions of corruption.

Evidence for the link between rent seeking and inequality is also wide. Corruption, one of the more egregious forms of rent seeking, has been shown to increase inequality (Gupta et al., 2002; Li et al., 2000; Murphy et al., 1993). Winters (2011) discusses evidence of rent appropriation by oligarchs which strengthens their power and entrenches inequality[3]. To be sure, these are forms of rent-seeking behavior where the powerful can appropriate from the weak [4]. They are able to do so because a skewed wealth distribution can result in differences in the allocation of property rights, voting rights (adult suffrage), public resources or education. These differences could arise for two reasons. First, inequality could reduce redistribution and public good provision (Rodriguez, 2004), because economic resources determine the ability to influence political outcomes (Acemoglu & Robinson, 2006). Second, the poor may lack the resources to push their political agenda, such as better public protection of property rights, more investment in overhead capital, etc. Thus, a system where the rich can take away from the less well-off also changes the allocation of wealth, perpetuating existing inequities.

None of the above papers, however, endogenizes the distribution of wealth as a function of rents and vice-versa. Alesina & Angeletos (2005) endogenize the wealth distribution by modelling rent seeking as embezzlement of tax money by bureaucrats in a voting model. They posit that more inequality leads to demands for bigger government (more progressive redistribution) which fosters more corruption. Assuming some bureaucrats are better at embezzling public funds, corruption leads to inequality which ends in a positive feedback loop between corruption and inequality. While this is one plausible mechanism for inequality-generating rent seeking, it is a weak one in so far as the evidence for inequality, higher public goods provision and taxation is mixed. Some research finds that inequality lowers public goods provision (Alesina et al., 1999; Anderson et al., 2008) while others find the opposite effect (Boustan et al., 2013; Fabre, 2018). A more general model of rent seeking arising from an unequal distribution of wealth has been presented in Chakraborty & Dabla-Norris (2006). However, the paper does not examine the effects of a change in inequality on

[2]See Stiglitz (2012) for a well-rounded discussion on the nature, causes and perils of inequality in the modern economy with rent seeking.

[3]We use economic and political power interchangeably in this paper. In so far as there is money in politics, one form of power at least weakly translates into another.

[4]The economic and politically powerful need not be the only classes that appropriate from others unfairly. In some cases, the poor may also engage in rent-seeking behavior such as robbery and stealing. Rent seeking from below may work to decrease inequality - the Robin Hood effect. We leave this avenue for future research.

rent seeking and vice-versa. This paper also relates to the literature on endogenous inequality which has been modeled before as a result of the combination of credit market imperfections and differences in skills or tastes (Banerjee & Newman, 1993; Matsuyama, 2000; Mookherjee & Ray, 2003) or increasing returns to scale (Engerman & Sokoloff, 1997; Freeman, 1996).

This paper presents a model of endogenous inequality with respect to rent seeking behavior. I model rent seeking in a general form, including rents from monopoly profits, quota restrictions, lobbying, campaign finance and feudal tax systems.[5] I start with a population where the only source of heterogeneity across agents is the level of wealth. Agents can choose to become either rentiers or non-rentiers based on their wealth. Rentiers incur a fee to collect the rents. Non-rentiers simply pay the rent and consume their net after-rents wealth. A fixed cost captures the increasing returns from rent seeking necessary to generate a split between the rentiers and the non-rentiers in terms of occupational choice.[6]

In equilibrium, more inequality gives rise to more agents choosing the rent seeking option when costs are fixed. Thus, rising inequality perpetuates rent seeking as a strategic response to the threat of being appropriated by the wealthy, pushing more and more agents at the margin into rent seeking. Rent seeking in this case implies a regressive redistribution of resources from the less endowed to the more endowed, increasing inequality even further. When the cost of rent seeking is endogenized to maximize a sovereign's revenue, costs increase with more inequality, reducing the proportion of agents who choose to be rentiers. Again, the resultant distribution is more unequal than before. Thus, a feedback loop is generated between inequality and rent seeking.

France under the Ancién French Regime is a prime example of such an economy with rent-seeking induced inequality. The Ancién French Regime (15th century - 18th century) was a period of high inequality in France. French society then was divided into three main estates - the clergy, the nobility and the bourgeoisie (consisting of merchants and craftsmen, the wealthiest members of the Third Estate) and peasants (landless tillers who rented land from the landed nobility and lived on very little). The nobility and clergy were entitled to collect taxes from the Third Estate. They themselves were exempt from these taxes, although they had to pay a regular fee to the King to maintain their position across generations.[7] In 1680, France established the Ferme Générale, a system of tax collection where some individuals bought the right to collect land tax on

[5]I discuss the definition and examples of rents and rent-seeking behavior in the Appendix Section A.

[6]Increasing returns in rent-seeking activities have been explained by Murphy et al. (1993). The occupational choice model was pioneered by Kanbur (1979) and used in the context of inequality and development by Banerjee & Newman (1993).

[7]According to an estimate, the top 10% of the population owned about 45% - 53% of the wealth (Morrisson & Snyder, 2000). The two estates comprised of 0.2% of a total population of about 28 million people when the French Revolution started.

behalf of the King. Exemptions from the tax could be bought if not already exempted. Given exceptional powers to collect the money, tax-farmers bore arms, conducted searches and imprisoned uncooperative citizens. The money collected over and above that specified in the contract with the government went to the tax farm (Anderson, 2007).[8]

In a modern setting, lobbying to reduce taxation, and to relax competition and other forms of regulation by special interest groups is another example of rent-seeking behavior with perverse implications for inequality. More inequality in the economy provides more resources and incentives to certain industries and special interest groups to pursue said lobbying. The sectors most prone to such lobbying are the sectors with the most rents, such as pharmaceuticals and healthcare, finance and insurance, and the digital economy more recently. Such rent-seeking behavior helps to concentrate wealth in fewer hands, leading to more inequality in turn. The increasing levels of market concentration, reduced competition, stagnant median wages, and rising profits for a few firms could be a direct or indirect result of the pursuit of rent-seeking activities in the US economy. Other examples of such positive relationship between inequality and rent seeking is corruption in developing countries that favors the rich, for example, politicians granting access to scarce national resources to those connected to them in India, Indonesia, and Russia. The rich class in such places keeps getting richer at the expense of the rest of the population through these political connections (Banerjee et al., 2001; Fisman, 2001; Sukhtankar, 2015; Winters, 2011).

The model used in Section (2) closely follows the pattern of appropriation and redistribution described above. We start with an initial distribution of wealth which can be a result of productivity differences or socio-economic status or both. There is a costly rent-seeking technology. Access to the technology allows agents to appropriate from other economic agents whilst protecting themselves. Due to the costly nature of the technology, only the wealthier are able to employ it. This regressive redistribution further skews the wealth distribution.

In order to raise more revenue the state often created more offices. The fee to buy these offices, the cost of the rent-seeking technology, must reflect the need for the state to balance the revenue-increasing effect of a higher fee with the negative impact on the demand for such offices. In Section (3), I allow for the parameters of the rent-seeking technology to change to reflect the sovereign's revenue maximization. I then study the impact of an increase in the

[8] This was such a profitable enterprise that each fermier-généraux, the tax farmers, paid the royal government up to 80 million livres for a six-year lease. In good economic times, when production and trading were up, some tax-farmers made several million livres per year. As a comparison, the taille, the oldest and most lucrative of France's state taxes, brought in about 20 million livres a year. By the reign of Louis XVI, the fermiers-généraux had become one of the wealthiest groups in France. They purchased grand homes along Paris' Place Vendôme, venal offices and noble titles.

fee for buying elite status on rent-seeking behavior and the subsequent effect on inequality.

I focus on a 2-period static model rather than the dynamic framework discussed in Bourguignon et al. (2007) to study the political economy closely relative to infinite period dynamics. There are rich comparative statics generated even in this setting, with insights for path dependence. The rest of the paper is organized as follows. Section 2 lays out the base model with fixed rent-seeking costs and its results. Section 3 endogenizes the costs from the sovereign's revenue-maximizing point and interprets the changes in results. Section 4 presents ideas for extending the results and concludes.

2 The Baseline Model of Rent Seeking

2.1 Environment: Regressive Redistribution

The economy consists of a continuum of agents, i, distributed over the interval $[0,1]$. All agents are endowed with some initial wealth w_i, which is drawn from the distribution $F : [\underline{w}, \infty) \longrightarrow [0,1]$, where $\underline{w} \geq 0$ is the minimum wealth. Agents have identical preferences and care about their net wealth. There is full information about the draws of wealth in the economy.

The costs and benefits of rent seeking Each agent i has the choice to pay a cost $\theta \in [0, \infty)$ from her endowment w_i to appropriate from those who don't make this payment. I will refer to agents who pay θ as rentiers (R) and those who don't pay θ as non-rentiers (NR). A rentier cannot appropriate from another rentier. Hence, θ serves the dual purpose of the cost of appropriation from non-rentiers and protection from other rentiers. The total rents collected from the non-rentiers are distributed equally among all the rentiers.[9]

All non-rentiers need to pay a fraction $n\gamma$ from their endowment. Here $\gamma \in [0,1]$ is a fixed appropriation rate and n is the mass of rentiers in the economy, $n \in [0,1]$. Thus, γ can be thought of as the maximum rate of appropriation when everyone is a rentier ($n = 1$).[10]

One could interpret the set of parameters $\{\theta,\gamma\}$ as the state of institutions in the economy. An economy with higher θ, all else equal, implies that it is costlier to appropriate or influence the government through lobbying or regulatory capture. Fewer agents are able to afford a higher value of θ, thereby

[9]This modelling choice arises naturally as a consequence of the assumption of equal payment of θ. In a later section, we relax the assumption of a fixed θ and let it be determined endogenously. However, we maintain the requirement that everybody still pays the same θ. An interesting case for future work is where rents are distributed proportionately to the initial draw of wealth or agents can choose different levels of θ or a combination of both.

[10]When everyone is a rentier ($n = 1$), everyone is protected. There is no redistribution in this case, only the sunk cost of paying for protection.

discouraging rent-seeking activities. Similarly, a large value of γ, all else equal, signifies a larger burden of direct or indirect appropriation on the non-rentier class. Taking the ratio of these two parameters gives us a comprehensive measure of the state of rent-s eeking institutions in the society: the effective cost of rent seeking ($\frac{\theta}{\gamma}$). Well-functioning polities that value an equitable sharing of resources will have a high value of θ and a low value of γ, thus, a very large effective cost of rent seeking. Polities with more tolerance towards rent-seeking activities or weaker institutions will have a lower effective cost of rent seeking.

2.2 Choice of Occupation

The assumptions above imply a type of occupational choice: agents draw their endowment from a known distribution and then decide whether to invest in θ or not. Investing in θ makes an agent a rentier and benefit from appropriation. Not paying θ makes her a non-rentier and hence vulnerable to appropriation. The assumption of increasing returns to θ implies that the wealthier agents engage in rent-seeking behavior in this economy.

Rewards We can now determine the net rewards to an agent from occupational choice. The net payoff for a non-rentier (U^{NR}) will be her initial wealth endowment, minus the appropriation rate:

$$U^{NR}(w|n,\gamma) = w(1 - n\gamma)$$

Each rentier receives an equal share of the total rents paid by the non-rentiers. Let $K(w^*)$ denote the expected wealth of the non-rentiers, where w^* is the cutoff level of wealth (to be determined endogenously) below which agents choose to be non-rentiers:

$$K(w^*) \equiv \int_{\underline{w}}^{w^*} wf(w)dw$$

Then the share of rents received by each rentier i will be the total expected rents divided by the mass of rentiers:

$$Rents_i = \frac{\gamma n K(w^*)}{n}$$

Thus, the net payoff for a rentier (U^R) is his wealth, w, plus the rents net of the fixed cost θ.

$$U^R(w|n,\theta,\gamma) = w - \theta + \gamma K$$

An agent chooses the occupation that generates a higher net expected payoff, which in turn depends on the proportion of rentiers (n) in the economy. Thus, an agent i will choose to be a rentier iff:

$$U_i^R - U_i^{NR} \geq 0$$

or

$$w_i \geq \frac{\frac{\theta}{\gamma} - K}{n}$$

Thus, the choice to be a rentier is a strategic choice that depends on each agent's belief about the other agents decisions. The more rentiers one believes to be in the economy, the lower the wealth level w_i above which agents have the incentives to become a rentier. A larger K (expected wealth of non-rentiers) again implies a lower threshold into the rent-seeking occupation due to bigger incentives in the form of bigger rents to be captured. Finally, the higher the effective cost of rent seeking ($\frac{\theta}{\gamma}$), the fewer agents will prefer to become rentiers.

2.3 Equilibrium (with θ as a sunk cost)

All agents in the economy simultaneously decide whether to invest in rent seeking (pay θ) or to be a non-rentier (pay γ). Agents have rational expectations about the proportion of agents who will choose the rentier occupation. A Nash equilibrium of this game can be one of the following three cases:

- $n = 0$ and $U^{NR} \geq U^R$
- $n = 1$ and $U^{NR} \leq U^R$
- $0 < n < 1$ and $U^{NR} = U^R$

Corner solutions, i.e., the first two cases, will arise for certain parameter values, which are characterized in the next section. The third case is an interior solution, with a threshold level of wealth (w^*) above which everyone in the distribution will want to be a rentier, for a given value of n^*. Therefore, w^* and n^* will jointly solve:

$$U^{NR}\{w^*, n^*\} = U^R\{w^*, n^*\} \tag{1}$$

Equation (1) gives us the equilibrium pair (w^*, n^*) for $0 < n^* < 1$. Now, if all individuals expect a proportion n of rentiers, and if this expectation is to be fulfilled, then n must also be identical to the proportion of agents with wealth above the critical level. Hence,

$$n^* = 1 - F(w^*) \tag{2}$$

Substituting (1) into (2) gives us a fixed point equation. Distributional assumptions on $F(w)$ and the set of institutional parameters $\{\gamma, \theta\}$ give conditions for existence, stability, and uniqueness of the equilibria. In the analysis below, I use the simple Pareto distribution to obtain a closed-form solution for the equilibrium variables $\{w^*, n^*\}$. The Pareto distribution is useful in giving the analysis a real-world paradigm since it is designed to closely approximate actual wealth distributions around the world (Jones, 2015; Piketty & Saez, 2014). The

Pareto inequality parameter (in this case $\alpha \in [1, \infty)$) also has a simple inverse relationship with the Gini coefficient, making the results easy to interpret[11].

2.4 Results

The simple Pareto distribution is defined by the cumulative distribution function $F(w) = 1 - (\frac{\underline{w}}{w})^{\alpha}$ with endowments w drawn over a support of $[\underline{w}, \infty)$ and α as the inequality or shape parameter. The expected rents received by each rentier in equilibrium are given by:

$$\gamma K(w^*) = \gamma w^m \left[1 - \left(\frac{\underline{w}}{w^*}\right)^{\alpha - 1}\right] \tag{3}$$

where $w^m = \frac{\alpha}{\alpha - 1}\underline{w}$ is the mean value of the distribution of endowments. Using the expected rents expression from equation (3) and the relationship between n^* and w^* from (2) in the equilibrium equation (1), we obtain the following equilibrium solution:

$$w^* = \begin{cases} \underline{w}, & \text{if } \frac{\theta}{\gamma} \leq \underline{w} \\ \underline{w}\left[\frac{w^m - \underline{w}}{w^m - \frac{\theta}{\gamma}}\right]^{\frac{1}{\alpha - 1}}, & \text{if } \underline{w} < \frac{\theta}{\gamma} < w^m \\ \infty, & \text{if } \frac{\theta}{\gamma} \geq w^m \end{cases} \tag{4}$$

and

$$n^* = \begin{cases} 0, & \text{if } \frac{\theta}{\gamma} \geq w^m \\ \left[\frac{w^m - \frac{\theta}{\gamma}}{w^m - \underline{w}}\right]^{\frac{\alpha}{\alpha - 1}}, & \text{if } \underline{w} < \frac{\theta}{\gamma} < w^m \\ 1, & \text{if } \frac{\theta}{\gamma} \leq \underline{w} \end{cases} \tag{5}$$

(Complete derivations are provided in Appendix C.1 and C.2).

The expressions for w^* and n^* in equations (4) and (5) imply that there will be some rentiers and some non-rentiers in the economy only when the effective cost of rent seeking is high enough such that not all agents can afford it and low enough such that at least some agents find it profitable. When the effective cost of rent seeking is higher than the mean wealth of the distribution, there is no rent seeking. Conversely, when the effective cost of rent seeking is lower than or equal to the lowest wealth in the distribution, then everyone will be able to afford it. The result will be a rentier state where everyone is protected and there is no redistribution. But the economy still pays the sunk cost of θ. Hence, we have the following result:

Proposition 1. *Equilibrium Rent Seeking*

[11] The properties of the general Pareto distribution used throughout in this paper are also provided in the Appendix, Section (B).

a. *Some Rent Seeking: There is an interior solution in the economy ($0 < n^* < 1$) when the effective cost of rent seeking lies between the mean and lower bound of wealth for the entire distribution ($\underline{w} < \frac{\theta}{\gamma} < w^m$).*

b. *All Rentiers, No Redistribution: $n^* = 1$ when $\frac{\theta}{\gamma} \leq \underline{w} \leq w^m$. Thus, when effective rent-seeking costs are less than or equal to the endowment of the poorest person, everyone will pay to become a rentier. Since everyone is protected, there is no redistribution in this economy.*

c. *All Non-Rentiers: $n^* = 0$ when $\underline{w} \leq w^m \leq \frac{\theta}{\gamma}$. In other words, when rent-seeking costs are higher than the mean wealth of the distribution, no one finds engaging in rent-seeking activities profitable.*

(Full proofs in Appendix C.2.)

It can be shown that $n^* = 0$ is an equilibrium strategy profile because $U^{NR} > U^R$ whenever $\frac{\theta}{\gamma} > w^m$. In such an economy the effective cost of rent seeking is so high that no one invests in it. Everyone takes home their initial draw of wealth. Similarly, it can also be shown that $n^* = 1$ is an equilibrium strategy profile because $U^R > U^{NR}$ whenever $\frac{\theta}{\gamma} < \underline{w}$. In this economy, the effective cost of rent seeking is very low and is affordable for everyone. Since being part of the rentier class also provides protection against the other rentiers, everyone chooses to invest in it, even though there are no rents to be collected. Thus, if everyone else becomes a rentier, it is rational for an agent to become a rentier as well. I discuss some examples of such rentier states later in Section (2.6). Thus, for the given parameters of the rent-seeking technology and the initial conditions given by the Pareto distribution parameters, the existence of an equilibrium is guaranteed. Moreover, the equilibrium is stable for each of the three cases discussed above.

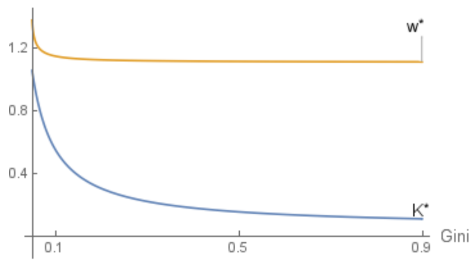
2.5 Inequality increases Rent Seeking

We now analyze the impact of an increase in inequality (decrease in α or increase in the Gini coefficient) on rent-seeking behavior (n^*) in this economy. In this setting, higher inequality (higher Gini or lower α) implies longer right tails of the Pareto distribution. This could be interpreted in two ways. One, there are more draws of wealthy agents. Two, the gap between the rich and the poor widens, i.e., relative to the lowest wealth, all other agents' wealth has increased. With the former interpretation, it is easy to see that adding more wealthy people will lead to more rent seeking. This is because the richer an agent, the more net return from the rent-seeking option. We show this later in Figure (3). The relationship is a function of the rent-seeking costs ($\frac{\theta}{\gamma}$) and nature of the wealth distribution, which can be fully characterized by the mean (w^m) and lower bound ($\underline{w}$).

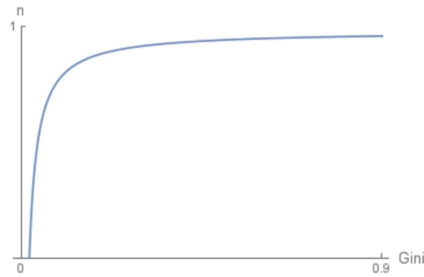
The derivative of n^* with respect to the inequality parameter α for interior solutions is:

$$\frac{dn^*}{d\alpha} = \frac{n^*}{(\alpha-1)^2}\left[ln\left(\frac{1}{x}\right) + \alpha\left(1 - \frac{1}{x}\right)\right] < 0 \tag{6}$$

Figure 1: Rent Seeking Dynamics with Inequality



(a) Equilibrium wealth threshold and expected non-rentier wealth as a function of the Gini coefficient



(b) Equilibrium proportion of rentiers as a function of Gini coefficient

where $x = \left[\frac{w^m - \frac{\theta}{\gamma}}{w^m - \underline{w}}\right]$ (derivation in Appendix C.3). The impact on the threshold wealth w^* and the corresponding expected non-rentier wealth K^* are the opposite. Figure (1a) maps the declining values of the threshold wealth level (w^*) and the expected non-rentier wealth (K^*) in equilibrium with respect to the Gini coefficient (also a function of α). Figure (1b) graphs the proportion of rentiers in equilibrium (n^*) as a function of the Gini. We have the following proposition:

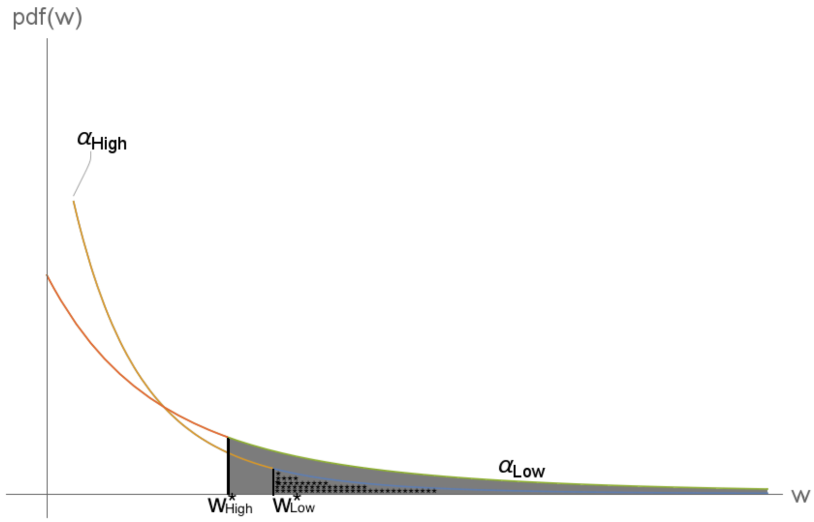
Proposition 2. *For $0 < n^* < 1$, more inequality in the economy ($\alpha \downarrow$) leads to a higher proportion of rentiers ($n^* \uparrow$), a lowering of the threshold wealth ($w^* \downarrow$), and the mean non-rentier wealth ($K^* \downarrow$).*

Proof. For $0 < n^* < 1$, we have $0 < x < 1 \Rightarrow \frac{1}{x} > 1$ in equation (6). The term outside the square brackets is positive. The term inside the square brackets depends on the relative values of $ln(1/x)$ and $1/x$. The logarithmic transformation of a variable is always smaller than the variable itself. So the term in the square bracket is dominated by the negative second term inside. Since w^* and K^* are negatively related to n^*, the remaining result follows. $\square$

The net wealth of society after rent seeking is $w^m - n^*\theta$, which is less than the initial wealth w^m, and θ is a sunk cost. We can also compare the net wealth of each agent before and after rent seeking. For non-rentiers, the ex-post wealth is $w_i(1 - n^*\gamma) < w_i$. For the rentiers, whether ex-post net wealth is greater or smaller than ex-ante wealth depends on the relative value of the costs (θ) and benefits (γK^*).

Proposition 3. *Both non-rentiers and rentiers are worse off than before ($U_i^j < w_i$ for $j = R, NR$).*

Figure 2: The marginal agent is pushed into rent-seeking activity as inequality rises



α_{High} denotes the distribution with lower inequality. The dotted area represents the mass of rentiers in the low-inequality economy, with w_{Low} being the threshold wealth. As inequality rises (α_{Low}), the marginal agent is pushed into rent seeking. The gray shaded area denotes the increased mass of rentiers in the high-inequality economy above the new threshold wealth w_{Low}.

Proof. Relative to θ, the net gain of rentiers is negative if

$$\gamma K^* - \theta < 0$$
$$\gamma\alpha\left(\frac{\theta}{\gamma} - \underline{w}\right) - \theta < 0$$

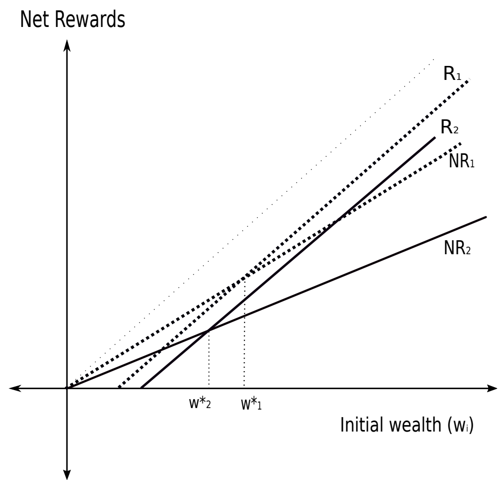
Rearranging the terms, we get $\frac{\theta}{\gamma} < w^m$, which is true for all interior solution, $0 < n^* < 1$. $\square$

As inequality increases and wealth becomes concentrated with fewer and fewer agents, there is more rent seeking in this economy. Higher inequality in the economy pulls the threshold rent-seeking wealth in the economy down by making rent seeking a more affordable option for the agent at the margin. This happens despite the returns to rent seeking, as captured by K, falling with more inequality. Non-rentiers just below the cut-off are pushed into rent seeking as inequality rises. These dynamics are captured in Figure (2).

2.6 Discussion

Why does an increase in inequality, with all else the same, lead to an increase in the proportion of rentiers? This is because the increase in inequality in this

Figure 3: Net Returns from Rent Seeking and Non-Rent Seeking



R and NR are the returns to rentiers and non-rentiers' respectively. The dotted lines denote returns functions in a low-inequality economy; the corresponding threshold for rent seeking is high (w_1^*). The solid lines denote returns functions in a high-inequality economy. High inequality lowers the return to both rentiers and non-rentiers. The burden is higher on non-rentiers. The marginal agent is pushed towards rent seeking. The corresponding threshold for rent seeking is low (w_2^*).

environment implies adding more people to the top of the wealth pyramid[12]. Once the rich class expands, for a given threshold wealth level, n increases. This changes the tradeoff for the marginal agent in two ways. As a non-rentier, she pays a higher appropriation rate ($n\gamma$). As a rentier, the pie is divided among more people, so she receives less. For the marginal agent, the increase in the rent burden on non-rentiers is greater than the loss from a reduction in share for rentiers. Thus, higher inequality burdens the non-rentiers disproportionately more than the rentiers in this set-up.

The mechanism becomes clear in Figure 3. As inequality increases, the non-rentiers' rewards fall from NR_1 to NR_2. The intersection of NR_2 and R_1 gives us the impact of adding more people to the top of the pyramid, keeping the threshold wealth constant. The higher rent rate on non-rentiers generates positive incentives for rent seeking and the threshold w^* falls. However, the share that each rentier receives is smaller, reducing rent-seeking incentives. This tradeoff is evident from the intersection of NR_2 and R_2, which moves w^* to the right, increasing the threshold wealth to become a rentier. It is also clear from the figure that there will always be a unique *interior* equilibrium for each initial condition if the net rewards functions are strictly increasing (or decreasing).

The type of rent-seeking behavior (regressive redistribution) assumed here captures three aspects of rent seeking as discussed in the social sciences literature. First, the motives for rent seeking depend strategically on the choices of others in the economy. For the agent at the margin, even though the net benefit from rent seeking ($\theta - \gamma K^*$) is lowered with more rentiers around, it is better than facing the increased probability of extortion (n^*) as a non-rentier. In other words, the loss of wealth due to insecure property rights is higher compared to the resources needed to secure protection and be part of the rentier club. Thus the marginal agent is pushed into rent-seeking behavior as inequality grows in the economy, as shown in Figure (2).

Secondly, all rentiers spend more resources net of what they gain, thus having a negative payoff, i.e.

$$\gamma K^* - \theta < 0 \tag{7}$$

Some forms of rent seeking are perceived as negative-sum games in the economics literature. The investment required in protection technology itself is a dead-weight loss to the economy. It diverts resources ($n^*\theta$) away from productive capacity. As such, total wealth in the economy decreases. However, with even rentiers worse off than if they didn't have to engage in rent seeking and pay θ, the implication is that rent seeking in this environment is unambiguously welfare-reducing or Pareto-inefficient. With more inequality and consequently more rentiers, the rate of wealth loss increases. In a dynamic setting, over time as the size of the pie shrinks, rent seeking may have implications

[12]One may want to study the effect of an increase in inequality by adding more people at both the top and the bottom. It can be implemented in the same framework by re-scaling all observations by the new lower bound and taking into account the resulting further decrease in α.

for growth. This will be especially true over the short-run to medium-run when rent-seeking costs are not flexible. This result matches intuitively with the literature on growth and rents, where under very diverse settings it has been shown that rent-seeking activities are growth-reducing (Murphy et al., 1991, 1993). Modern economies that have amassed wealth through cartelization of natural resource rents, for example, the Gulf countries, have grown slower than the economies relying on innovation and productivity increase to foster economic growth (Stiglitz, 2012). Such economies, also known as rentier states, are an example of economies where everyone chooses to be a rentier. The cost of rent seeking is low compared to wealth in the economy. As such almost everyone who has citizenship rights chooses to invest in these protection and rent-seeking technologies. Such economies are characterized by increasing inequality and low growth rates.

Third, building on the last two points, the rent-seeking equilibrium above is a like a prisoner's dilemma. It is a dominant strategy for an agent to pay her respective dues, the γn^* rent rate for non-rentiers and the $\theta - \gamma K^*$ for rentiers, given her draw of wealth. This kind of rent-seeking behavior becomes a tradition: everybody agrees it is bad for them, but it requires a collective effort by society to move out of it. In a dynamic setting, the welfare-enhancing strategy would be to coordinate and agree to not invest in θ at all. Without an infinitely repeated game, however, the possibility of coordination is limited. Moreover, another strand of research in economics says that chances of coordination decrease as the number of players involved increase. In an entire economy, such costly coordination could be possible with a major institutional change, such as a change in the government regime or better investments in property rights.

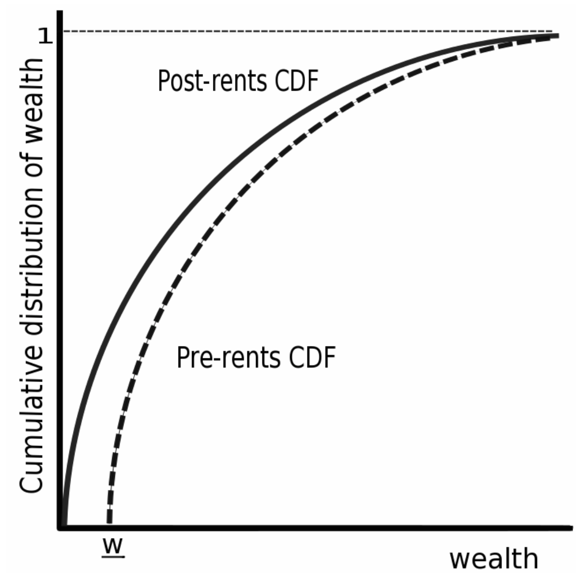
2.7 Rent Seeking increases Inequality

We have so far shown that a more unequal initial distribution increases the proportion of rentiers in the economy, leading to more regressive redistribution. It is intuitive that a regressive redistribution will lead to a more unequal society. Here I present results using a formal analytical proof on how inequality can feed into itself in the presence of rent-seeking activities. Since any redistribution changes the characteristics of the statistical distribution, I use the Gini coefficient to compare initial and ex-post inequality. In this respect, the simple Pareto distribution is useful as it has a closed-form parametric solution for the Gini coefficient.

Proposition 4. *Gini coefficient for the ex-post rent-seeking distribution of wealth is greater than the initial Gini coefficient.*

Proof. We begin by partitioning the initial wealth distribution into two parts: non-rentiers' (W_{NR}) and rentiers' wealth (W_R). Let V_j be a linear transformation of w for $j = NR, R$. We compute the CDF for the new transformed variable V_j by joining the CDF's for each of the partitions, conditioned over

Figure 4: The CDF Before And After The Redistribution.



The dotted line is the cumulative distribution function of the initial distribution of wealth. The solid line is the cdf of the new distribution of wealth after redistribution. The initial cdf stochastically dominates (first order) the new.

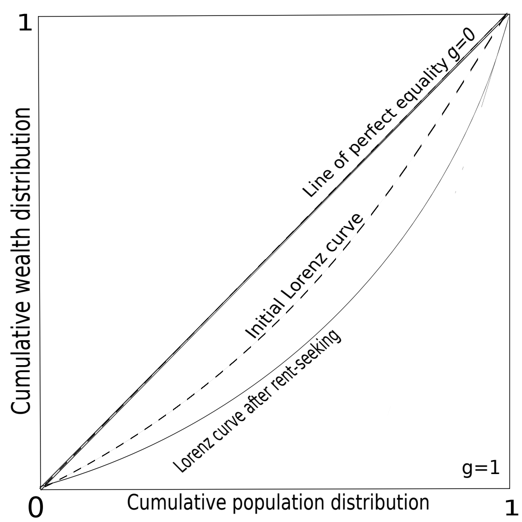
their support; denote this new CDF as $F_V(w)$. From here it is straightforward to compute the Lorenz curve for the new CDF: $L(F_V)$, and the subsequent Gini coefficient: $G(F_V)$. Comparing the new 'ex-post' Gini coefficient with the initial Gini ($\bar{G} = \frac{1}{2\alpha-1}$), it can be shown that the ex-post Gini ($G(F_V)$) is greater than the initial Gini coefficient. (Detail derivations in Appendix D.) $\square$

Thus, any economy which is characterized by the kind of rent-seeking technology used here will end up with higher inequality. Figure 4 captures the lowering of the overall wealth of all agents in the economy. We already showed in the preceding section that higher inequality is responsible for more rent-seeking agents. Figure 5 captures the Lorenz curves corresponding to the same pre- and post- rent-seeking economies. The transfer of wealth from lower wealth to higher wealth agents leaves the economy more unequal than before.

2.8 Discussion

Th equilibrium discussed here is a function of the Pareto distribution's parameters. This implies that the system has multiple equilibria, with each equilibrium corresponding to unique initial conditions (in the space of permissible α, $\underline{w}$ values). Any distribution with some initial inequality in the allocation of resources leads to a more unequal ex-post distribution. The only distributional allocation that leads to no subsequent increase in inequality is the one with perfect

Figure 5: The Lorenz Curve with Pre- and Post- Rent Seeking distributions



The Lorenz curve of the economy after rents have been distributed is further away from the line of perfect equality than before the redistribution. The Gini coefficient is the ratio of the area between the Lorenz curve and the line of perfect equality with respect to the total area under the triangle (= 1/2). The Lorenz curve closer to the line of perfect equality represents the more egalitarian economy.

equality.

Proposition 5. *When $\alpha \to \infty$, we have the following two cases of rent seeking:*

- $n^* = 0$ *when* $\frac{\theta}{\gamma} \geq \underline{w}$
- $n^* = 1$ *when* $\frac{\theta}{\gamma} < \underline{w}$

Proof. The equilibrium value of rent seeking (n^*) can be expressed as a function of α:

$$n^* = \left(\alpha(1 - \frac{\theta}{\underline{w}\gamma}) + \frac{\theta}{\underline{w}\gamma}\right)^{\frac{\alpha}{\alpha-1}} \tag{8}$$

As $\alpha \to \infty$, the power function tends to unity. The value in round brackets tends to either $+\infty$ or 0 or $-\infty$ when $\frac{\theta}{\gamma} < \underline{w}$ or $\frac{\theta}{\gamma} = \underline{w}$ or $\frac{\theta}{\gamma} > \underline{w}$ respectively. From Proposition (1), this implies a value of $n^* = 1$ for the first case or $n^* = 0$ for the last two cases. $\square$

Thus, a redistribution of resources that levels the playing field will break down the rent seeking equilibrium, thereby bringing the economy out of the high inequality equilibrium.

As discussed earlier, ideally, one would want to extend the above model to a dynamic framework. An overlapping-generations model seems the ideal framework to study the comparative statics over multiple time periods. The challenge to develop a dynamic model stems from the discontinuity in the pdf of the ex-post wealth distribution, which is a piece-wise Pareto distribution. Hence, an analytical solution is no longer elegant or even possible. One either needs to change the rent-extraction structure or use simulation to develop the OLG framework and study the Gini coefficient of the resulting distribution.

3 Rent Seeking with Endogenous Cost

The last section gave us some insights about how inequality affects the size of the rentier population when the costs are fixed. In this section, I endogenize the effective cost of rent seeking by letting the sovereign choose the optimal size of the rent-seeking class. This is a natural extension in a static setting where the sovereign state sets the costs and determines the returns to rent seeking to optimize their revenue[13]. The state faces the trade-off on the intensive versus

[13]Another way to endogenize costs would be to let all agents invest in a θ to maximize their return non-cooperatively (contest). For the rentiers, it could be used to bring in rents, while for the non-rentiers it could provide some protection from rentiers. modelling this requires two things: first, a protection technology (contest success function) and second, a third party that provides the protection such as a police or a government. It would also raise concerns for additional modelling features such as voting and/or collective action problems. Instead I use θ to capture the differential cost of investment between rentiers and non-rentiers. γ is then an increasing but concave function of θ, capturing the dependency of rents on the rent-seeking technology. Preliminary work suggests there will be multiple equilibria in this setting for each initial condition, with some stable and some unstable. This is the subject of another paper I am working on.

extensive margin since a higher θ implies more direct revenue per agent but reduces the size of the rent-seeking class.

I assume that the rate of rent extraction depends on the cost of rent seeking. This is plausible because the amount of resources spent on rent seeking may influence the rate of rent extraction, just as investing resources in fiscal capacity affects the state's ability to tax citizens[14]. To obtain closed form solutions, I assume the following rent-extraction function:

$$\gamma = \beta\theta^{1-\epsilon} \tag{9}$$

Here, $\epsilon \in (0,1)$ and $\beta \in \Re_+$ such that γ is a proportion. Thus, the elasticity of γ with respect to θ is given by $1-\epsilon$: a higher value of ϵ implies that the extraction rate is less sensitive to changes in θ. From the last section, we know that the sovereign's revenue is given by $n\left(\frac{\theta}{\gamma}\right)^*\theta$. In what follows, I extend the framework to a two-stage game, where first, the state decides the revenue-maximizing cost of appropriation (θ), anticipating the size of the rentier class based on $\frac{\theta}{\gamma}$. In the second stage, agents choose whether to be rentiers or non-rentiers.

3.1 Central Planner's Choice

The state faces the following optimization problem:

$$\underset{\theta}{\text{Maximize}} \quad n\left(\frac{\theta}{\gamma}\right)\theta \quad \text{subject to} \quad \gamma = \beta\theta^{1-\epsilon}$$

Let the revenue-maximizing θ be denoted by θ^* and the corresponding effective cost of rent seeking be $\left(\frac{\theta}{\gamma}\right)^*$. Knowing this ratio, agents face the same optimization problem as in the last section with exogenous costs. The equilibrium size of the rent-seeking class will be given by:

$$n\left(\frac{\theta^*}{\gamma}\right) = \left[\frac{w^m - \frac{\theta}{\gamma}^*}{w^m - \underline{w}}\right]^{\frac{\alpha}{\alpha-1}}$$

From backward induction, the state's optimization problem reduces to the following simple problem:

$$\underset{\theta}{\text{Maximize}} \quad \theta\left[\frac{w^m - \frac{\theta}{\beta\theta^{1-\epsilon}}}{w^m - \underline{w}}\right]^{\frac{\alpha}{\alpha-1}}$$

[14] Fiscal capacity is the ability of the state to raise revenues from their own sources in order to pay for a basket of goods and services. For literature on fiscal capacity, see Besley & Persson (2009, 2011, 2013)

3.2 Equilibrium with endogenous costs

From the first-order conditions, the revenue-maximizing value of θ is given by

$$\theta^* = \left[\frac{\alpha \underline{w} \beta}{\alpha - 1 + \alpha\epsilon}\right]^{\frac{1}{\epsilon}} \tag{10}$$

The corresponding revenue-maximizing value of $\frac{\theta}{\gamma}$ and n^* are given by

$$\frac{\theta}{\gamma} = \frac{\alpha \underline{w}}{\alpha - 1 + \alpha\epsilon} \tag{11}$$

and

$$n^*(\alpha) = \left[\frac{\alpha^2 \epsilon}{\alpha - 1 + \alpha\epsilon}\right]^{\frac{\alpha}{\alpha-1}} \tag{12}$$

3.3 Inequality decreases Rent Seeking

We can now analyze how rent-seeking activities change in the economy when inequality increases. Once again, we can analyze the effect of a change in inequality by differentiating the expressions of interest with respect to the inequality parameter.

Proposition 6. *The optimal cost of rent seeking chosen by the sovereign to maximize her revenue increases with more inequality.*

Proof. The derivative of θ with respect to α is

$$\frac{d\theta^*}{d\alpha} = \frac{1}{\epsilon} y(\alpha)^{\frac{1}{\epsilon}-1} \frac{dy}{d\alpha} \tag{13}$$

where $y(\alpha) = \frac{\alpha \underline{w} \beta}{\alpha - 1 + \alpha\epsilon} > 0$ and $\frac{dy}{d\alpha} = \left(\frac{-1}{(\alpha - 1 + \alpha\epsilon)^2}\right) \underline{w}\beta < 0$. Hence,

$$\frac{d\theta^*(\alpha)}{d\alpha} < 0 \tag{14}$$

(Detailed derivation in appendix F.1). □

Proposition 7. *The proportion of rentiers decreases as inequality increases.*

Proof. The derivative of n^* with respect to α is:

$$\frac{dn^*(\alpha)}{d\alpha} = \frac{n^*}{\alpha - 1}\left(1 - \frac{1}{(\alpha - 1 + \alpha\epsilon)} + \frac{-lnf(\alpha)}{(\alpha - 1)}\right) > 0 \tag{15}$$

for all values of ϵ and α. (Full derivation in Appendix F.2). □

All rentiers optimize by increasing investments in θ when endowments become more unequal. As a result, they are able to appropriate a higher fraction of resources from the non-rentiers. The concavity of γ makes the effective cost of rent seeking increase in equilibrium with more inequality. However, the proportion of rentiers ($n^*(\alpha)$) in the economy shrinks. Therefore, with more inequality, we have a smaller proportion of rentiers, with each rentier spending more on appropriation.

3.4 Rent Seeking increases Inequality

With the above results, we can still say something about the level of ex-post inequality in the economy relative to the initial one.

Proposition 8. *The Gini coefficient for the ex-post rent-seeking distribution of wealth is greater than the initial Gini coefficient.*

Proof. In the section with the exogenous $\frac{\theta}{\gamma}$, we proved the Lorenz curve dominance of the initial distribution over the ex-post distribution for any value of $\frac{\theta}{\gamma}$ such that n^* is interior (or $\underline{w} < \frac{\theta}{\gamma} < w^m$). Here, $\left(\frac{\theta}{\gamma}\right)^* = \frac{\alpha \underline{w}}{\alpha - 1 + \alpha\epsilon}$. For $\alpha\epsilon > 0$, this value is smaller than the mean wealth w^m. For $\alpha > \alpha - 1 + \alpha\epsilon$ or $\alpha\epsilon < 1$, this value is greater than $\underline{w}$. Since rent seeking increases inequality for all values of $\underline{w} < \frac{\theta}{\gamma} < w^m$, the result holds for $0 < \alpha\epsilon < 1$. □

The above result shows the robustness of the result to changes in rent-seeking institutions. The class of rentiers may shrink (as shown before) but they still command a greater proportion of society's wealth. Unless rent-seeking costs are so exorbitantly high as to eliminate all rent-seeking behavior, rents will always work to redistribute wealth regressively.

One can also see more clearly how different metrics that are used to measure rent seeking in the economy change with inequality. On the one hand, we have a lower fraction of agents opting to become rentiers, as opposed to what we had in the case of exogenous costs. On the other hand, the per rentier expense or investment into rent seeking increases, as does the rent rate. Thus at the pecuniary level, one would observe bigger amounts of wealth or resources spent on rent seeking and bigger distortions in the economy due to rising inequality. But the club of rentiers will consequently be more exclusive and limited to a few very wealthy. Any empirical analysis thus must be careful in assessing inequality's impact evaluation on rent seeking as the interpretation is sensitive to the metric of rent seeking used.

4 Conclusion

This paper answers the question whether more inequality indeed increases rent-seeking activities in the economy and vice versa. For exogenous costs, the answer to the above question is yes. Rent-seeking behavior in a society increases inequality. More inequality encourages more agents to become rentiers, appropriating the wealth of non-rentiers. The feedback loop creates a path of ever-growing inequality and rent seeking. With endogenous costs, the results are more mixed. In this case, more inequality reduces the proportion of rentiers, although they are each now wasting more resources on rent-seeking activities. Rent seeking still increases inequality in this society.

The simplicity of the setting offers clear insights regarding the welfare implications of such a regressive redistribution. Rent-seeking activities are unambiguously Pareto inefficient and lead to the destruction of wealth. This aspect of the model is in line with the view that rent seeking is a negative-sum game. Developing a dynamic model with a more direct link between inequality, rent seeking, and more inequality will require some modelling variations. Alternatively, using simulation for such an environment will lead to more clear answers in this direction. An inequality metric comparable across distributions such as the Gini coefficient can be used to draw inferences on the impact on ex-post inequality.

A possible extension with centralized planning and voting can provide the likelihood of multiple equilibria in this economy. Consider the same framework but with two parties competing for votes. Their election platform is setting the optimal level of institutions $\{\theta,\ \gamma\}$ in the economy. The median voter theorem can be invoked in this setting with some additional assumptions. As this economy becomes more unequal, the mean and median wealth go up. At the same time, the equilibrium threshold of wealth w^* dividing rentiers from non-rentiers goes down. For a critical value of $\alpha^\star$ the median agent will switch from being a non-rentier to a rentier. Supposing that rentiers are pro- weaker institutions, a more unequal society will demand weaker institutions.

Another possibility for future work is a non-cooperative game among the rentiers for sharing the rents pie. A contest with infinite agents makes the analysis complex. However, there has been recent progress on the topic of large games which may be exploited to study the dynamics in such a setting. Preliminary work shows the existence of multiple equilibria for each set of initial conditions when rentiers have to contest for a share of the rents pie.

A third possibility arises by generalizing the rent-seeking technology to include factors of production such as capital and labor. When capital is relatively cheaper, rents are appropriated by the rich, capital-owning class. When labor is cheaper, the lower income class engages in appropriation through violence, kidnapping, stealing or robbing.

The analysis presented here renders itself to several empirical applications. The case of rentier states, as noted before, and their political economy can be understood better using the framework used here. This paper also speaks to the vast literature on corruption, connections and political kickbacks that perpetuate inequalities in developing economies. There is evidence from some developing countries with long stable rules of dictators, for example, Suharto in Indonesia, that show an increase in corruption under the dictatorial rule, favoring allocation of scarce resources to certain groups that could provide kickbacks (Fisman, 2001). There is also evidence of increased within-sector inequality and a general worsening of institutions over the same time period in these places. A future extension of this paper will work on such an empirical application, linking sectoral inequality with measures of corruption and institutions. Lastly, this paper and its results can add to the analysis on intra- or inter-generational mobility. Countries with more progressive taxation schemes, better institutions of accountability and campaign finance management do better in distributing their national wealth more equitably. More concrete empirical evidence on these inter-linkages between inequality, rent seeking, and institutions is clearly needed.

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APPENDIX

A Definition and examples of concepts

Types of rent-seeking behaviors Traditionally, rents are described as the return to a factor of production irrespective of any effort on the part of the owner. Rent-seeking behavior is spending resources to transfer resources from others to oneself without compensating them, usually using a favorable decision on some public policy. This broad definition over time has come to encompass monopoly profits, quota-rents from import restrictions and profits from the exclusive rights to mine a natural resource. It may also include direct and indirect transfers and subsidies from the government, profits from laws that limit competition or poor enforcement of existing competition laws, and exclusive rights that allow corporations to pass on costs to the rest of society. In some countries, rent-seeking behavior can also take its most egregious form - corruption - where bureaucrats frequently take bribes for providing services that are part of their usual job description or siphoning off scarce resources from the public exchequer.

Cost of rent-seeking behavior This transfer of rents typically comes at a cost to the rent seeker, be it funds or time spent on lobbying efforts, financing political campaigns, maintaining connections with those in power, or on other forms of influence. In return, rent seekers get favorable legislation enacted for exclusive mining, drilling, or distribution rights. Revolving-door lobbying has costs including but not limited to the promise of lucrative industry positions after completion of the political tenure (Blanes i Vidal et al., 2012). Bargaining over a higher share of rents could also take the form of rent seeking, with costly lawyers or bribes to the jury. The latter amounts to judicial corruption and may be restricted depending upon the state of institutions in the country, but the former is perfectly legal and openly practiced . In most countries with the kind of bureaucratic corruption described earlier, agents have to first bribe the insiders to secure these lucrative bureaucratic positions, expecting a valuable future stream of payments in return. In countries with weak law enforcement and poor institutions, the costs of rent seeking may take the form of traditional arms keeping, as in by gangs and paid criminals. Historically, the feudal lords had to pay monies to the king for rights to collect taxes, maintain an army as well as a big house with the status appropriate for their station (Goubert, 1974). For each of these rent-seeking activities that favor rent seekers, there is an increased burden on the rest of the population.

Regressive Redistribution The above forms of rent-seeking behavior are characterized by a regressive redistribution of resources - from smaller firms to bigger corporations, or from peasants to the elites. I model this rent-seeking behavior as a transfer of wealth from the non-rentier agents to the rentiers. The rate of appropriation (subsequently, γ) is fixed as a proportion of the non-rentiers' wealth[15]. The higher the wealth, the bigger the loss from paying rents. The costs of rent seeking, on the other hand, are modeled as a fixed cost (hereafter, θ) to obtain the increasing returns necessary to drive a wedge between the rich and poor. While it is possible that these costs also have a variable component[16], I keep costs fixed for simplicity of exposition[17]. θ also includes the cost of investing in protection from having to pay rents. In modelling rent seeking in this manner, I capture its essential attribute of regressive redistribution while maintaining features such as strategic incentives in a negative-sum game.

I abstract from production to focus on the distributive aspect of a rent-seeking society. To model rent seeking in an economy with production requires considerations such as distortion of incentives, prices, markups, etc. The advantage is to be able to capture the impact of rent-seeking activities on growth. This has been the subject of much research already and is, therefore, not discussed in this paper.

In this model, $<\theta, \gamma>$ represent the state of institutions in society, with a higher cost of rent seeking implying stronger institutions. For example, if bureaucrats, judges, and politicians are incorruptible, or if the penalty for being caught accepting bribes is prohibitively high, a rentier will have to pay a lot to influence the system in her favor, implying a higher θ and/or lower γ. Institutions also change over time but that is likely a much slower process. In the baseline model, I take up the case of a given θ as the rentier's choice, with a predetermined appropriation rate γ. In the extension, θ is chosen by the sovereign to maximize her revenue. γ is modelled as an increasing concave function of θ to represent rent-seeking capacity, i.e., bigger expenditures on rent-seeking activity should increase the capacity to appropriate. In this scenario, the sovereign chooses θ (and by extension γ), keeping in mind that a higher θ implies a U-shaped taxation curve - revenues fall before increasing. This is because a higher θ discourages rent seeking.

Inequality Traps An inequality trap refers to a system of economic, political, and social structures that lead to persistent inequality in the distribution of resources in wealth, power, and social status (Tilly, 1998). Inequality traps are similar to poverty traps in that they keep the poor from getting out of the

[15]While rentiers also usually pay an appropriation cost, their burden is smaller owing to the rents they accrue.

[16]For examples of models with rent seeking as an increasing function of ability or talent, see Murphy et al. (1991) and Murphy et al. (1993).

[17]An example of corruption that hurt the public exchequer without changing the efficient allocation of resource is the '2G Scam' in India. Bureaucrats gave away the second-generation spectrum at throwaway prices to phony companies affiliated with their relatives. But Sukhtankar (2015) shows that subsequently the spectrum was auctioned off efficiently by these companies. The only concern was the transfer of resources to private agents that should have gone to the national chequers.

cycle of poverty[18]. Poverty traps occur at an individual level while inequality traps characterize the whole distribution. In an inequality trap, the entire distribution is stable such that even the rich (the right tails of the distribution) are protected from downward mobility (Rao et al., 2006).

Bourguignon et al. (2007) define inequality traps using two conditions: first, the relative positions in the distribution be persistent across time; second, that this be a result of the features of the overall distribution. The second condition implies that the circumstances that each agent finds herself in is a direct consequence of her position in the distribution of resources. For example, if the children of the poor remain poor because they go to bad schools and the quality of schools reflects the economic status of parents, then we have an equilibrium with an inequality trap. Another example directly related to rent seeking arises in industries with government-granted licences such as patents, mining rights, spectrum allocation, multi-sided markets, and network externalities. If only wealthier firms are able to purchase the licences and grants, and access to a licence generates bigger rents, then we again have an equilibrium with an inequality trap.

In general, market power or ownership of scarce resources enables rent seekers to entrench their advantage by engaging in behavior such as lobbying for lower regulation or against redistributive policies. A redistribution of initial resources or regulation of market power breaks down the higher inequality equilibrium. Hence, an important part of formalizing an inequality trap is the existence of multiple equilibria, at least one of which does not result in the inferior inequality distribution.

Banerjee et al. (2001) describes an inequality trap in Maharashtra's sugar cooperatives due to rent-seeking behavior by larger farmers who exert disproportionate control within the cooperatives. They show that cooperatives that have relatively more homogeneous land holdings are less susceptible to exploitative sugarcane pricing. In more heterogeneous cooperatives, large farmers are able to depress sugarcane prices to inefficiently low levels and siphon off the excess retained earnings.

B Pareto Properties

- Support: $w \in [\underline{w}, \infty)$, where $\underline{w} \geq 1$
- Pdf: $f(w) = \frac{\alpha \underline{w}^{\alpha}}{w^{\alpha+1}}$, for $w \geq \underline{w}$
- Cdf: $F(w) = 1 - (\frac{\underline{w}}{w})^{\alpha}$ for $w \geq \underline{w}$
- Mean: $w^m = \frac{\alpha \underline{w}}{\alpha - 1}$, for $\alpha > 1$

[18]For recent reviews of the literature on poverty traps, see Barrett et al. (2016) and Kraay & McKenzie (2014). For environment and education induced poverty traps, see Ikefuji & Horii (2007) and Horii & Sasaki (2012)

- Median: $w^{med} = \underline{w}(2)^{\frac{1}{\alpha}}$
- Gini coefficient: $G(\alpha) = \frac{1}{2\alpha - 1}$, for $\alpha > 1$

The simple Pareto Distribution is defined by two parameters: the shape parameter α, and the location parameter $\underline{w}$. We restrict the analysis to $\alpha > 1$ because mean wealth $w^m \to \infty$ if $0 < \alpha < 1$ and $\underline{w} \geq 1$. With more inequality (i.e., a lower α), there are more draws of w in the right tail. Both the mean (w^m) and the median (w^{med}) increase as a result. One way to interpret the decrease in α and the subsequent increase in inequality is that, relative to the poorest person's wealth, the wealth of the rich increases (if the poorest person's wealth is the normalized lower bound of the wealth distribution, $\underline{w}$).

C Proofs

C.1 Expected Non-Rentier Wealth K

To derive the expected non-rentier wealth in equation 1, substitute the Pareto $f(w) = \alpha \frac{\underline{w}^\alpha}{w^{\alpha+1}}$ into $K(w'|f(w)) = \int_{\underline{w}}^{w'} w f(w) dw$:

$$
\begin{aligned}
K(w'|\underline{w}, \alpha) &= \int_{\underline{w}}^{w'} w\alpha \frac{\underline{w}^\alpha}{w^{\alpha+1}} dw \\
&= \left(\frac{\alpha \underline{w}}{\alpha - 1}\right) \left[\frac{w'^{\alpha-1} - \underline{w}^{\alpha-1}}{w'^{\alpha-1}}\right] \\
&= w^m \left[1 - \left(\frac{\underline{w}}{w'}\right)^{\alpha-1}\right]
\end{aligned}
$$

where w^m is the mean of the distribution. $K(w'|\underline{w}, \alpha)$ is never greater than the mean wealth of this economy since $\underline{w} \leq w'$ for all w' by construction, including the equilibrium wealth, w^*. Same follows for $\gamma K^* < w^m$. K can also be expressed as a function of n by using the equilibrium relationship between n and w, i.e., $n^* = 1 - F(w^*)$ or $w^* = \underline{w} n^{*-\frac{1}{\alpha}}$

$$
K(n^*|\underline{w}, \alpha) = w^m \left[1 - n^{*\frac{\alpha-1}{\alpha}}\right]
$$

For $n = 0$, $K = w^m$, i.e., when there are no rentiers, the expected wealth of the non-rentiers is the same as that of the whole distribution.

C.2 Equilibrium closed form solutions of w^*, n^* and K^*

Substituting $K(w|\underline{w},\alpha) = w^m\left[1-\left(\frac{\underline{w}}{w}\right)^{\alpha-1}\right]$ and $n = 1-F(w)$ into $\frac{\theta}{\gamma} = w^*n^* + K^*$, we get

$$\frac{\theta}{\gamma} = w^*\left(\frac{\underline{w}}{w^*}\right)^{\alpha} + w^m\left[1-\left(\frac{\underline{w}}{w^*}\right)^{\alpha-1}\right]$$

Rearranging the terms, we get the interior solution

$$w^* = \underline{w}\left[\frac{w^m-\underline{w}}{w^m-\frac{\theta}{\gamma}}\right]^{\frac{1}{\alpha-1}}$$

To fully specify the solution:

$$w^* = \begin{cases} \underline{w}, & \text{if } \frac{\theta}{\gamma} \leq \underline{w} \\ \underline{w}\left[\frac{w^m-\underline{w}}{w^m-\frac{\theta}{\gamma}}\right]^{\frac{1}{\alpha-1}}, & \text{if } \underline{w} < \frac{\theta}{\gamma} < w^m \\ \infty, & \text{if } \frac{\theta}{\gamma} \geq w^m \end{cases} \tag{16}$$

Let $x = \left[\frac{w^m-\frac{\theta}{\gamma}}{w^m-\underline{w}}\right]$. Then w^* can be expressed as $w^* = \underline{w}x^{-\frac{1}{\alpha-1}}$

An alternative expression for x can be obtained by collecting either the α term:

$$x = \frac{\theta}{\gamma\underline{w}} - \left(\frac{\theta}{\gamma\underline{w}}-1\right)\alpha$$

or the $\frac{\theta}{\gamma}$ term:

$$x = \alpha - \left(\frac{\alpha-1}{\underline{w}}\right)\frac{\theta}{\gamma}$$

Substitute w^* from (16) into $n^* = 1-F(w^*)$:

$$\begin{aligned} n^* &= x^{\frac{\alpha}{\alpha-1}} \\ &= \left[\frac{w^m-\frac{\theta}{\gamma}}{w^m-\underline{w}}\right]^{\frac{\alpha}{\alpha-1}} \end{aligned}$$

Specifying n^* fully, we have

$$n^* = \begin{cases} 0, & \text{if } \frac{\theta}{\gamma} \geq w^m \\ \left[\frac{w^m-\frac{\theta}{\gamma}}{w^m-\underline{w}}\right]^{\frac{\alpha}{\alpha-1}}, & \text{if } \underline{w} < \frac{\theta}{\gamma} < w^m \\ 1, & \text{if } \frac{\theta}{\gamma} \leq \underline{w} \end{cases} \tag{17}$$

Substitute equation (17) into $K^* = w^m\left[1-n^{*\frac{\alpha-1}{\alpha}}\right]$ and simplify to get

$$K^* = \begin{cases} w^m, & \text{if } \frac{\theta}{\gamma} \geq w^m \\ \alpha\left(\frac{\theta}{\gamma} - \underline{w}\right), & \text{if } \underline{w} < \frac{\theta}{\gamma} < w^m \\ 0, & \text{if } \frac{\theta}{\gamma} \leq \underline{w} \end{cases} \tag{18}$$

There is an linear inverse relationship between K^* and inequality (α). K^* also falls with the distance between the effective cost of rent seeking and the wealth lower bound. A smaller difference between cost and lowest wealth means a larger fraction of rentiers to divide the pie, along with a smaller fraction of non-rentiers, thus decreasing the size of the pie K^*.

Corner solutions stability check

The no-rent seeking equilibrium is stable if $U_i^{NR} > U_i^R$ for all i. When $n^* = 0$, $U_i^{NR} = w_i$ and $U_i^R = w_i - \theta + \gamma K$. For $\frac{\theta}{\gamma} > w^m$ (the parametric condition that gives $n^* = 0$), it is also true that $\frac{\theta}{\gamma} > w^m > K$ (as noted in the derivation of K in section C.1. Therefore, $\theta > \gamma K$ which implies that $U_i^{NR} > U_i^R$ for all i. Hence, the corner solution with no rentiers is stable.

The full-rent seeking equilibrium is stable if $U_i^{NR} < U_i^R$ for all i. When $n^* = 1$, $U_i^{NR} = w_i(1-\gamma)$ and $U_i^R = w_i - \theta$. $U_i^R > U_i^{NR}$ if $\theta < w_i\gamma$ or $\frac{\theta}{\gamma} < w_i$ for all i. Since $\frac{\theta}{\gamma} < \underline{w}$ is the parametric condition to obtain $n^* = 1$, the effective cost of rent-seeking is less than all possible values that wealth can take. Hence proved.

C.3 Comparative Statics

Taking logs of both sides in $n^* = x(\alpha)^{\frac{\alpha}{\alpha-1}}$, where $x(\alpha) = \frac{\theta}{\gamma\underline{w}} - \left(\frac{\theta}{\gamma\underline{w}} - 1\right)\alpha$, we have

$$ln(n^*) = \frac{\alpha}{\alpha - 1} lnx(\alpha)$$

Differentiating both sides with respect to α and suppressing $x(\alpha)$:

$$\begin{aligned} \frac{1}{n^*}\frac{dn^*}{d\alpha} &= lnx\frac{d\left(\frac{\alpha}{\alpha-1}\right)}{d\alpha} + \left(\frac{\alpha}{\alpha-1}\right)\frac{1}{x}\frac{dx}{d\alpha} \\ \Rightarrow \frac{dn^*}{d\alpha} &= \frac{n^*}{(\alpha-1)^2}\left[-lnx - \alpha(\alpha-1)\frac{\left(\frac{\theta}{\gamma\underline{w}} - 1\right)}{x}\right] \\ &= -\frac{n^*}{(\alpha-1)^2}\left[lnx + \alpha\left(\frac{1}{x} - 1\right)\right] \\ &= \frac{n^*}{(\alpha-1)^2}\left[ln\frac{1}{x} + \alpha\left(1 - \frac{1}{x}\right)\right] \end{aligned}$$

Negative Sum game

Net wealth (v^m) = wealth of non-rentiers + wealth of rentiers

$$\begin{aligned} v^m &= \int_{\underline{w}}^{w^*} w(1-\gamma n^*)dF(w) + \int_{w^*}^{\infty} (w-\theta+\gamma K^*)dF(w) \\ &= w^m - \gamma n^* K^* - (\theta - \gamma K^*)n^* \\ &= w^m - \theta n^* \end{aligned}$$

D Rent Seeking increases Inequality: Exogenous θ

From the last section, we know that both rentiers and non-rentiers are worse off than before, i.e. both lose some part of their wealth net of gains in the rent-seeking process. The normalized total (mean) wealth of society falls from w^m to $w^m - n^*\theta$. In this section, I check how rent-seeking activity transforms the wealth distribution.

One way is to parametrically see that happens to the wealth random variable post rent seeking.[19] If the ex-post random variable is a linear transformation of the ex-ante random variable, then the Pareto distribution's properties are preserved. We may then compare the pre- and post distributions' Pareto properties to comment on the effect on inequality.

Outline of proof:

1. The wealth of both rentiers and non-rentiers is a linear transformation of the initial draw of wealth. Rentiers' wealth is $w - \theta + \gamma K^*$. Non-rentiers' wealth is $(1-\gamma n^*)w$.

2. Any linear transformation of a random variable preserves the distribution. Therefore, we can construct the ex-post distribution as a joint of two different Pareto distributions.

3. Once the complete ex-post distribution is constructed, I can compute the new Lorenz curves as a function of the new cdf's.

4. Integrating the two Lorenz curves gives the Gini coefficient of the new distribution. The Gini of the old distribution is $\frac{1}{2\alpha-1}$. Comparing the two sheds light on the size of inequality relative to the initial.

[19]The other way to compare the pre and post distributions would be to simulate and see if the ex-post distribution resembles a known distribution, and compare its properties with the initial Pareto distribution. If the ex-post distribution were also a Pareto, we could compare the parameters (location and shape) of the two distributions. The location clearly shifts to the left. It'd be of more interest to know what happens to the inequality (shape) parameter.

D.1 Two Linear Transformations of Wealth

Non-rentiers' wealth: Non-rentiers pay a fraction $(n^*\gamma)$ of their initial wealth in equilibrium. Their final take home wealth is then $(1 - n^*\gamma)w$. Let V denote the ex-post wealth and v the random variable, just as W denotes initial wealth and w the random variable. Thus, $v = (1 - n^*\gamma)w$ for $w \in [\underline{w}, w^*]$. For simplicity, let us use $T = (1 - n^*\gamma)$ to denote this transformation. So, $V = TW$ for $w \in [\underline{w}, w^*]$.

Rentiers' wealth:: Rentiers pay θ and gain γK^* in return. Their net wealth is therefore $w - \theta + \gamma K^*$. Let us use $c = \theta - \gamma K^*$ to denote this linear transformation. Therefore, the rentiers' wealth can be denoted as $V = W - c$ for $w \in [w^*, \infty)$.

There is continuity at w^* by definition. As in, the ex-post distribution has $Tw^* = w^* - c$ since the equilibrium (w^*) was computed using this as a rule.

Thus, we can integrate this information and write the ex-post random variable of wealth (V) as:

$$V = \begin{cases} TW, & \text{if } \underline{w} < w < w^* \\ W - c, & \text{if } w^* < w. \end{cases} \tag{19}$$

The new support of the distribution is $v \in [T\underline{w}, \infty)$.

D.2 General rule for transformation of Pareto variable

Let $V = a + hW$ where $a \in \Re$ and $h > 0$. Let

$$pdf_W(w) = \frac{\alpha \underline{w}^\alpha}{w^{\alpha+1}}$$

and

$$cdf_W(w) = 1 - \left(\frac{\underline{w}}{w}\right)^\alpha$$

.

Then, the distribution of the linear transformed variable V can be written as follows:

$$\begin{aligned} cdf_V(w) &= P\{V \leq w\} \\ &= P\{a + hW \leq w\} \\ &= P\{W \leq \frac{w-a}{h}\} \\ &= P_W\left(\frac{w-a}{h}\right) \\ &= cdf_W\left(\frac{w-a}{h}\right) \\ &= 1 - \left(\frac{h\underline{w}}{w-a}\right)^\alpha, \text{ if } w > a + h\underline{w} \end{aligned}$$

Therefore,

$$\boxed{cdf_V(w) = 1 - \left(\frac{h\underline{w}}{w-a}\right)^{\alpha}, \text{ for } w > a + h\underline{w}} \tag{20}$$

The corresponding pdf will be

$$pdf_V(w) = \frac{\alpha(h\underline{w})^{\alpha}}{(w-a)^{\alpha+1}} \tag{21}$$

For non-rentiers ($\underline{w} < w \leq w^*$), $V_{NR} = TW$, implying $h_{NR} = T$ and $a_{NR} = 0$. For rentiers ($w \geq w^*$), $V_R = W - c$, implying $a_R = -c$ and $h_R = 0$.

D.3 Corresponding Pareto CDF

Let F be the initial Pareto distribution of wealth, $F = 1 - (\frac{\underline{w}}{w})^{\alpha}$. Let $F_{NR}(w)$ and $F_R(w)$ be the ex-post non-rentiers' and rentiers' wealth distributions respectively, as a function of initial wealth. Then,

$$f_V(w) = \begin{cases} \frac{\alpha(T\underline{w})^{\alpha}}{w^{\alpha+1}}, & \text{if } \underline{w} \leq w \leq w^* \\ \frac{\alpha\underline{w}^{\alpha}}{(w+c)^{\alpha+1}}, & \text{if } w^* < w. \end{cases} \tag{22}$$

and

$$F_V(w) = \begin{cases} 1 - \left(\frac{T\underline{w}}{w}\right)^{\alpha}, & \text{if } \underline{w} \leq w \leq w^* \\ 1 - \left(\frac{\underline{w}}{w+c}\right)^{\alpha}, & \text{if } w^* < w. \end{cases} \tag{23}$$

The area under f_V over $[T\underline{w}, Tw^*]$ will be the same as the area under the f_W curve over $[\underline{w}, w^*]$, i.e. $1 - n^*$. Alternatively,

$$F_W(w^*) = F_V(Tw^*) = 1 - n^* \tag{24}$$

Correspondingly, the inverse cdf will be

$$v(F) = \begin{cases} T\underline{w}(1-F)^{-\frac{1}{\alpha}}, & \text{if } 0 \leq F \leq (1-n^*) \\ \underline{w}(1-F)^{-\frac{1}{\alpha}} - c, & \text{if } (1-n^*) < F \leq 1. \end{cases} \tag{25}$$

where T:

$$T = 1 - \gamma n = 1 - \gamma x^{\frac{\alpha}{\alpha-1}}$$

and c:

$$c = \theta - \gamma K = \theta - \gamma w^m (1 - n^{\frac{\alpha-1}{\alpha}})$$

Let $v_{NR}(F)$ denote the inverse cdf for non-rentiers and $v_R(F)$ for rentiers. The pdf of v is discontinuous at Tw^*, while the cdf is continuous but not differentiable.

D.4 Lorenz Curve

The Lorenz curve is a function of the cdf F, and maps the ratio of cumulative wealth upto F and total wealth under the distribution. Specifically,

$$L(F) = \frac{\int_0^F v(F')dF'}{\int_0^1 v(F')dF'}$$

where $v(F')$ is the inverse cdf of F. Lorenz curve values range is given by $[0,1]$ just like that for F. If wealth is equally distributed, then half the population will have exactly half the wealth of the economy. Thus, the Lorenz curve value corresponding to $F = 0.5$ will be 0.5. We can make similar assessments for other values of $F \in [0,1]$. Hence, the 45° line plotting $L(F)$ represents the Lorenz curve corresponding to an equal wealth distribution. For unequal distributions, the Lorenz curve will be below from the 45° line.

In the context of a Pareto distribution, as used throughout this paper, the Lorenz curve can be computed to be

$$L(F) = [1 - (1-F)^{\frac{\alpha-1}{\alpha}}] \tag{26}$$

Thus $\alpha = 1 \Rightarrow L(F) = 0$ should denote high inequality and $\alpha \to \infty \Rightarrow L(F) = F$ should denote perfect equality. The $\alpha \to \infty$ Pareto's Lorenz curve lies on the 45° line. As the inequality parameter (α's) value decreases, inequality increases, and the Lorenz curve moves away from the 45° line.

For the purpose of this proof, we need to compute the following Lorenz function:

$$L(F) = \begin{cases} \frac{\int_0^F v_{NR}(F')dF'}{\int_0^1 v(F')dF'} & \text{, if } 0 \le F \le (1-n^*) \\ \frac{\int_0^{(1-n^*)} v_{NR}(F')dF' + \int_{(1-n^*)}^F v_R(F')dF'}{\int_0^1 v(F')dF'} & \text{, if } (1-n^*) < F \le 1. \end{cases}$$

The denominator of the Lorenz function is the normalized total wealth of society. For the ex-post distribution:

$$den\{L(F)\} = \int_0^1 v(F')dF' = w^m - \theta n^*$$

As was evident from the previous section, the total ex-post wealth (normalized) falls from w^m to $w^m - \theta n^*$. The new total normalized wealth is positive for interior solutions, i.e. $w^m > \frac{\theta}{\gamma}$ because that implies $w^m > \gamma w^m > \theta > n^*\theta$, since γ and n^* are weakly less than unity.

For the numerator for non-rentiers:

$$
\begin{aligned}
num_{NR}\{L(F)\} &= \int_0^F v_{NR}(F')dF' \\
&= \int_0^F \{T\underline{w}(1-F')^{-\frac{1}{\alpha}}\}dF' \\
&= T\underline{w}\int_0^F (1-F')^{-\frac{1}{\alpha}}dF'
\end{aligned}
$$

Using chain rule of integration, let $y = 1 - F'$. Then, $dF' = -dy$. Suitably changing the limits of integration, we get

$$
\begin{aligned}
num_{NR}\{L(F)\} &= -T\underline{w}\int_1^{1-F} y^{-\frac{1}{\alpha}}dy \\
&= Tw^m[1-(1-F)^{\frac{\alpha-1}{\alpha}}]
\end{aligned}
$$

The Lorenz curve for the non-rentiers' part of the population is given by

$$
L_{NR}(F) = \frac{Tw^m[1-(1-F)^{\frac{\alpha-1}{\alpha}}]}{w^m - \theta n^*} \tag{27}
$$

Ceteris paribus, for a given value of F, a higher n^* implies a lower Lorenz curve (further away from the equality diagonal).

The numerator of the Lorenz curve for the rentiers can be computed from eqn (25) using:

$$
\begin{aligned}
num_R\{L(F)\} &= \int_0^{(1-n^*)} v_{NR}(F')dF' + \int_{(1-n^*)}^F v_R(F')dF' \\
&= \int_0^{(1-n^*)} \{T\underline{w}(1-F')^{-\frac{1}{\alpha}}\}dF' + \int_{(1-n^*)}^F \{\underline{w}(1-F')^{-\frac{1}{\alpha}} - c\}dF'
\end{aligned}
$$

For the first term, we can use the derivation for non-rentiers sum of wealth and plug $(1-n)$ in place of F. We break down the second term into two parts and use the chain rule for the first part of the two.

$$
\begin{aligned}
num_R\{L(F)\} &= Tw^m[1-(1-(1-n))^{\frac{\alpha-1}{\alpha}}] + \int_{(1-n^*)}^F \{\underline{w}(1-F')^{-\frac{1}{\alpha}} - c\}dF' \\
&= Tw^m[1-n^{\frac{\alpha-1}{\alpha}}] - w^m[(1-F)^{\frac{\alpha-1}{\alpha}} - n^{\frac{\alpha-1}{\alpha}}] - c[F-1+n]
\end{aligned}
$$

The final Lorenz function that we will use to compute Gini coefficients is

$$
L(F(n)) = \begin{cases} \frac{T(n)w^m\{1-(1-F)^{\frac{\alpha-1}{\alpha}}\}}{w^m-\theta n} & \text{, if } 0 \le F \le (1-n) \\ \frac{T(n)w^m(1-n^{\frac{\alpha-1}{\alpha}})+c(n)(1-n-F)+w^m[n^{\frac{\alpha-1}{\alpha}}-(1-F)^{\frac{\alpha-1}{\alpha}}]}{w^m-\theta n} & \text{, if } (1-n) < F \le 1. \end{cases} \tag{28}
$$

Like the cdf of v, the Lorenz function is also continuous at $F' = 1 - n^*$.

$L(F')$ is below $L(F)$

For $F' = 1 - n^*$, cumulative wealth as a ratio of total wealth is

$$L(F') = \frac{Tw^m(1 - n^{\frac{\alpha-1}{\alpha}})}{w^m - \theta n^*}$$

To see that $L(F) > L(F')$ at $1-n^*$, we need to check whether $(1-n^{\frac{\alpha-1}{\alpha}}) > \frac{Tw^m(1-n^{\frac{\alpha-1}{\alpha}})}{w^m-\theta n^*}$. Rearranging the terms, we get $w^m > \frac{\theta}{\gamma}$, which is true for all interior values of n^*.

D.5 Lorenz dominance and the Gini coefficient

The above discussion on the Lorenz curve brings us to the last step here: prove that the ex-post Lorenz curve lies below the initial Lorenz curve. Another way to show the above, which may be modelled using computation, is comparing the respective Gini coefficients. The Gini is the ratio of the area between the Lorenz curve and the 45° line to the total area of triangle under the Lorenz curve. When there is perfect equality, the Lorenz lies on the 45° line, and Gini coefficient is 0. Higher inequality Lorenz curves lie below lower inequality ones.

I start by showing that the ex-post Lorenz curve lies below the initial Lorenz curve at all points. I do this in two parts, one for the non-rentier part and one for the rentier part of the ex-post Lorenz curve. The initial Lorenz curve has the same shape all over.

Part 1: Non-rentiers

$$L(F') < L(F)$$
$$\frac{Tw^m\{1-(1-F)^{\frac{\alpha-1}{\alpha}}\}}{w^m - \theta n^*} < 1-(1-F)^{\frac{\alpha-1}{\alpha}}$$
$$\frac{\theta}{\gamma} < w^m$$

from the last section, and which is true for all interior values of n^*.
Part 2: Rentiers

$$L(F') < L(F)$$
$$\frac{Tw^m(1-x) + c(1-n^*) - cF + w^m[x-(1-F)^{\frac{\alpha-1}{\alpha}}]}{w^m - \theta n^*} < 1-(1-F)^{\frac{\alpha-1}{\alpha}}$$

Rearranging the terms, we get

$$(1-F)^{\frac{1}{\alpha}} < \frac{\theta n^*}{(\theta - \gamma w^m(1-x))}$$

which when simplified gives us:

$$(1-F) < \left(\frac{\theta}{\gamma \underline{w}}\right)^{\alpha} n^*$$

Therefore, if $(1-F) < \left(\frac{\theta}{\gamma \underline{w}}\right)^{\alpha} n^*$, then the ex-post Lorenz is lower than the initial Lorenz everywhere. For the rentiers' part of the Lorenz curve, the largest value that the LHS can take is when F is smallest, i.e., $F = 1 - n^*$. Proving the above inequality for the largest LHS will prove it for the remaining part of the curve since the RHS is fixed.

$$n^* < \left(\frac{\theta}{\gamma \underline{w}}\right)^{\alpha} n^* \Rightarrow \underline{w} < \frac{\theta}{\gamma}$$

which is true for all interior values of n^*. Hence proved.

D.5.1 Gini

I also show that the area under the $L(F)$ curve is greater than area under $L(F')$ curve, i.e. the Gini coefficient is higher for the ex-post distribution.

Let the area under $L(F) = [1 - (1-F)^{\frac{\alpha-1}{\alpha}}]$ curve be denoted by B:

$$\begin{aligned} B = Area\{L(F)\} &= \int_0^1 L(F)dF \\ &= \int_0^1 [1 - (1-F)^{\frac{\alpha-1}{\alpha}}]dF \\ &= \frac{\alpha-1}{2\alpha-1} \end{aligned}$$

Corresponding Gini coefficient is

$$\begin{aligned} Gini(F) = 1 - 2B &= 1 - 2\frac{\alpha-1}{2\alpha-1} \\ &= \frac{1}{2\alpha-1} \end{aligned}$$

which is a known relationship for the simple Pareto distribution used throughout this paper.

The $L(F')$ curve is given by:

$$L(F') = \begin{cases} \frac{Tw^m\{1-(1-F')^{\frac{\alpha-1}{\alpha}}\}}{w^m - \theta n} & \text{, if } 0 \leq F \leq (1-n) \\ \frac{Tw^m(1-n^{\frac{\alpha-1}{\alpha}}) + c(1-n-F') + w^m[n^{\frac{\alpha-1}{\alpha}} - (1-F')^{\frac{\alpha-1}{\alpha}}]}{w^m - \theta n} & \text{, if } (1-n) < F \leq 1. \end{cases}$$

Area under the $L(F')$ curve can be computed as:

$$\begin{aligned}B' &= Area\{L(F')\}\\ &= \underbrace{\int_0^{1-n} L(F')dF'}_{B_1'/(w^m-\theta n)} + \underbrace{\int_{1-n}^{1} L(F')dF'}_{B_2'/(w^m-\theta n)}\end{aligned}$$

Now,

$$\begin{aligned}B_1' &= \int_0^{1-n} Tw^m\{1-(1-F')^{\frac{\alpha-1}{\alpha}}\}dF'\\ &= Tw^m\left((1-n) - \frac{\alpha}{2\alpha-1}[1-n^{\frac{2\alpha-1}{\alpha}}]\right)\end{aligned}$$

and

$$\begin{aligned}B_2' = \int_{1-n}^{1} &\left(Tw^m(1-n^{\frac{2\alpha-1}{\alpha}}) + c(1-n) - cF' + w^m[n^{\frac{2\alpha-1}{\alpha}} - (1-F')^{\frac{\alpha-1}{\alpha}}]\right)dF'\\ &= n\{w^m(1-\frac{\alpha}{2\alpha-1}n^{\frac{2\alpha-1}{\alpha}}) - \frac{n}{2}\left(\theta+\gamma w^m(1-n^{\frac{2\alpha-1}{\alpha}})\right)\}\end{aligned}$$

Lets compare areas under the Lorenz curve for F and F' separately for the domains $[0, 1-n]$ and $[1-n, 1]$.

Let A_{NR} be the area under the $L(F)$ curve and $B_{NR} = \frac{B_1'}{w^m-\theta n^*}$ be the area under the $L(F')$ curve respectively from $[0, 1-n]$.

$$\begin{aligned}A_{NR} &= \int_0^{1-n} L(F)dF\\ &= (1-n) - \frac{\alpha}{2\alpha-1}[1-n^{\frac{2\alpha-1}{\alpha}}]\end{aligned}$$

Now there would be greater inequality in the ex-post distribution if $A_{NR} > B_{NR}$, i.e.

$$\begin{aligned}(1-n) - \frac{\alpha}{2\alpha-1}[1-n^{\frac{2\alpha-1}{\alpha}}] &> \frac{Tw^m\left((1-n)-\frac{\alpha}{2\alpha-1}[1-n^{\frac{2\alpha-1}{\alpha}}]\right)}{w^m-n\theta}\\ &\Rightarrow w^m - n\theta > Tw^m\end{aligned}$$

This holds because the numerator is a positive quantity being area under a curve.

$$\begin{aligned}w^m(1-T) &> n\theta\\ \Rightarrow w^m &> \frac{\theta}{\gamma}\end{aligned}$$

which is true for interior solution of n^*.

Similarly, let A_R be the area under the $L(F)$ curve and B_R be the area under $L(F')$ curve respectively from $[1-n^*, 1]$.

$$\begin{aligned} A_R &= \int_{1-n}^{1} L(F)dF &(29)\\ &= n\left(1 - \frac{\alpha}{2\alpha-1} n^{\frac{\alpha-1}{\alpha}}\right) &(30)\end{aligned}$$

Now there would be greater inequality in the ex-post distribution if $A_R > B_R$, i.e.

$$\begin{aligned} n\left(1 - \frac{\alpha}{2\alpha-1} n^{\frac{\alpha-1}{\alpha}}\right) &> \frac{\{w^m n(1 - \frac{\alpha}{2\alpha-1} n^{\frac{\alpha-1}{\alpha}}) - \frac{n^2}{2}\left(\theta + \gamma w^m (1 - n^{\frac{\alpha-1}{\alpha}})\right)\}}{w^m - n\theta} \\ \Rightarrow -n\theta A_R &> -\frac{n^2}{2}\left(\theta + \gamma w^m (1 - n^{\frac{\alpha-1}{\alpha}})\right) \\ \Rightarrow \theta A_R &< \frac{n}{2}\left(\theta + \gamma w^m (1 - n^{\frac{\alpha-1}{\alpha}})\right)\end{aligned}$$

Replacing the value of A_R from equation 29,

$$\begin{aligned} \Rightarrow \theta\left(1 - \frac{\alpha}{2\alpha-1} n^{\frac{\alpha-1}{\alpha}}\right) &< \frac{n}{2}\left(\theta + \gamma w^m (1 - n^{\frac{\alpha-1}{\alpha}})\right) \\ \Rightarrow \underline{w}(1 - \frac{1}{2\alpha}) &< \underline{w} < \frac{\theta}{\gamma}\end{aligned}$$

which holds for all interior solutions. Hence the inequality holds and we can say that $A_R > B_R$. Since both A_{NR} and A_R are respectively greater than B_{NR} and B_R, the following is also true:

$$\begin{aligned} A_{NR} + A_R &> B_{NR} + B_R \\ \Rightarrow 1/2 - (A_{NR} + A_R) &< 1/2 - (B_{NR} + B_R) \\ \Rightarrow Gini(F) &< Gini(F')\end{aligned}$$

Hence, inequality increases in the ex-post distribution.

E The n and G fixed point for equilibrium distribution of wealth

E.1 $n^*(G)$

$G = \frac{1}{2\alpha-1}$ implies that $\alpha = \frac{1+G}{2G}$. For interior values of n^*,

$$
\begin{aligned}
n^* &= \left[\frac{w^m - \frac{\theta}{\gamma}}{w^m - \underline{w}}\right]^{\frac{\alpha}{\alpha-1}} \\
&= \left[\frac{\frac{\alpha}{\alpha-1}\underline{w} - \frac{\theta}{\gamma}}{\frac{\alpha}{\alpha-1}\underline{w} - \underline{w}}\right]^{\frac{\alpha}{\alpha-1}} \\
&= \left[\frac{\frac{\alpha}{\alpha-1} - \frac{\theta}{\gamma\underline{w}}}{\frac{\alpha}{\alpha-1} - 1}\right]^{\frac{\alpha}{\alpha-1}} \\
&= \left[\frac{\frac{\alpha}{\alpha-1} - \frac{\theta}{\gamma\underline{w}}}{\frac{1}{\alpha-1}}\right]^{\frac{\alpha}{\alpha-1}} \\
&= \left[\alpha - \frac{(\alpha-1)\theta}{\gamma\underline{w}}\right]^{\frac{\alpha}{\alpha-1}} \\
&= \left[\frac{1+G}{2G} - \frac{(\frac{1+G}{2G} - 1)\theta}{\gamma\underline{w}}\right]^{\frac{1+G}{1-G}} \\
&= \left[\frac{1+G}{2G} - (\frac{1-G}{2G})\frac{\theta}{\gamma\underline{w}}\right]^{\frac{1+G}{1-G}} \\
&= \left[\frac{(1+G) - \frac{\theta}{\gamma\underline{w}}(1-G)}{2G}\right]^{\frac{1+G}{1-G}}
\end{aligned}
$$

Therefore,

$$
n^* = \left[\left(\frac{1 + \frac{\theta}{\gamma\underline{w}}}{2}\right) - \left(\frac{\frac{\theta}{\gamma\underline{w}} - 1}{2G}\right)\right]^{\frac{1+G}{1-G}} \tag{31}
$$

As G $\uparrow$s, the term in [] $\uparrow$s. But the power term $(\frac{1+G}{1-G})$ also $\uparrow$s, which works to reduce the value of n^* since $[] < 1$. Overall, the [] term wins, and $\frac{\partial n^*}{\partial G} > 0$. As G $\uparrow$s (the Lorenz curve shifts down), w^* moves to the left (more rent-seeking, n^* $\uparrow$s). More inequality leads to more agents choosing to be rentiers.

E.2 $G^*(n)$

Now we'll show that there exists a unique G for every n. That is, when rent seeking increases (w^* or n^* changes), it makes the Gini change as well (wealth distribution more unequal). For this we need to show that the a higher n^* corresponds to a lower Lorenz curve and a higher Gini coefficient.

We know that for a simple Pareto distribution, Lorenz curve is given by $L(F) = [1 - (1-F)^{\frac{\alpha-1}{\alpha}}]$ and $G = \frac{1}{2\alpha-1}$. For the economy with redistribution, we need the inverse cdf(n) $\rightarrow$ Lorenz(n) $\rightarrow$ Gini(n).

The transformed inverse cdf from equation (25):

$$v(F) = \begin{cases} T(n)\underline{w}(1-F_v)^{-\frac{1}{\alpha}}, & \text{if } 0 \leq F \leq (1-n^*) \\ \underline{w}(1-F_v)^{-\frac{1}{\alpha}} - c, & \text{if } (1-n^*) < F \leq 1. \end{cases}$$

where $T = 1 - \gamma n^*$, and $c = \theta - \gamma K^* = \theta - \gamma w^m(1 - n^{\frac{\alpha-1}{\alpha}})$.

The new Lorenz curve is given by equations (28):

$$L(F(n)) = \begin{cases} \frac{T(n)w^m\{1-(1-F)^{\frac{\alpha-1}{\alpha}}\}}{w^m-\theta n} & , \text{if } 0 \leq F \leq (1-n) \\ \frac{T(n)w^m(1-n^{\frac{\alpha-1}{\alpha}})+c(n)(1-n-F)+w^m[n^{\frac{\alpha-1}{\alpha}}-(1-F)^{\frac{\alpha-1}{\alpha}}]}{w^m-\theta n} & , \text{if } (1-n) < F \leq 1. \end{cases}$$

Integrating Lorenz to obtain Gini

Continuity Check 1. If $F = 1 - n$,

$$L(NR) = \frac{Tw^m(1-n^{\frac{\alpha-1}{\alpha}})}{w^m - \theta n}$$

$$\begin{aligned} L(R) &= \frac{T(n)w^m(1-n^{\frac{\alpha-1}{\alpha}}) + c(n)(0) + w^m[n^{\frac{\alpha-1}{\alpha}} - (n)^{\frac{\alpha-1}{\alpha}}]}{w^m - \theta n} \\ &= \frac{T(n)w^m(1-n^{\frac{\alpha-1}{\alpha}})}{w^m - \theta n} \\ &= L(NR) \end{aligned}$$

The Lorenz curve is continuous at $F = 1 - n$ and everywhere else, and is bounded between 0 and 1. Thus the integral must exist and be finite. $F : n[0,1] \to G[0,1]$.

Let $g(n)$ denote Gini as a function of n (not n^*). Then,

1. for every n, there must be a unique Gini coefficient.
2. the Lorenz for a higher n will be lower everywhere, so there can't be 2 Gini's associated with an 'n'.
3. The area $A(L)$ (in the fig 1.1 in the notebook) is the Gini associated with the Lorenz curve L. $A(L(n)) = g(L(N))$.
4. We know the $L(n)$ function now. and that there's only 1 L associated with an n.

From the figure 1.1 in notebook, the area under the Lorenz curve is $B = B_{NR} + B_R$.

$$B = \int_0^{(1-n)} L_{NR}(F(n))dF + \int_{(1-n)}^1 L_R(F(n))dF$$

$$G(n) = 2A(n) = 2\left(\frac{1}{2} - B(n)\right)$$

*We will check whether there is a unique L_R and L_{NR} later.

$$\begin{aligned}
B_{NR}(n) &= \int_0^{(1-n)} L_{NR}(F)dF \\
&= \int_0^{(1-n)} \frac{T(n)w^m\{1-(1-F)^{\frac{\alpha-1}{\alpha}}\}}{w^m-\theta n}dF \\
&= \frac{T(n)w^m}{w^m-\theta n}\int_0^{(1-n)}\{1-(1-F)^{\frac{\alpha-1}{\alpha}}\}dF \\
&= \frac{T(n)w^m}{w^m-\theta n}\left[\int_0^{(1-n)} 1dF - \int_0^{(1-n)}(1-F)^{\frac{\alpha-1}{\alpha}}dF\right] \\
&= \frac{T(n)w^m}{w^m-\theta n}\left[\{F\}_0^{1-n} - (-)\int_1^n y^{\frac{\alpha-1}{\alpha}}dy\right] \\
&= \frac{T(n)w^m}{w^m-\theta n}\left[(1-n)+\{\frac{y^{1+\frac{\alpha-1}{\alpha}}}{1+\frac{\alpha-1}{\alpha}}\}_1^n\right] \\
&= \frac{T(n)w^m}{w^m-\theta n}\left[(1-n)+\frac{\alpha}{2\alpha-1}(n^{\frac{2\alpha-1}{\alpha}}-1)\right]
\end{aligned}$$

Therefore,

$$B_{NR}(n) = \frac{T(n)w^m}{w^m-\theta n}\left[1-n-\frac{\alpha}{2\alpha-1}\left(1-n^{\frac{2\alpha-1}{\alpha}}\right)\right] \tag{32}$$

Similarly,

$$
\begin{aligned}
B_R(n) &= \int_{1-n}^{1} L_R(F)dF \\
&= \int_{1-n}^{1} \frac{T(n)w^m(1-n^{\frac{\alpha-1}{\alpha}}) + c(n)(1-n-F) + w^m[n^{\frac{\alpha-1}{\alpha}} - (1-F)^{\frac{\alpha-1}{\alpha}}]}{w^m - \theta n} dF \\
&= \frac{T(n)w^m(1-n^{\frac{\alpha-1}{\alpha}})}{w^m-\theta n}\int_{1-n}^{1} 1dF + \frac{c(n)}{w^m-\theta n}\{\int_{1-n}^{1}(1-n)dF - \int_{1-n}^{1} FdF\} - \frac{w^m}{w^m-\theta n}\{\int_{1-n}^{1}(1-
\\
&= \frac{T(n)w^m(1-n^{\frac{\alpha-1}{\alpha}})(n)}{w^m-\theta n} + \frac{c(n)}{w^m-\theta n}\{(1-n)\int_{1-n}^{1} 1dF - \int_{1-n}^{1} FdF\} - \frac{w^m}{w^m-\theta n}\{-\int_{n}^{0} y^{\frac{\alpha-1}{\alpha}}dy -
\\
&= \frac{T(n)w^m(1-n^{\frac{\alpha-1}{\alpha}})n}{w^m-\theta n} + \frac{c(n)}{w^m-\theta n}\{(1-n)n - \frac{[F^2]_{1-n}^{1}}{2}\} - \frac{w^m}{w^m-\theta n}\{-[\frac{y^{1+\frac{\alpha-1}{\alpha}}}{1+\frac{\alpha-1}{\alpha}}]_n^0 - n^{\frac{\alpha-1}{\alpha}}n\} \\
&= \frac{T(n)w^m(1-n^{\frac{\alpha-1}{\alpha}})n}{w^m-\theta n} + \frac{c(n)}{w^m-\theta n}\{(1-n)n - \frac{1}{2}\{1-(1-n)^2\}\} - \frac{w^m}{w^m-\theta n}\{\frac{\alpha}{2\alpha-1}(n^{\frac{2\alpha-1}{\alpha}}) - n
\\
&= \frac{1}{w^m-\theta n}\left[T(n)w^m(n-n^{\frac{2\alpha-1}{\alpha}}) + c(n)\{n-n^2-\frac{1}{2}+\frac{1}{2}+\frac{n^2}{2}-\frac{2n}{2}\} - w^m\{(\frac{2\alpha-1}{\alpha}-1)n^{\frac{2\alpha-1}{\alpha}}\}
\right. \\
&= \frac{1}{w^m-\theta n}\left[T(n)w^m(n-n^{\frac{2\alpha-1}{\alpha}}) + c(n)(-\frac{n^2}{2}) + w^m\left(\frac{\alpha-1}{2\alpha-1}\right)n^{\frac{2\alpha-1}{\alpha}}\right] \\
&= \frac{T(n)w^m(n-n^{\frac{2\alpha-1}{\alpha}}) - c(n)(\frac{n^2}{2}) + w^m n^{\frac{2\alpha-1}{\alpha}}\left(\frac{\alpha-1}{2\alpha-1}\right)}{w^m-\theta n}
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
B_R(n) &= \frac{n}{w^m-\theta n}\left[T(n)w^m(1-n^{\frac{\alpha-1}{\alpha}}) - \frac{c(n)n}{2} + w^m\left(\frac{\alpha-1}{2\alpha-1}\right)n^{\frac{\alpha-1}{\alpha}}\right] \quad (33) \\
g(n) &= 1 - 2(B_{NR}(n) + B_R(n))
\end{aligned}
$$

$$
\begin{aligned}
g(n) &= 1 - \frac{2}{w^m-\theta n}\left[T(n)w^m\{(1-n) - (\frac{\alpha}{2\alpha-1})(1-n^{\frac{2\alpha-1}{\alpha}})\} + T(n)w^m(n-n^{\frac{2\alpha-1}{\alpha}}) - \frac{c(n)n^2}{2} + w^m(
\right. \\
&= 1 - \frac{2}{w^m-\theta n}\left[T(n)w^m\{(1-\frac{\alpha}{2\alpha-1}) + \frac{\alpha}{2\alpha-1}n^{\frac{2\alpha-1}{\alpha}}\} - T(n)w^m n^{\frac{2\alpha-1}{\alpha}} - c(n)\frac{n^2}{2} + w^m(\frac{\alpha-1}{2\alpha-1})n
\right. \\
&= 1 - \frac{2}{w^m-\theta n}\left[T(n)w^m\{(\frac{\alpha-1}{2\alpha-1}) + (\frac{\alpha}{2\alpha-1}-1)n^{\frac{2\alpha-1}{\alpha}}\} + w^m(\frac{\alpha-1}{2\alpha-1})n^{\frac{2\alpha-1}{\alpha}} - c(n)\frac{n^2}{2}\right] \\
&= 1 - \frac{2}{w^m-\theta n}\left[T(n)w^m\{\frac{\alpha-1}{2\alpha-1} - (\frac{\alpha-1}{2\alpha-1})n^{\frac{2\alpha-1}{\alpha}}\} + w^m(\frac{\alpha-1}{2\alpha-1})n^{\frac{2\alpha-1}{\alpha}} - c(n)\frac{n^2}{2}\right]
\end{aligned}
$$

Therefore,

$$
g(n) = 1 - \frac{2}{w^m-\theta n}\left[T(n)w^m(frac\alpha - 12\alpha - 1)(1-n^{\frac{2\alpha-1}{\alpha}}) + w^m(\frac{\alpha-1}{2\alpha-1})n^{\frac{2\alpha-1}{\alpha}} - c(n)\frac{n^2}{2}\right] \quad (34)
$$

E.3 Fixed Point: $n^{**} = n(G(n))$

Equations (31) and (34) are two equations in two unknowns and are non-linear systems. Solving the two gives us the fixed point of the system: n^{**} and G^{**}, i.e. distribution-wide (or general) equilibrium rent seeking and inequality. $n \to G \to n$. What does $n(G(n))$ look like? Is there a unique curve? Is it compact and continuous? If $n(G(n))$ is a unique, compact, and continuous curve, then we have our fixed points $\to$ social equilibrium.

For notational sake, it is helpful to clarify that n^* is the partial equilibrium (at the agent level) in this model. Given the rent-seeking technology, agents choose to be rentiers or non-rentiers. This choice gives us $n^*(G)$. But once redistribution happens based on $n^*(G)$, the G itself changes. This change is characterized by $g(n)$. The general (or social) equilibrium in this economy will occur when $n^*(G)$ and $g(n)$ are in harmony, i.e., the fixed point. We need to know if such a fixed point exists, and if so, is it interior? If it is indeed an interior solution, we may be able to look at comparative statics of the general equilibrium.

To do so, let $g(n) = 1 - 2[\Gamma(n)]$, where, from equation (34)

$$\Gamma(n) = \frac{\left[T(n)w^m(\frac{\alpha-1}{2\alpha-1})(1 - n^{\frac{2\alpha-1}{\alpha}}) + w^m(\frac{\alpha-1}{2\alpha-1})n^{\frac{2\alpha-1}{\alpha}} - c(n)\frac{n^2}{2}\right]}{w^m - \theta n} \tag{35}$$

Then,

$$1 + G = 2(1 - \Gamma(n))$$

and

$$1 - G = 2\Gamma(n)$$

So,

$$\frac{1+G}{1-G} = \frac{1-\Gamma(n)}{\Gamma(n)} \tag{36}$$

Rewriting equation (31),

$$n = \left[\frac{\frac{1-\Gamma(n)}{\Gamma(n)} - \frac{\theta}{\gamma \underline{w}}}{\frac{1-\Gamma(n)}{\Gamma(n)} - 1}\right]^{\frac{1-\Gamma(n)}{\Gamma(n)}} = f(n) \tag{37}$$

Now, we guess and verify.

E.3.1 $n = 0$?

$$\begin{aligned}
\Gamma(n=0) &= \frac{1}{w^m}\left[T(0)w^m(\frac{\alpha-1}{2\alpha-1})(1-0)+0-0\right] \\
&= \frac{[1 * w^m(\frac{\alpha-1}{2\alpha-1})]}{w^m} \\
&= \frac{\alpha-1}{2\alpha-1}
\end{aligned}$$

$$\begin{aligned}
\frac{1-\Gamma(n=0)}{\Gamma(n=0)} &= \frac{1-\frac{\alpha-1}{2\alpha-1}}{\frac{\alpha-1}{2\alpha-1}} \\
&= \frac{\frac{\alpha}{2\alpha-1}}{\frac{\alpha-1}{2\alpha-1}} \\
&= \frac{\alpha}{\alpha-1}
\end{aligned}$$

In eqn (37), the RHS for $n = 0$ will be

$$f(n=0) = \left[\frac{\frac{\alpha}{\alpha-1}-\frac{\theta}{\gamma \underline{w}}}{\frac{\alpha}{\alpha-1}-1}\right]^{\frac{\alpha}{\alpha-1}} \tag{38}$$

$$= \left[\frac{w^m-\frac{\theta}{\gamma}}{w^m-\underline{w}}\right]^{\frac{\alpha}{\alpha-1}} \tag{39}$$

As we've seen from Section 2.4 (check this reference), $n^* = 0$ iff $w^m \leq \frac{\theta}{\gamma}$. For $w^m > \frac{\theta}{\gamma}$, $LHS(n=0) < RHS(n^* > 0)$. So $n = 0$ is a general equilibrium iff $w^m = \frac{\theta}{\gamma}$.

Proposition 9. *$n = 0$ is a fixed point iff $w^m = \frac{\theta}{\gamma}$.*

E.3.2 $n = 1$?

$$\begin{aligned}
\Gamma(n=1) &= \frac{1}{w^m-\theta}\left[(1-\gamma)w^m(\frac{\alpha-1}{2\alpha-1})(0)+w^m(\frac{\alpha-1}{2\alpha-1})(1)-c(n)/2\right] \\
&= \frac{w^m(\frac{\alpha-1}{2\alpha-1})-\theta/2}{w^m-\theta}
\end{aligned}$$

So,

$$\begin{aligned}
\frac{1-\Gamma(n=1)}{\Gamma(n=1)} &= \frac{(w^m-\theta)-(w^m(\frac{\alpha-1}{2\alpha-1})-\theta/2)}{(w^m(\frac{\alpha-1}{2\alpha-1})-\theta/2)} \\
&= \frac{w^m(\frac{\alpha}{2\alpha-1})-\theta/2}{w^m(\frac{\alpha-1}{2\alpha-1})-\theta/2}
\end{aligned}$$

In eqn (37), the RHS for $n = 1$ will be

$$f(n=1) = \left[\frac{\frac{w^m(\frac{\alpha}{2\alpha-1})-\theta/2}{w^m(\frac{\alpha-1}{2\alpha-1})-\theta/2} - \frac{\theta}{\gamma \underline{w}}}{\frac{w^m(\frac{\alpha}{2\alpha-1})-\theta/2}{w^m(\frac{\alpha-1}{2\alpha-1})-\theta/2} - 1} \right]^{\frac{w^m(\frac{\alpha}{2\alpha-1})-\theta/2}{w^m(\frac{\alpha-1}{2\alpha-1})-\theta/2}} \tag{40}$$

Proposition 10. *From the above and eqn (37), we can say that for any* $\Gamma(n)$*,* $n = 1$ *will be a fixed point iff* $\frac{\theta}{\gamma} = \underline{w}$ *when the [] term collapses to 1.*

E.3.3 $0 < n < 1$?

Now let us check for interior value of $\underline{w} < \frac{\theta}{\gamma} < w^m$.

Intermediate value theorem: Brower's Fixed Point Theorem: If $(f(X_0) - X_0)(f(X_1) - X_1) < 0$, then an interior fixed point exists. Let $X_0 \equiv n = 0$ and $X_1 \equiv n = 1$. Then, $f(n=0) = \left[\frac{w^m - \frac{\theta}{\gamma}}{w^m - \underline{w}}\right]^{\frac{\alpha}{\alpha-1}}$ from eqn (38) and $f(n=1) = \left[\frac{\frac{w^m(\frac{\alpha}{2\alpha-1})-\theta/2}{w^m(\frac{\alpha-1}{2\alpha-1})-\theta/2} - \frac{\theta}{\gamma \underline{w}}}{\frac{w^m(\frac{\alpha}{2\alpha-1})-\theta/2}{w^m(\frac{\alpha-1}{2\alpha-1})-\theta/2} - 1} \right]^{\frac{w^m(\frac{\alpha}{2\alpha-1})-\theta/2}{w^m(\frac{\alpha-1}{2\alpha-1})-\theta/2}}$ from eqn (40).

Then, $f(X_0) - X_0$ will become

$$f(n=0) - 0 = \left[\frac{w^m - \frac{\theta}{\gamma}}{w^m - \underline{w}}\right]^{\frac{\alpha}{\alpha-1}} \tag{41}$$

which is $> 0 \ \forall \ \underline{w} < \frac{\theta}{\gamma} < w^m$, and $f(X_1) - X_1$ will become

$$f(n=1) - 1 = \left[\frac{\frac{w^m(\frac{\alpha}{2\alpha-1})-\theta/2}{w^m(\frac{\alpha-1}{2\alpha-1})-\theta/2} - \frac{\theta}{\gamma \underline{w}}}{\frac{w^m(\frac{\alpha}{2\alpha-1})-\theta/2}{w^m(\frac{\alpha-1}{2\alpha-1})-\theta/2} - 1} \right]^{\frac{w^m(\frac{\alpha}{2\alpha-1})-\theta/2}{w^m(\frac{\alpha-1}{2\alpha-1})-\theta/2}} - 1 \tag{42}$$

To evaluate the RHS of eqn (42), we need only check whether $[] \gtreqless 1$ since the power term is > 1.

Proof.

$$\frac{\frac{w^m(\frac{\alpha}{2\alpha-1})-\theta/2}{w^m(\frac{\alpha-1}{2\alpha-1})-\theta/2} - \frac{\theta}{\gamma \underline{w}}}{\frac{w^m(\frac{\alpha}{2\alpha-1})-\theta/2}{w^m(\frac{\alpha-1}{2\alpha-1})-\theta/2} - 1} \gtrless 1$$

$$\frac{w^m(\frac{\alpha}{2\alpha-1})-\theta/2}{w^m(\frac{\alpha-1}{2\alpha-1})-\theta/2} - \frac{\theta}{\gamma \underline{w}} \gtrless \frac{w^m(\frac{\alpha}{2\alpha-1})-\theta/2}{w^m(\frac{\alpha-1}{2\alpha-1})-\theta/2} - 1$$

$$-\frac{\theta}{\gamma \underline{w}} \gtrless -1$$

$$\frac{\theta}{\gamma \underline{w}} \lesseqgtr 1$$

$$\frac{\theta}{\gamma} \lesseqgtr \underline{w}$$

$w^m > \frac{\theta}{\gamma} > \underline{w}$ for all interior values of n, [] < 1. Thus, $f(n = 1) - 1 < 1$. $\square$

Since $(f(X_0) - X_0) > 0$ and $(f(X_1) - X_1) < 0$, the system satisfies the conditions for an interior fixed point to exist ($0 < n < 1$) for parameter values that satisfy: $w^m > \frac{\theta}{\gamma} > \underline{w}$.

F Endogenizing θ and γ

Let

$$\gamma(\theta) = \beta\theta^{1-\epsilon} \tag{43}$$

where $0 < \epsilon < 1$ and $\beta \in R_+$ such that γ is a proportion.

Then,

$$\frac{\theta}{\gamma} = \frac{1}{\beta}\theta^{\epsilon} \tag{44}$$

In the second stage, n^* is given by

$$n^* = \left[\frac{w^m - \frac{1}{\beta}\theta^{\epsilon}}{w^m - \underline{w}}\right]^{\frac{\alpha}{\alpha-1}} \tag{45}$$

The revenue as a function of θ:

$$Rev = \left[\frac{w^m - \frac{1}{\beta}\theta^{\epsilon}}{w^m - \underline{w}}\right]^{\frac{\alpha}{\alpha-1}} \theta \tag{46}$$

Simplifying the above expression:

$$\begin{aligned} Rev &= \left[\alpha - \frac{(\alpha - 1)}{\beta \underline{w}}\theta^{\epsilon}\right]^{\frac{\alpha}{\alpha-1}} \theta \\ &= x^{\frac{\alpha}{\alpha-1}}\theta \end{aligned}$$

The first-order condition is given by

$$\begin{aligned} &\frac{dRev}{d\theta} = 0 \\ &\Rightarrow \frac{\alpha}{\alpha - 1}[x]^{\frac{\alpha}{\alpha-1}-1} - \frac{(\alpha - 1)\epsilon}{\beta \underline{w}}\theta^{\epsilon-1}\theta + x^{\frac{\alpha}{\alpha-1}} = 0 \\ &\Rightarrow x^{\frac{\alpha}{\alpha-1}} = \left(\frac{\alpha\epsilon}{\beta \underline{w}}\theta^{*\epsilon}\right)x^{\frac{1}{\alpha-1}} \end{aligned}$$

Therefore, the value of θ which maximizes the revenue is:

$$\theta^* = \left[\frac{\alpha \underline{w} \beta}{\alpha - 1 + \alpha\epsilon}\right]^{\frac{1}{\epsilon}} \tag{47}$$

This value of θ^* is smaller than $(\beta w^m)^{\frac{1}{\epsilon}}$ for $\alpha\epsilon > 0$.

The corresponding value of $\frac{\theta^*}{\gamma}$ is

$$\frac{\theta^*}{\gamma} = \frac{\theta^{*\epsilon}}{\beta} = \frac{\alpha \underline{w}}{\alpha - 1 + \alpha\epsilon} < \frac{\alpha \underline{w}}{\alpha - 1} = w^m$$

Thus, the revenue-maximizing value of the effective cost of rent seeking is less than the mean wealth of the economy. Recall that when the effective cost of rent seeking is greater than the mean wealth, rent seeking is not profitable for anyone, thereby giving the corner solution of no rent seeking. We obtain an interior solution when $\frac{\theta^*}{\gamma}$ greater than $\underline{w}$ as well. This will happen when $\alpha\epsilon < 1$. Since $\epsilon < 1$ and $\alpha > 1$, an interior solution exists for some values of α, for a given ϵ.

Equilibrium n^*:

$$\begin{aligned} n^* &= \left(\alpha - \frac{(\alpha - 1)}{\underline{w}\beta}\theta^{*\epsilon}\right)^{\frac{\alpha}{\alpha-1}} \\ &= \left(\frac{\alpha^2\epsilon}{\alpha - 1 + \alpha\epsilon}\right)^{\frac{\alpha}{\alpha-1}} \end{aligned}$$

For what values of α and ϵ will $0 \leq n^* \leq 1$? When $0 \leq \alpha^2\epsilon \leq \alpha - 1 + \alpha\epsilon$.

$$\begin{aligned} \alpha^2\epsilon &\leq \alpha - 1 + \alpha\epsilon \\ \alpha^2\epsilon - \alpha\epsilon &\leq \alpha - 1 \\ \alpha\epsilon(\alpha - 1) &\leq (\alpha - 1) \\ \alpha\epsilon &\leq 1 \end{aligned}$$

Hence, $0 \leq n^* \leq 1$ when $0 \leq \alpha\epsilon \leq 1$.

F.1 Comparative statics with endogenous θ and γ

We will do comparative statics for interior values of n^*, i.e., $0 \leq \alpha\epsilon \leq 1$. How does the equilibrium cost per person (θ^*) and the effective cost of rent seeking ($\frac{\theta}{\gamma}$) change with α?

$$\theta^* = y(\alpha)^{\frac{1}{\epsilon}} \tag{48}$$

where $y(\alpha) = \frac{\alpha \underline{w} \beta}{\alpha - 1 + \alpha\epsilon}$. So,

$$\frac{d\theta^*}{d\alpha} = \frac{1}{\epsilon} y(\alpha)^{\frac{1}{\epsilon}-1} \frac{dy}{d\alpha} \tag{49}$$

Hence, $\frac{d\theta^*}{d\alpha} < 0$ if $\frac{dy}{d\alpha} < 0$.

$$\begin{aligned}
\frac{dy}{d\alpha} &= \frac{d\left(\frac{\alpha}{\alpha-1+\alpha\epsilon}\right)\underline{w}\beta}{d\alpha} \\
&= \left(\frac{-1}{(\alpha - 1 + \alpha\epsilon)^2}\right)\underline{w}\beta \\
&< 0
\end{aligned}$$

Thus, as inequality increases, the cost of rent seeking per agent also increases. Similarly, the effective cost of rent seeking $(\frac{\theta^\epsilon}{\beta})$ also increases with more inequality.

F.2 n^* dynamic with inequality

Let $n^* = f(\alpha)^{g(\alpha)}$, where $f(\alpha) = \frac{\alpha^2\epsilon}{\alpha-1+\alpha\epsilon}$ and $g(\alpha) = \frac{\alpha}{\alpha-1}$. Taking logs on both sides, we have $ln(n^*) = g(\alpha) ln f(\alpha)$. Differentiating both sides w.r.t. α, we get:

$$\frac{1}{n^*}\frac{dn^*}{d\alpha} = g(\alpha)\frac{1}{f(\alpha)}f'(\alpha) + g'(\alpha) ln f(\alpha)$$

Now,

$$\begin{aligned}
f'(\alpha) &= \epsilon\frac{2\alpha(\alpha - 1 + \alpha\epsilon) - \alpha^2(1+\epsilon)}{(\alpha - 1 + \alpha\epsilon)^2} \\
&= \epsilon\frac{2\alpha^2 - 2\alpha + 2\alpha^2\epsilon - \alpha^2 - \alpha^2\epsilon)}{(\alpha - 1 + \alpha\epsilon)^2} \\
&= \epsilon\frac{\alpha^2 - 2\alpha + \alpha^2\epsilon}{(\alpha - 1 + \alpha\epsilon)^2}
\end{aligned}$$

and

$$g'(\alpha) = \frac{(\alpha-1)-\alpha}{(\alpha-1)^2} = \frac{-1}{(\alpha-1)^2}$$

Putting things together, we get

$$\begin{aligned}
\frac{1}{n^*}\frac{dn^*}{d\alpha} &= \left(\frac{\alpha}{\alpha-1}\right)\left(\frac{\alpha-1+\alpha\epsilon}{\alpha^2\epsilon}\right)\left(\epsilon\frac{\alpha^2 - 2\alpha + \alpha^2\epsilon}{(\alpha - 1 + \alpha\epsilon)^2}\right) - \frac{1}{(\alpha-1)^2} ln f(\alpha) \\
\Rightarrow \frac{dn^*}{d\alpha} &= \frac{n^*}{\alpha-1}\left(1 - \frac{1}{(\alpha-1+\alpha\epsilon)} + \frac{-ln f(\alpha)}{(\alpha-1)}\right)
\end{aligned}$$

We are interested in values of $f(\alpha)$ such that $0 \leq f(\alpha) \leq 1$ (for $0 \leq n^* \leq 1$). therefore, for the purposes of this study, $lnf(\alpha) \leq 0$.

Now, $n^{*\prime} \lesseqgtr 0$ iff

$$\begin{aligned}
1 - \frac{1}{(\alpha - 1 + \alpha\epsilon)} + \frac{-lnf(\alpha)}{(\alpha - 1)} &\lesseqgtr 0 \\
1 - \frac{1}{(\alpha - 1 + \alpha\epsilon)} &\lesseqgtr \frac{lnf(\alpha)}{(\alpha - 1)} \\
\frac{\alpha + \alpha\epsilon - 2}{\alpha - 1 + \alpha\epsilon} &\lesseqgtr \frac{lnf(\alpha)}{(\alpha - 1)} \\
\frac{(\alpha - 1)(\alpha + \alpha\epsilon - 2)}{\alpha - 1 + \alpha\epsilon} &\lesseqgtr lnf(\alpha)
\end{aligned}$$

On the LHS, $(\alpha - 1) > 0$ and the denominator is also positive. On the RHS, $lnf(\alpha) \leq 0$. So, $n^{*\prime} > 0$ if $(\alpha + \alpha\epsilon - 2) > 0$. Hence, $n^{*\prime} > 0$ for all values of α such that $\alpha + \alpha\epsilon > 2$ which simplifies to $\alpha > \frac{2}{(1+\epsilon)}$. Re-arranging in terms of the elasticity of γ with respect to θ, we get $n^{*\prime} > 0$ if

$$1 - \epsilon < 2(\frac{\alpha - 1}{\alpha}) \tag{50}$$

Thus, the proportion of rentiers is likely to decline with more inequality ($n^{*\prime} > 0$) when the elasticity of rent rate with respect to θ is low. Likewise, for very high values of elasticity $1 - \epsilon$, n^* is likely to rise with more inequality.